Design and simulation of a control for the opening and closing of the side ventilation windows in a greenhouse

E. M. Gutiérrez-Arias[a], J. E. Flores-Mena[a,*], G. Perez-Osorio[b],
M. M. Morín-Castillo[a], G. Pantle-Cuautle[a], and E. J. Córdova-Gutiérrez[a]

[a]*Facultad de Ciencias de la Electrónica, Benemérita Universidad Autónoma de Puebla,
Av. San Claudio y 18 Sur, Ciudad Universitaria, Colonia Jardines de San Manuel, 72570. Puebla, Pue., México.*
[b]*Facultad de Ingeniería Química, Benemérita Universidad Autónoma de Puebla,
Av. San Claudio y 18 Sur, Ciudad Universitaria, Colonia Jardines de San Manuel, 72570. Puebla, Pue., México.*
[*]*e-mail: jefloresmena@gmail.com*

Received 5 August 2020; accepted 17 September2020

Optimal control for the opening and closing of the side ventilation windows of a greenhouse can be obtained from a mathematical model of the crop and the greenhouse. In the greenhouse model, the control input is the ventilation, and to carry out the instrumentation in the immediate future, this term we related with the aperture of the lee and windward side ventilation windows. We consider a model with four states variables: the structural biomass of leaves, the structural biomass of fruit, the nonstructural biomass (nutrients), and the carbon dioxide. Even though the control of carbon dioxide concentration inside the greenhouse is not directly addressed in this study, optimal control of the opening and closing of vents significantly complements the regulation of the carbon dioxide concentration. To apply the optimal control theory, we select a functional cost to increase the benefit of the farmer.

Keywords: Optimal control; dynamic model; microclimate; performance index; opening and closing of the side ventilation windows.

PACS: 46.80.+j; 45.80.+r; 46.15.Cc; 07.10.-h; 02.30.Yy

DOI: https://doi.org/10.31349/RevMexFis.67.100

1. Introduction

For several years now, the modeling of greenhouses became the subject of many research works [1–9]. These works consist mainly in the study of energy phenomena related to changes in the internal climate of greenhouses. Greenhouse cultivation [10] has become an important mean of modern agriculture production, due to the advantages in aspects of extending growth season and improving potential yield and quality. A greenhouse can protect plantations from bad weather and create a favorable artificial environment for crop growth by some control strategies implementation such as heating, ventilation, fogging, and CO_2 enrichment. Generally, the creation of the favorable environment requires accurate regulations of the environmental variables. Therefore, greenhouse climate control plays an important role in the greenhouse production process. However, greenhouse climate control is still a challenging task due to the inherent complexity of the greenhouse climate. Commonly, many control approaches for nonlinear system need a precise system model to be used to design an efficient controller. However, the lack of an accurate greenhouse climate model with a simple structure is still a challenger to restrict the study of greenhouse climate control so that the control performances of many control approaches derived based on an inaccurate model [1, 11–13] are undesirable. Actually, during the last three decades, a big effort was devoted to developing an adequate greenhouse climate model for greenhouse management [2–4, 14], and many greenhouse climate models have been developed. However, although some complete models can predict the greenhouse climate well, they usually have a complex structure and many state variables, so that it is hard to use them to synthesize a suitable control law. From the perspective of system control, the existing greenhouse climate models still have the two drawbacks: a) some model is too simple so that it cannot describe the highly nonlinear couple relations among inner variables of greenhouse environment, which implies that it cannot correctly predict the inside environment [11], and b) some complete models with many state variables are adverse to design a controller, and their computation is usually expensive to worsen the real-time performance of the control process [3]. A viable solution to overcome these drawbacks is to reconstruct a simple model without loss of simulation accuracy. The main physical processes are heat and mass exchanges among various components of the greenhouse. Hence, thermodynamics is a useful tool to analyze and describe these physical processes.

Wireless sensor networks (WSN) or electronics devise can be used to monitor and control many parameters of the environment in a greenhouse [15–18]. Wireless sensor nodes could be deployed and communicate with a central base station to measure and transmit the sensed required environment factors. The WSN for agriculture applications is a well prospective research subject, and it will draw a lot of attention in the years to come. This type of proposal is based on electronic systems without approaching the dynamic systems or the theory of optimal control.

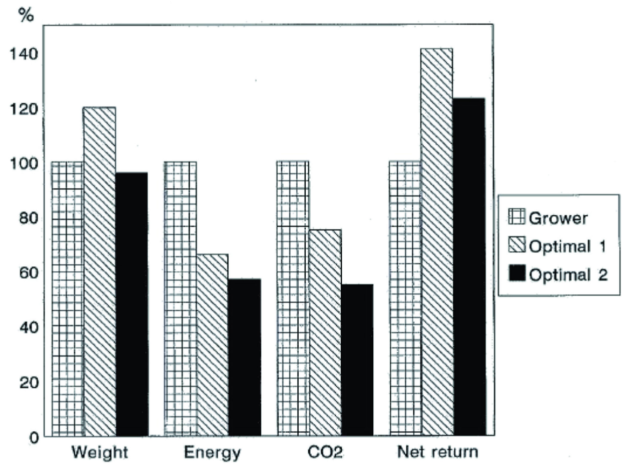

FIGURE 1. Types of control.

Figure 1 shows three different types of control for the crop: the first one is a traditional method of the farmer, the second one and the last one are optimal controllers. Note that two last have a better impact on energy saving, production, and total gain [19].

A dynamic optimization tool of Matlab based on optimal control theory was used to obtain time trajectories of the energy flux that minimizes total external energy input over the year while maintaining greenhouse air temperature and humidity between grower defined bounds [20, 21]. By giving the grower the lead in defining the bounds, the method stays as close as possible to the daily practice of its growth and experience, and no crop production models and market prices are needed.

A simulation model has been developed [22] to predict the performance of a greenhouse that is heated with a heat-pipe system. The model is validated with experimental data and is found to be in close agreement. The simulation can provide estimations of the influence of the maximum height, the heating power required in cold weather, and the heat losses from the greenhouse. In this article, no crop model is necessary.

An optimal control to regulate the open and closing of the side ventilation windows of the greenhouse can be obtained from a mathematical model such model integrates the dynamic model of the crop and the dynamic model of the greenhouse. A performance criterion was selected appropriately in order to apply the optimal control theory and obtain the trajectories that maximize the benefit of the crop and minimize the energy consumption to reach the optimal open and closing of the greenhouse windows. Through Matlab, an algorithm was built, which gives a solution for the optimal control problem, and we realized a simulation throughout a period of 5 days.

One of the main objectives is to contribute to the optimal control problem and its implementation in real-time. The tomato crop has been chosen because it is one of the most important crops in our country and is the second farm product consumed in the world. To achieve the objective, we part from the tomato and greenhouse mathematical set model considering the variable of plant and fruit dry weight, the availability of nutrients, the quantity of carbon dioxide, and the mechanism of opening and closing of the windows.

2. A dynamic model of the crop and greenhouse

Assume that a greenhouse microclimate is considered as a lumped parameter system, *i.e.*, a greenhouse is treated as a homogeneous block (a perfectly stirred tank), which means that the inside air is well mixed. Therefore, all the heat and mass fluxes can be described per square meter. Besides crop canopy is also viewed as a big leaf. We can see that a greenhouse mainly includes the following 5 classes of variables:

i) The greenhouse microclimate state variables below screen: concentration C_{CO_2} (mg/m^3).

ii) Crop growth state variables and actions: dry matter of crop W_L (mg/m^2), dry matter of fruit W_F (mg/m^2), photosynthesis rate P(mg/(sm^2).

iii) Outside climate variables: wind speed v (m/s) , etc.

iv) Control input variables: the opening of vents to leeward u_v^{Aplsd} (m), opening of vents to windward u_v^{Apwsd} (m).

v) Physical parameters of material and structure: greenhouse ground area A_g (m^2), greenhouse air volume V_g (m^3).

The book (Economics-based Optimal Control of Greenhouse Tomato Crop Production, R.F. Tap, Thesis Wageningen Agricultural University. ISBN 90-5808-236-9. 2000. Page 31) describes the calibration and validation of the greenhouse tomato crop production model developed for optimal control purposes. The calibration of the greenhouse and crop model was performed sequentially; first, the tomato model is calibrated, and then the greenhouse model is calibrated using the outputs of the tomato model as inputs. This way, the mutual influence of the different models is partly taken into account in the calibration process were resulting in a more accurate description of the overall system behavior. Calibration parameters were chosen based on insight in the model and sensitivity analysis of the model outputs to parameter change. The calibration results were evaluated using the parameter covariance matrices and their eigenvalue decomposition. Both models were validated using independent experimental data. Calibration and validation results showed that the performance of the greenhouse model and the tomato model is comparatively reasonable to the experimental ones. Figure 2 (extracted from Tap) shows the comparison of both results for the fruit dry weight. The complete results are observed in the same Ref. [23].

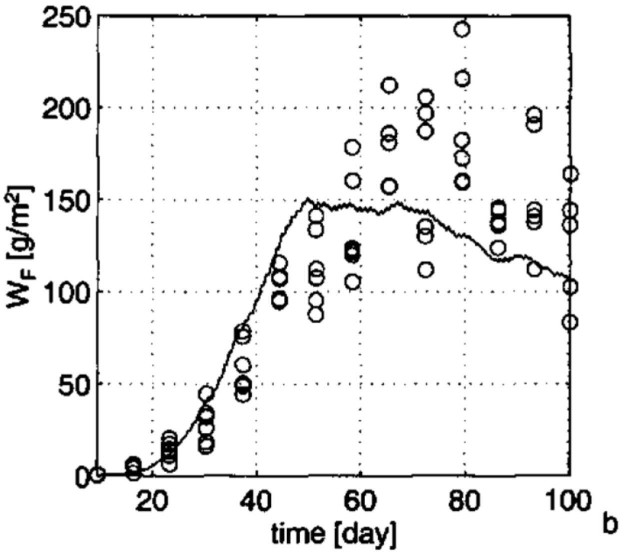

FIGURE 2. Results of simultaneous calibration for the fruit dry weight. ◯ replicates of measurements –simulations.

2.1. Dynamic model of the Crop

The chosen crop here as a case is tomato. This is a generative crop that poses larger challenges to the model as compared with other crops. Moreover, it is of larger economic interest. The crop model is a three-state model, with the assimilates and the fruit and leaf biomass as states. The greenhouse climate model is a relatively simple lumped parameter model, with the CO_2 inner concentration as a state. It is called a big leaf-big fruit model because it makes no distinction between the number of leaves and fruits. The model works with state-space and describes the evolution of the biomass of the leaves and the fruits after the first sprout. The model in space states of the tomato crop has three principle states [19]:

1. Non-structural Biomass (Nutrients).
2. Leaves Structural Biomass.
3. Fruits Structural Biomass.

2.1.1. Biomass balance of nutrients

Nutrients are being produced by photosynthesis. The gross canopy photosynthesis rate in dry matter per unit area is P. Nutrients are converted to leaves and fruits; this is known as growth. Leaves and fruits have a demand for nutrients, which will be honored if there are sufficient nutrients available. We denote W_B as the total nutrients in the plant, and it is expressed as dry weight per area unit. The biomass balance equation of nutrients is the following:

$$\frac{dW_B}{dt} = P - h\{\cdot\}\left(\frac{1+\theta_V}{z}G_L^{\text{dem}} + (1+\theta_F)G_F^{\text{dem}}\right) - h\{\cdot\}\left(\frac{R_L}{z} + R_F\right). \tag{1}$$

The biomass balance equation of nutrients (1) can take two values depending on the nutrients abundance $h\{\cdot\}$, where the first expression is taken when $h\{\cdot\} = 1$ (abundance of nutrients), and the second one is taken when $h\{\cdot\} = 0$ (lack of nutrients).

$$\frac{dW_B}{dt} = \begin{cases} P - \frac{(1+\theta_V)}{z}G_L^{\text{dem}} - (1+\theta_F)G_F^{\text{dem}} - \frac{R_L}{z} - R_{F,} \\ P, \end{cases} \tag{2}$$

where R_F respiration needs of fruits, θ_V additional amount of nutrients needs for one unit of a structural vegetative parts, G_L^{dem} unit area growth demand of levels, θ_F additional amount of nutrients needs for one unit of structural fruit parts, G_F^{dem} units area growth demand of fruit, z total vegetative parts and $h\{\cdot\}$ nutrients abundance.

2.1.2. Biomass balance of leaves

The leaf growth is equal to the number of nutrients converted to structural leaf biomass in the plant, and it is given by $h\{\cdot\}\,G_L^{\text{dem}}$. The model does not incorporate an extra state for stem and roots, but the factor z assumes that each increment in the leaf will be accompanied by an increment in stem and roots. If there are no sufficient assimilates (nutrients), growth stops; normally, the assimilates are used for the maintenance, but in lack of nutrients, maintenance in the model goes at the expense of structural parts (leaves and fruit). The mass balance for the canopy leaf biomass per unit area, WL, reads

$$\frac{dW_L}{dt} = h\{\cdot\}\,G_L^{dem} - (1 - h\{\cdot\})R_L - H_L. \tag{3}$$

Depending on the abundance of nutrients $h\{\cdot\}$, the biomass leaf balance Eq. (3) can take two values:

$$\frac{dW_L}{dt} = \begin{cases} G_L^{dem} - H_L, & if \quad h\{\cdot\} = 1, \\ -R_L - H_L, & if \quad h\{\cdot\} = 0, \end{cases} \tag{4}$$

where H_L is the leaf picking rate.

2.1.3. Biomass balance of fruit

Similarly to the biomass of leaf case, the growth of fruits in the plant from the nutrients is given by $h\{\cdot\}\,G_F^{\text{dem}}$. The term $G_{\text{dem}}F$ depends principally on the pivotal temperature, cultivation temperature level, and the reference temperature,

$$\frac{dW_F}{dt} = h\{\cdot\}\,G_F^{\text{dem}} - (1 - h\{\cdot\})R_F - H_F. \tag{5}$$

Finally, the Eq. (5) of biomass balance of fruits can take two different values depending on nutrient abundance $h\{\cdot\}$, where H_f is the fruit harvest rate:

$$\frac{dW_F}{dt} = \begin{cases} G_F^{\text{dem}} - H_F, & if \quad h\{\cdot\} = 1, \\ -R_F - H_F, & if \quad h\{\cdot\} = 0, \end{cases} \tag{6}$$

2.2. Dynamic model of the greenhouse

The dynamic model of the greenhouse covers many aspects, including ground heat balance, heating system, the mass balance of water vapor and carbon dioxide, however, for this work, only the opening and closing of the side ventilation windows are considered.

2.2.1. Dynamic model of the greenhouse

The balance of carbon dioxide energy within the greenhouse is given by the equation:

$$\frac{V_g}{A_g}\frac{d\mathrm{C}_{\mathrm{CO}_2}}{dt} = -\eta P + \eta R - \varphi^{\text{vent}}_{\mathrm{CO}_2} + u_{\mathrm{CO}_2}. \tag{7}$$

The total mass of CO_2 in the greenhouse is $V_g\mathrm{C}_{\mathrm{CO}_2}$, where V_g is the greenhouse air volume, and $\mathrm{C}_{\mathrm{CO}_2}$ is the concentration. The rate of change of the CO_2 mass per unit greenhouse ground area A_g equals the amount taken up by photosynthesis, ηP, where η is the amount of CO_2 needed to form one unit of biomass plus the amount returned by respiration ηR minus the loss by ventilation $(\varphi^{vent}_{\mathrm{CO}_2})$ plus the supply (u_{CO_2}). P and R are the photosynthesis and the total crop respiration.

The modeling of a greenhouse involves a large number of variables and parameters. Some of the main variables are the carbon dioxide concentration, the temperature inside the greenhouse, the water vapor mass balance, the lighting, etc.

The Eq. (7) relative to the concentration of carbon dioxide inside the greenhouse does not depend directly on the temperature or the mass balance of water vapor; however, it involves the term that controls the opening and closing of vents, the reason why this equation was chosen, in addition to simplifying the objectives of this work.

Loss of carbon dioxide mass by ventilation:

$$\varphi^{vent}_{\mathrm{CO}_2} = u_v(\mathrm{C}_{\mathrm{CO}_2} - \mathrm{C}^o_{\mathrm{CO}_2}), \tag{8}$$

here $\mathrm{C}^o_{\mathrm{CO}_2}$ (kg/m^3) is the carbon dioxide concentration on the outside greenhouse.

Carbon dioxide supply:

$$u_{\mathrm{CO}_2} = u^{V_p}_{\mathrm{CO}_2}\varphi^{\max}_{\mathrm{CO}_2}, \tag{9}$$

where: $u^{V_p}_{\mathrm{CO}_2}$ is the opening supply valve. $\varphi^{\max}_{\mathrm{CO}_2}$ is the maximum flow rate of carbon dioxide, respectively.

In this greenhouse model, the position of the carbon dioxide supply valve is a control input, and it is replaced by a sinusoidal function. While u_v is the flow rate of the volumetric ventilation per unit area of the greenhouse, defined by:

$$u_v = \left(\frac{pv1u_v^{Aplsd}}{1+pv2u_v^{Aplsd}} + pv3 + pv4u_v^{Apwsd}\right)v + pv5, \tag{10}$$

where $pv1$, $pv2$, $pv3$, $pv4$, and $pv5$ are fit parameters, to see [24], v is the wind speed, u_v^{Aplsd} is the opening of vents to leeward, and u_v^{Apwsd} is the opening to windward.

In this model, Eq. (10) relates the opening and closing of the vents, which is the control input. That is the purpose of this work.

Assuming an abundance of nutrients, the dynamic model of the crop can be represented by 3 differential equations, while the dynamic model of the microclimate is represented by one differential equation, which describes its behavior based on the energy balance of carbon dioxide.

$$\begin{cases} \dot{W}_L = G^{\text{dem}}_L - H_L \\ \dot{W}_F = G^{\text{dem}}_F - H_F \\ \dot{W}_B = P - \frac{(1+\theta)}{z}G^{\text{dem}}_L - (1+\theta_F)G^{\text{dem}}_F - \frac{R_L}{z} - R_F \\ \frac{V_g}{A_g}\dot{\mathrm{C}}_{\mathrm{CO}_2} = -\eta P + \eta R - \varphi^{\text{vent}}_{\mathrm{CO}_2} + u_{\mathrm{CO}_2} \end{cases} \tag{11}$$

3. General formulation of the optimal control problem

The optimal control of any system has to be based on three concepts: the dynamic model of the system, a functional and the system restrictions. In matrix notation, the equation of state is represented as follows

$$\dot{x} = f(x(t), u(t), t), \tag{12}$$

where $x(t)$ is the state vector, $u(t)$ is the control signal, and t is the time. A standard is required to help evaluate the performance of the system; normally, the function is defined by

$$J = \phi(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t)dt, \tag{13}$$

where t_0 and t_f are the initial and final time, ϕ and L are scalar functions, t_f can be fixed or free. Starting at the initial state $x(t_0) = x_0$ and applying the control signal $u(t)$ for $t \in [t_0, t_f]$, it makes the system follow some trajectory of states. Then the performance index assigns a unique real number for each trajectory of the system. The fundamental problem of optimal control is to determine an admissible control u^*, which makes Eq. (12) follow one admissible trajectory x^* that minimizes the performance measure showed in the Eq. (13). Then, u^* is called optimal control, and x^* is an optimal trajectory.

Necessary conditions for a solution. Adding the restrictions given by expression (12) to the performance index Eq. (13) with a vector of Lagrange multipliers variants at time $\Psi(\cdot)$ as follows

$$J = \phi(x(t_f)) + \int_{t_0}^{t_f} \Big\{L(x,u,t) + \Psi^T\left[f(x,u,t) - \dot{x}\right]\Big\}dt. \tag{14}$$

Defining the Hamiltonian Scalar Function $H(x(t),\, u(t),\, \Psi(t),\, t)$ as

$$H(x,u,\Psi,t) = L(x(t),u(t),t) + \Psi^T(t) f(x(t),u(t),t). \qquad (15)$$

By integrating the parts the Eq. (14), you get

$$J = \phi(x(t_f)) - \Psi^T(t_f)x(t_f) + \Psi^T(t_0)x(t_0) + \int_{t_0}^{t_f} \left\{ H(x(t),u(t),\Psi(t),t) + \dot{\Psi}^T(t)x(t) \right\} dt. \qquad (16)$$

Now consider an infinitesimal variation in $u(t)$, $\delta(t)$. This variation produces a variation in the trajectory of the states $\delta x(t)$ and a variation in the performance index δJ. This last variation can be calculated as follows

$$\delta J = \left[\left(\frac{\partial \phi}{\partial x} \Psi^T \right) \delta x \right]_{t=t_0} + \left[\Psi^T \delta x \right]_{t=t_0} + \int_{t_0}^{t_f} \left[\left(\frac{\partial H}{\partial x} + \Psi^T \right) \delta x + \frac{\partial H}{\partial u} \delta u \right] dt. \qquad (17)$$

To avoid having to determine the functions $\delta x(t)$ produced by $\delta u(t)$, choose the multipliers $\Psi(t)$ such that the coefficients of $\delta x(t)$ y $\delta x(t_f)$ disappear in the equation above. Choosing then

$$\dot{\Psi}^T = -\frac{\partial H}{\partial x} = -\frac{\partial L}{\partial x} - \Psi^T \frac{\partial f}{\partial x}, \qquad (18)$$

with the border conditions

$$\Psi^T(t_f) = \frac{\partial \phi}{\partial x}(t_f). \qquad (19)$$

Then δJ transforms into

$$\delta J = \Psi^T(t_0)\delta x(t_0) + \int_{t_0}^{t_f} \frac{\partial H}{\partial u} \delta u dt. \qquad (20)$$

If $x(t_0)$ is specified, then $\delta x(t_0) = 0$. For a stationary solution $\delta J = 0$ is required for an arbitrary variation $\delta u(t)$. This happens only if,

$$\frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \dot{\Psi}^T \frac{\partial f}{\partial u} = 0,$$

this equation is called the stationary condition.

The equations of Ψ, $\dot{\Psi}^T(t_f)$, and the previous are the Euler-Lagrange equations of the calculation of variations. Then to find the control vector function $u(t)$ that produces a stationary value of the performance index J the differential equations must be solved (states and co-states),

$$\dot{x}(t) = f(x(t),u(t),t), \qquad (21)$$

$$\dot{\Psi}(t) = -\frac{\partial H^T}{\partial x} = -\frac{\partial L^T}{\partial x} - \frac{\partial f^T}{\partial x}\Psi, \qquad (22)$$

where $u(t)$ is determined from the stationary condition. The conditions in the border for these differential equations are separated. This means some are defined in $t = t_0$ and some in $t = t_f$. $x(t_0)$ is specified by the condition,

$$\Psi(t_f) = \left[\frac{\partial \phi}{\partial x}(t_f) \right]^T. \qquad (23)$$

This is a Two-Point Boundary Value Problem. Note that the state equations and the co-states are coped because $u(t)$ depends on $\Psi(t)$ through the stationary condition and the co-states depend on $x(t)$ and $u(t)$.

4. Design of the control law

Before carrying out the design of the control law, it is necessary to perform the synthesis of the control, which consists of choosing a performance index, from which it is possible to obtain a system of attached state variables. In this way, the initial and final conditions of the system are obtained. In order to perform the synthesis of the control, it is necessary to know the values of the parameters involved so that the system is close to reality, for what the substitution in Eq. (11) is performed with the values of the existing climatic conditions in the state of Puebla and Eq. (24) shows the integrated model microclimate-cultivation:

$$\begin{cases} \dot{W}_L = 2.2996 \times 10^{-6} W_L \\ \dot{W}_F = 4.3925 \times 10^{-6} W_F \\ \dot{W}_B = P - 5.39 \times 10^{-6} W_L - 5.92 \times 10^{-6} W_F \\ 3\dot{\mathrm{C}}_{\mathrm{CO_2}} = 1.0266(R-P) - \varphi_{\mathrm{CO_2}}^{vent} + u_{\mathrm{CO_2}}, \end{cases} \qquad (24)$$

where,

$$P = \frac{3.7192 \times 10^{-11} W_L^{2.511}}{1.6353 \times 10^{-9} + 4.0439 \times 10^{-5} W_L^{2.511}} \qquad (25)$$

and

$$R = 1.5942 \times 10^{-6} W_F + 0.485 \times 10^{-6} W_L + 1.668 \times 10^{-7}. \qquad (26)$$

Based on the knowledge of the problem and the requirements of the system, the following performance index is considered:

$$J = \frac{1}{2} \Bigg[- W_L^2(tf) - W_F^2(tf) + W_B^2(tf) + \mathrm{C}_{\mathrm{CO_2}}^2(tf) + \int_{to}^{tf} a \left[-W_F^2 - W_L^2 + W_B^2 + \mathrm{C}_{\mathrm{CO_2}}^2 \right] + \beta(\varphi^2) \Bigg] dt. \qquad (27)$$

Performance index is known as the General Index for Optimum Control Systems, which includes the variables of state, the control, and scalar functions a and β, scalar functions that can be adjusted to the consideration of the designer.

The Hamiltonian scalar function is defined following the performance index, which depends on the state variables, the control input, and the new vector of Lagrange multipliers.

$$H(x(t),u(t),\Psi(t),t) = L(x(t),u(t),t) + \Psi^T(t) f(x(t),u(t),t). \quad (28)$$

Therefore, the H function is as follows:

$$\begin{aligned} H(x(t),u(t),\Psi(t),t) &= \frac{1}{2}a\Big[\Big(-W_L^2(t) - W_F^2(t) + W_B^2(t) + C_{CO_2}^2(t)\Big) + \beta\left(\varphi^2(t)\right)\Big] \\ &+ 2.2996\times10^{-6}W_L(t)\Psi_1(t) \\ &+ 4.3925\times10^{-6}W_F(t)\Psi_2(t) \\ &+ \left[P - 5.39\times10^{-6}W_L(t) - 5.92\times10^{-6}W_F(t)\right]\Psi_3(t) \\ &+ \frac{1}{3}\left[1.0266(R-P) - \varphi\right]\Psi_4(t). \end{aligned} \quad (29)$$

The Hamiltonian scalar function allows us to obtain a new system of differential equations, which is formed by the attached variables. Consequently, the system of attached state variables is expressed as:

$$\begin{cases} \psi_1 = -aW_L + 2.2996\times10^{-6}\Psi_1 + \frac{\partial P}{\partial W_L}\Psi_2 \\ \quad -5.39\times10^{-6}\Psi_2 + \frac{1}{3}\frac{\partial(R-P)}{\partial W_L}\Psi_4(1.0266) \\ \psi_2 = -aW_F + 4.3925\times10^{-6}\Psi_2 - 5.92\times10^{-6}\Psi_3 \\ \quad +\frac{1}{3}\frac{\partial R}{\partial W_F}\Psi_4(1.0266) \\ \psi_3 = aW_B \\ \psi_4 = a\mathrm{C}_{\mathrm{CO_2}} \end{cases}. \quad (30)$$

To find the control vector function $u(t)$ that produces a steady value of the performance function *J*, the following system of differential equations must be solved:

$$\begin{cases} \dot{x} = f(x(t),u(t),t) \\ \psi(t) = -\frac{\partial H^T}{\partial x} \end{cases}, \quad (31)$$

where $u(t)$ is determined from the stationary condition; therefore, the Hamiltonian function is partially derived concerning to the control signal, which results in:

$$\frac{\partial H}{\partial \varphi} = \frac{\partial L}{\partial \varphi} + \Psi^T\frac{\partial f}{\partial \varphi} \quad \text{then} \quad \varphi_{\mathrm{CO_2}}^{\mathrm{vent}} = \frac{1}{3*\beta}\Psi_4(t). \quad (32)$$

From (32), it can be seen that the control depends on the fourth attached state variable $\Psi_4(t)$ at each time point.

5. Simulation results

In this section, we present the results of the simulation of the control law using the parameters shown in Table I. The complete system representation is:

$$\begin{cases} \dot{W}_L = 2.2996\times10^{-6}W_L \\ \dot{W}_F = 4.3925\times10^{-6}W_F \\ \dot{W}_B = P - 5.39\times10^{-6}W_L - 5.92\times10^{-6}W_F \\ 3\dot{\mathrm{C}}_{\mathrm{CO_2}} = 1.0266(R-P) - \varphi_{\mathrm{CO_2}}^{\mathrm{vent}} + u_{CO_2} \\ \psi_1 = -aW_L + 2.2996\times10^{-6}\Psi_2 - 5.39\times10^{-6}\Psi_2 \\ \quad +\frac{1}{3}\frac{\partial(R-P)}{\partial W_L}\Psi_4(1.0266) \\ \psi_2 = -aW_F + 4.3925\times10^{-6}\Psi_2 - 5.92\times10^{-6}\Psi_3 \\ \quad +\frac{1}{3}\frac{\partial R}{\partial W_F}\Psi_4(1.0266) \\ \psi_3 = aW_B \\ \psi_4 = aC_{CO_2} \end{cases}. \quad (33)$$

It is not possible to use the system (6) because there are only initial conditions for the state variables, and, for the attached state variables, there are only final conditions, so it is necessary to make use of the inverse time. Once the partial derivatives in (33) have been determined,

$$\frac{\partial P}{\partial W_L} = \frac{C_1(2.511W_L^{2.511})(2.511W_L^{1.511})}{\left(\frac{C_2}{C_3} + W_L^{2.511}\right)}, \quad (34)$$

$$\frac{\partial(R-P)}{\partial W_L} = C_5 + \frac{C_1(2.511W_L^{1.511})(2.511W_L^{1.511})}{\left(\frac{C_2}{C_3} + W_L^{2.511}\right)^2}, \quad (35)$$

$$\frac{\partial R}{\partial W_F} = C_4, \quad (36)$$

$$P = C_1\left(\frac{W_L^{2.551}}{\frac{C_2}{C_3} + W_L^{2.551}}\right) \quad (37)$$

and

$$R = C_4W_F + C_5W_L + C_6, \quad (38)$$

where: $C_1 = 3.7192\times10^{-11}$, $C_2 = 1.6353\times10^{-9}$, $C_3 = 4.0439\times10^{-5}$, $C_4 = 1.5942\times10^{-6}$, $C_5 = 0.4856\times10^{-6}$, and $C_6 = 1.668\times10^{-7}$.

It is possible to solve the system (33) and graph the behavior in the Matlab software. The behavior of the state variables, is shown in Fig. 3.

Figure 4 describes the control behavior, how a function of the fourth variable attached Ψ_4; therefore, the necessary substitutions are made to convert that behavior to the function of the opening and closing of the side ventilation windows; besides it is considered that the opening to leeward and windward are done at the same time. Clearly, the selection of the mechanism to open and close the vents does not affect the structure of the validated model,

$$u_v^{Aplsd} = u_v^{Apwsd} = \tilde{u}_v. \quad (39)$$

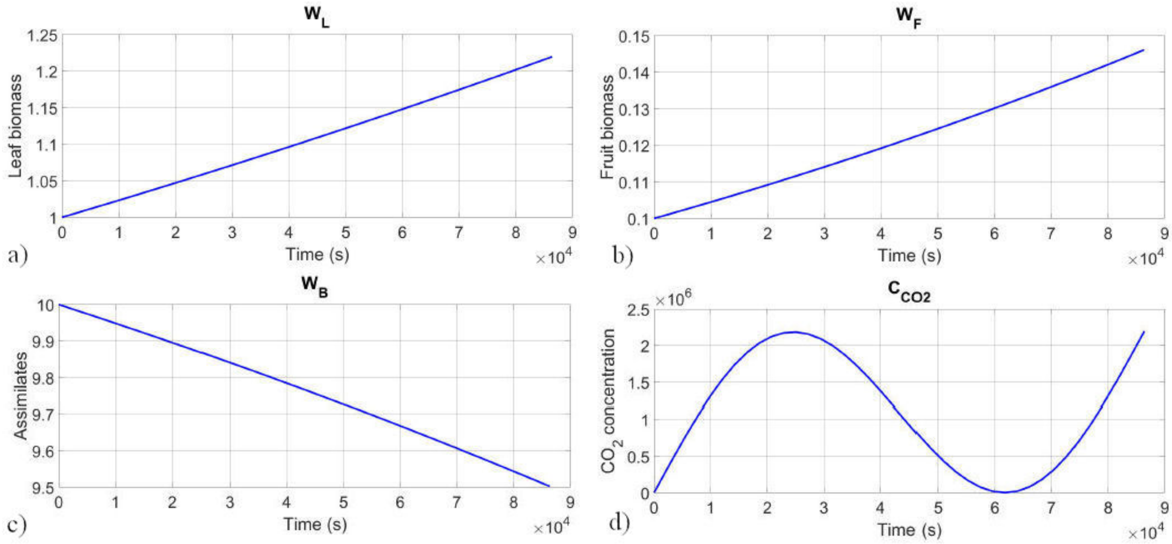

FIGURE 3. a) Leaf biomass behavior. b) Fruit biomass behavior. c) Assimilates behavior. d) CO_2 behavior. System response.

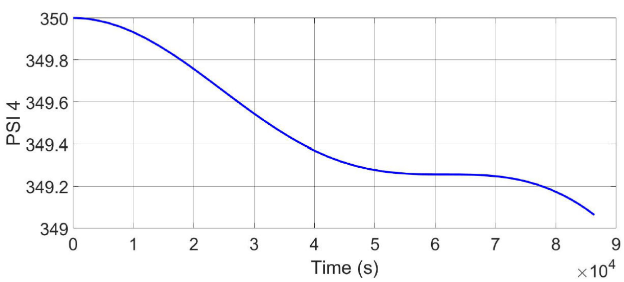

FIGURE 4. Graph of the attached state variable Ψ_4.

Substituting Eq. (39) into Eq. (10), we obtain that

$$u_v = \left(\frac{pv1\tilde{u}_v}{1+pv2\tilde{u}_v} + pv3 + pv4\tilde{u}_v\right) v + pv5. \qquad (40)$$

TABLE I. Physical parameters related to the microclimate.

Variable	Value	Description
η	0.7	Heat absorbed in relation to the total energy of the net radiation received.
$\frac{V_g}{A_g}$	3	Reason for volume of the greenhouse per unit area.
$C^o_{CO_2}$	1.6637	Concentration of carbon dioxide outside the greenhouse.
pv1	7.17×10^{-5}	Parameter
pv2	0.01556	Parameter
pv3	2.71×10^{-5}	Parameter
pv4	6.32×10^{-5}	Parameter
pv5	7.40×10^{-5}	Parameter

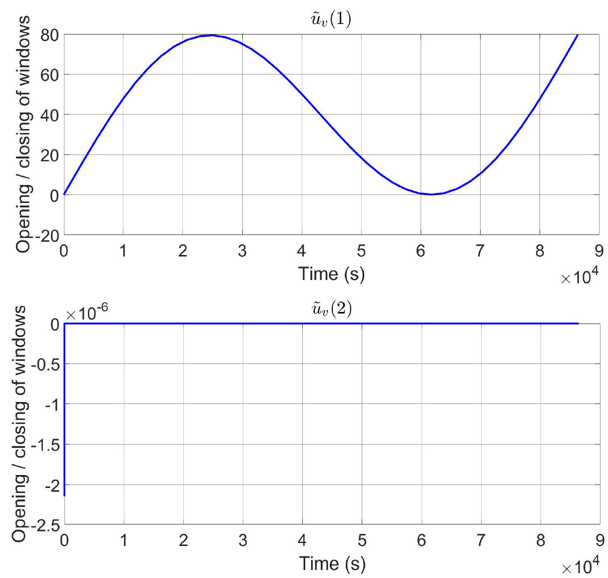

FIGURE 5. a)Graph of solution $x(1)$. b)Graph of solution $x(2)$. Graphs of control output applied to the side ventilation windows.

Using Eq. (40) in Eq. (8), we obtain the following equation

$$\varphi^{\text{vent}}_{\text{CO}_2} = \left[\left(\frac{pv1\tilde{u}_v}{1+pv2\tilde{u}_v} + pv3 + pv4\tilde{u}_v\right) v + pv5\right] \times \left(\text{C}_{\text{CO}_2} - \text{C}^o_{\text{CO}_2}\right). \qquad (41)$$

In Eq. (32), it is observed that the control is in function of Ψ_4

$$\varphi^{\text{vent}}_{\text{CO}_2} = \frac{1}{3}\Psi_4(t). \qquad (42)$$

The substitution of (42) in (40) is made in order to obtain $\tilde{u}_v$, which is the reference signal for the opening and closing of the side ventilation windows (30).

$$\frac{1}{3}\Psi_4(t) = \left[\left(\frac{pv1\tilde{u}_v}{1+pv2\tilde{u}_v} + pv3 + pv4\tilde{u}_v\right) v + pv5\right] \times \left(\mathrm{C}_{\mathrm{CO}_2} - \mathrm{C}^o_{\mathrm{CO}_2}\right). \quad (43)$$

To obtain variable of interest $\tilde{u}_v$, it is observed that in order to obtain its values, it is necessary to apply the general formula for quadratic equations, besides making the substitution in Eq. (44) with the values in Table I,

$$\tilde{u}_v^2 \left(pv2 * pv4\right) + \tilde{u}_v \Bigg[\left(pv1 + pv4\right) - \frac{pv2}{v}\left(\frac{\Psi_4(t)}{3\left(C_{CO_2} - C^o_{CO_2}\right)} pv3 - pv5\right)\Bigg] - \frac{1}{v}\left(\frac{\Psi_4(t)}{3\left(C_{CO_2} - C^o_{CO_2}\right)} - pv3 - pv5\right) = 0. \quad (44)$$

In Fig. 5, it is possible to see the 2 solutions of the quadratic equation; besides it is observed that the behavior of the first graph resembles more to the reality, reason why that result will be applied to the control system for the side ventilation windows.

6. Conclusions

This paper considered the integrated model of crop-microclimate, a situation that does not perform other research work because, generally it take into account only the model of the microclimate. The application of the theory of optimal control allowed the design of the law of control for the opening and closing of the side ventilation windows, the creation of an algorithm in Matlab solved the contour problem and allowed the simulation of the variables of state of the integrated dynamic system crop-microclimate. Although the objective of this work is not to regulate the inner optimum concentration of carbon dioxide in the greenhouse, by including such variable in the states, the optimum concentration of carbon dioxide was also obtained to the inner; therefore, the control law optimum for the opening and closing of the side ventilation windows can contribute to the regulation of this concentration. The implementation of the electronic device will provide economic benefits in saving energy consumption.

Acknowledgments

JMGA and JEFM are grateful for the support given by VIEP-BUAP (project ING-2019).

1. A. M. Abdel-Ghany and T. Kozai, Dynamic modeling of the environment in a naturally ventilated, fog-cooled greenhouse, *Renewable Energy*, **31** (2006) 1521-1539. DOI: `10.1016/j.renene.2005.07.013`
2. G. C. Monteil, Issanchou, and M. Amouroux, Modele energetique de la serre agricole, *Journal Physique III*, **1** (1991) 429-454, `https://doi.org/10.1051/jp3:1991130`
3. M. K. Ghosal and G. N. Tiwari, Mathematical modeling for greenhouse heating by using thermal curtain and geothermal energy, *Solar Energy*, **76** (2004) 603-613, `https://doi.org/10.1016/j.solener.2003.12.004`
4. M. J. Gupta and P. Chandr, Effect of greenhouse design parameters on conservation of energy for greenhouse environmental control, *Energy*, **27** (2002) 777-794, `https://doi.org/10.1016/S0360-5442(02)00030-0`
5. F. Lafont and J. F. Balmat, Optimized fuzzy control of a greenhouse, *Fuzzy Set and Systems*, **128** (2002) 47-59, `https://doi.org/10.1016/S0165-0114(01)00182-8`
6. W. Luo, H. F. Zwart, J. Dail, X. Wang, C. Stanghellini, and C. Bu, Simulation of greenhouse management in the subtropics, Part I : Model validation and scenario study for the winter season, *Biosystems Engineering*, **90** (2005) 307-318, `https://doi.org/10.1016/j.biosystemseng.2004.11.008`
7. P. Salgado and J. B. Cunha, Greenhouse climate hierarchical fuzzy modelling, *Control Engineering Practice* **13** (2005) 613-628, `https://doi.org/10.1016/j.conengprac.2004.05.007`
8. G. Singh, P. P. Singh, P. P. S. Lubana, and K. G. Singh, Formulation and validation of a mathematical model of the microclimate of greenhouse, *Renewable Energy*, **31** (2006) 1541-1560, `https://doi.org/10.1016/j.renene.2005.07.011`
9. T. Takakura, A. Jordann, and L. Boyd, Dynamic simulation of plant growth and environment in the greenhouse, *Trans. ASAE*, **14** (1971) 964-971, `https://doi.org/10.13031/2013.38432`
10. Yuanping Su, Lihong Xu, *Greenhouse Climate Model for Control Design*, 2015 IEEE 15th International Conference on Environment and Electrical Engineering (EEEIC), (2015) 1-6. doi:`10.1109/EEEIC.2015.7165318`
11. N. Bennis, J. Duplaix, G. Enea, M. Haloua, and H. Youlal, *Greenhouse climate modelling and robust control*, *Computers and Electronics in Agriculture*, **61** (2008) 96-107, `https://doi.org/10.1016/j.compag.2007.09.014`
12. S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, (Society for Industrial and Applied Mathematics, Philadelphia, 1994), pp. 148-150.
13. M. S. Branicky, Multiple lyapunov functions and other analysis tools for switched and hybrid systems, IEEE *Transactions on automatic control*, **43** (1998) 475-482, DOI: `10.1109/9.664150`
14. Z. S. Chalabi, B. J. Bailey, and D. J. Wilkinson, A real time optimal control algorithm for greenhouse heating, *Computers and electronics in Agriculture*, **15** (1996) 1-13, `https://doi.org/10.1016/0168-1699(95)00053-4`

15. Marwa Mekki, Osman Abdallah, Magdi Amin B. M., Moez Eltayeb, Tafaoul Abdalfatah, Amin Babiker, *Greenhouse monitoring and control system based on wireless Sensor Network*, International Conference on Computing, Control, Networking, Electronics and Embedded Systems Engineering (ICCNEEE), (2015). doi:10.1109/ICCNEEE.2015.7381396

16. Wei Cao, Ge Xu, Yaprak, E., Lockhart, R., Taiqian Yang, Yanqing Gao, *Using Wireless Sensor Networking (WSN) to Manage Micro-Climate in Greenhouse*, International Conference on Mechtronic and Embedded Systems and Applications, MESA 2008. IEEE/ASME: (2008) 636 - 641. DOI: 10.1109/MESA.2008.4735706

17. Takehiko Hoshi, Ryosuke Ohata, Katsuyoshi Watanabe and Ryuji Osuka, *A Gadget-Based Information Management System for Environmental Measurement and Control in Greenhouses*, SICE Annual Conference, (2011), 2801-2805, https://ieeexplore.ieee.org/document/6060458

18. Jin Yulong, Yang Jiaqiang, Design an Intelligent Environment Control System for Greenhouse Based on RS485, in *Second International Conference on Digital Manufacturing and Automation*, **1** (2011) 361-364, DOI Bookmark:10.1109/ICDMA.2011.95

19. G. van Straten, G. van Willigenburg, E. van Henten, R. van Ooteghem, *Optimal Control of Greenhouse Cultivation*, 1st ed. (Taylor and Francis Group, , Boca Raton, 2011), pp. 123.

20. P.J.M. Van Beveren, J. Bontsema, G. Van Straten, E.J. Van Henten, Minimal heating and cooling in a modern rose greenhouse, *Applied Energy*, **137** (2014) 97-109, https://doi.org/10.1016/j.apenergy.2014.09.083

21. P.J.M. van Beveren, J. Bontsema, G. van Straten, E.J. van Henten, Optimal control of greenhouse climate using minimal energy and grower defined bounds, *Applied Energy*, **159** (2015) 509-519, https://doi.org/10.1016/j.apenergy.2015.09.012

22. Jun Du, Pradeep Bansal, Bo Huang, Simulation model of a greenhouse with a heat-pipe heating system, *Applied Energy*, **93** (2012) 268-276, https://doi.org/10.1016/j.apenergy.2011.12.069

23. R. F. Tap, Economics-based Optimal Control of Greenhouse Tomato Crop Production. Dynamic model for optimal control of greenhouse tomato crop production, (Thesis Wageningen Agricultural University. ISBN 90-5808-236-9. 2000), pp. 31.

24. E. J. van Henten, Thesis: Greenhouse climate management: An optimal control approach, (Wageningen Wageningen University Research, Netherlands, ISBN 9789054853213-329, 1994).