Numerical solutions of the Maung-Norbury-Kahana equation with the coulomb potential in momentum space

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In this paper, the numerical solutions of the Maung-Norbury-Kahana equation which has the complicated form of the eigenvalues are presented. Taken as examples, the bound states e^+e^-, $\mu^+\mu^-$ and μ^+e^- are discussed by employing the Maung-Norbury-Kahana equation with the Coulomb potential.

Keywords: MNK equation; virtual constituents; binding energy.

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1. Introduction

The Bethe-Salpeter equation [1] is based on the relativistic field theory and is an appropriate tool to deal with bound states. In comparison with the four-dimensional Bethe-Salpeter equation [2–5], the three-dimensional reductions of it are relatively easy to be handled [1, 6–11]. In Ref. [12], it was shown that there exist infinite versions of the reduced Bethe-Salpeter equation. One of them is the Maung-Norbury-Kahana (MNK) equation [8].

The MNK equation is covariant, obeys the unitarity relation and possesses a one-body limit. It is a proportionally off-mass-shell equation and is a relativistic, three-dimensional equation for bound states with two constituents. Moreover, the MNK equation gives a physically meaningful prescription of how the constituents go off-mass-shell in the intermediate states. The MNK equation allows the components of bound states to go off-mass-shell proportionally to their masses. In this paper, the MNK equation is solved numerically and is applied to discuss the equal-mass systems (positronium and true muonium) and the unequal-mass system (muonium).

The paper is organized as follows. In Sec. 2, the MNK equation is reviewed and the spinless MNK equation is derived. In Sec. 3, the spinless MNK equation is solved numerically and the discussions are presented. The conclusion is in Sec. 4.

2. Maung-Norbury-Kahana equation

In this section, the MNK equation is reviewed and the spinless MNK equation is derived. To discuss the bound states, e^+e^-, $\mu^+\mu^-$ and μ^+e^-, the MNK equation with the Coulomb potential is needed. The logarithmic singularity in the momentum-space Coulomb potential is removed by the Landé subtraction method.

2.1. Reduction of the Bethe-Salpeter equation

The Bethe-Salpeter equation in momentum space reads [1, 6]

$$\chi_P(p) = S_1^F(p_1) \int \frac{d^4p'}{(2\pi)^4} \times K(P,p,p')\,\chi_P(p')S_2^F(-p_2), \tag{1}$$

where

$$p = \eta_2 p_1 - \eta_1 p_2, \quad P = p_1 + p_2. \tag{2}$$

In order to have a correct one-body limit in three-dimensional reduced equations, the Wrightmann-Gordon choice [13] of η_1 and η_2 should be applied,

$$\eta_1 = \frac{s + m_1^2 - m_2^2}{2s}, \quad \eta_2 = \frac{s - m_1^2 + m_2^2}{2s}, \tag{3}$$

where $s = P^2$. In Eq. (1), $S_i^F(p_i)$ are the full fermion propagators. We will approximate the full propagators $S_i^F(p_i)$ by free propagators [5]

$$S_i(p_i) = \frac{i}{\not{p}_i - m_i + i\epsilon}, \tag{4}$$

where m_1 and m_2 are interpreted as effective masses for the fermion and antifermion.

We introduce components of the relative momentum $p = p_\parallel + p_\perp$ parallel and perpendicular to the bound-state momentum P by [11, 14–16]

$$\hat{P} = \frac{P}{M}, \quad M = \sqrt{P^2}, \quad p_l = p \cdot \hat{P}, \quad p = p_\parallel + p_\perp,$$
$$p_\parallel = p_l \hat{P}, \quad p_\perp = p - p_l \hat{P}, \quad d^4p = dp_l d^3p_\perp, \tag{5}$$

where $p_\parallel$ is the longitudinal part and $p_\perp$ is the transverse part. In the rest frame of the bound state with momentum

$P=(M,\mathbf{0})$, $p_l=p^0$, $p_\parallel=(p^0,\mathbf{0})$ and $p_\perp=(0,\mathbf{p})$. The projection operators can be written in covariant form

$$\Lambda_i^{\pm}(p_\perp)=\frac{\omega_i\pm H_i(p_\perp)}{2\omega_i},\quad H_i(p_\perp)=\hat{P}(m_i-\not{p}_\perp),$$
$$\omega_i=\sqrt{m_i^2+\varpi^2},\quad \varpi=\sqrt{-p_\perp^2} \tag{6}$$

with the properties

$$\Lambda_i^{\mp}(p_\perp)\Lambda_i^{\pm}(p_\perp)=0,\quad \Lambda_i^{+}(p_\perp)+\Lambda_i^{-}(p_\perp)=1,$$
$$\Lambda_i^{\pm}(p_\perp)\Lambda_i^{\pm}(p_\perp)=\Lambda_i^{\pm}(p_\perp),$$
$$H_i(p_\perp)\Lambda_i^{\pm}(p_\perp)=\pm\omega_i\Lambda_i^{\pm}(p_\perp). \tag{7}$$

In this paper the covariant instantaneous approximation is employed [14], in which the approximated kernel is independent of the change of the longitudinal component of the relative momentum,

$$K(P,p,p')\to K(p_\perp,p'_\perp)=iV(p_\perp,p'_\perp). \tag{8}$$

It is a good approximation for a system composed of heavy and light constituents or of two heavy constituents which can move relativistically as a whole. It will reduce to the instantaneous approximation in the rest frame of the bound state.

Introduce the notation for later convenience

$$\psi_P(p_\perp)=\int\frac{dp_l}{2\pi}\chi_P(p),$$
$$\Gamma(p_\perp)=\int\frac{d^3p'_\perp}{(2\pi)^3}V(p_\perp,p'_\perp)\psi_P(p'_\perp), \tag{9}$$

where $\psi_P(p_\perp)$ is the Salpeter wave function. Using Eqs. (4), (6), (8) and (9), the Bethe-Salpeter equation (1) becomes

$$\chi(p)=G_0(P,p)i\Gamma(p_\perp), \tag{10}$$

where

$$\begin{aligned}G_0(P,p)=g_0(P,p)\big[&\Lambda_1^+(p_\perp)(p_{10}+\omega_1)\\&+\Lambda_1^-(p_\perp)(p_{10}-\omega_1)\big]\ \hat{P}\otimes\hat{P}\\&\times\big[\Lambda_2^+(-p_\perp)(p_{20}-\omega_2)\\&+\Lambda_2^-(-p_\perp)(p_{20}+\omega_2)\big]\end{aligned} \tag{11}$$

and

$$g_0(P,p)=\frac{1}{p_1^2-m_1^2+i\epsilon}\frac{1}{p_2^2-m_2^2+i\epsilon}. \tag{12}$$

In Refs. [7] and [8], $g_0(P,p)$ is given as

$$g_0(P,p)\Rightarrow-2\pi i\frac{\delta^+\left[f(\iota)\right]}{p_1^2-m_1^2+p_2^2-m_2^2+i\epsilon}, \tag{13}$$

where $f(\iota)$ is defined as

$$f(\iota)=(p_1^2-m_1^2)\frac{1+\iota}{2}-(p_2^2-m_2^2)\frac{1-\iota}{2}. \tag{14}$$

In the above equation, ι is the parameter describing the relative virtuality of two components in bound state. When $\iota=1$, the constituent 1 is on-shell and constituent 2 are off-shell arbitrarily; vice versa, when $\iota=-1$, the constituent 2 is on-shell with another constituent's virtuality arbitrary. For the MNK equation,

$$\iota=\frac{m_1-m_2}{m_1+m_2}. \tag{15}$$

Eq. (13) can be simplified as

$$g_0(P,p)=-2\pi i\frac{\delta\left(p_l-p_l^+\right)/W}{p_1^2-m_1^2+p_2^2-m_2^2}, \tag{16}$$

where

$$W=\sqrt{(1-\iota^2)M^2+2\iota\left[(1+\iota)\omega_1^2-(1-\iota)\omega_2^2\right]} \tag{17}$$

and

$$p_l^+=\begin{cases}\left[-\left(1+\iota\eta_1-\iota\eta_2\right)M+W\right]/(2\iota),\\ \left[-\left(1+\iota\eta_1-\iota\eta_2\right)M-W\right]/(2\iota).\end{cases} \tag{18}$$

If constituent 1 takes positive energy as $0\leq\iota\leq1$ and constituent 2 takes positive energy as $-1\leq\iota<0$, p_l^+ should be

$$p_l^+=\frac{W-\left(1+\iota\eta_1-\iota\eta_2\right)M}{2\iota},\quad -1\leq\iota\leq1. \tag{19}$$

After integrating over p_l, we have from Eq. (16)

$$\tilde{g}_0(P,p_\perp)=\frac{-i/W}{\left[(\eta_1M+p_l^+)^2-\omega_1^2+(\eta_2M-p_l^+)^2-\omega_2^2\right]}, \tag{20}$$

From Eqs. (11), (16) and (20), we have

$$\begin{aligned}\tilde{G}_0(P,p_\perp)=\tilde{g}_0(P,p_\perp)\big[&\Lambda_1^+(p_\perp)(\eta_1M+p_l^++\omega_1)\\&+\Lambda_1^-(p_\perp)(\eta_1M+p_l^+-\omega_1)\big]\ \hat{P}\otimes\hat{P}\\&\times\big[\Lambda_2^+(-p_\perp)(\eta_2M-p_l^+-\omega_2)\\&+\Lambda_2^-(-p_\perp)(\eta_2M-p_l^++\omega_2)\big]\end{aligned} \tag{21}$$

Using Eqs. (9), (19) and (21), Eq. (10) reduces to the MNK equation

$$\psi_P(p_\perp)=\tilde{G}_0(P,p_\perp)\int\frac{d^3p'_\perp}{(2\pi)^3}iV(p_\perp,p'_\perp)\psi_P(p'_\perp). \tag{22}$$

The MNK equation has been understood physically meaningful: when masses of constituents are not equal but comparable, this kind of choice of ι [Eq. (15)] promises that the heavier particle is less virtual while the lighter massive particles is further off-mass-shell. For the bound states composed of equally massive constituents, the constituents will be put equally off-mass-shell.

Assuming

$$\eta_1M+p_l^++\omega_1\gg\eta_1M+p_l^+-\omega_1,$$
$$\eta_2M-p_l^++\omega_2\gg\eta_2M-p_l^+-\omega_2, \tag{23}$$

we have from Eq. (22)

$$\frac{W\left[(\eta_1 M+p_l^+)^2-\omega_1^2+(\eta_2 M-p_l^+)^2-\omega_2^2\right]}{(\eta_1 M+p_l^++\omega_1)(\eta_2 M-p_l^++\omega_2)}\psi_P(p_\perp)$$
$$=\Lambda_1^+(p_\perp)\not{P}\int\frac{d^3p'_\perp}{(2\pi)^3}V(p_\perp,p'_\perp)$$
$$\times\psi_P(p'_\perp)\not{P}\Lambda_2^-(-p_\perp). \quad (24)$$

Neglecting any reference to the spin degrees of freedom of the involved bound-state constituents, we have the spinless MNK equation from Eq. (24)

$$\frac{W\left[(\eta_1 M+p_l^+)^2-\omega_1^2+(\eta_2 M-p_l^+)^2-\omega_2^2\right]}{(\eta_1 M+p_l^++\omega_1)(\eta_2 M-p_l^++\omega_2)}\psi_P(p_\perp)$$
$$=\int\frac{d^3p'_\perp}{(2\pi)^3}V(p_\perp,p'_\perp)\psi_P(p'_\perp), \quad (25)$$

where ι is in Eq. (15). Eq. (25) describes the semirelativistic bound states composed to two spinless constituents which are virtual according to Eqs. (14) and (15). Following the approaches in Refs. [6] and [24], the spin-independent terms and spin-dependent terms can be obtained from Eq. (24).

2.2. Landé subtraction method

In this paper, the Coulomb potential is considered. The Coulomb potential reads in the momentum space

$$V(\mathbf{p},\mathbf{p}')=-\frac{4\pi\alpha}{(\mathbf{p}-\mathbf{p}')^2}, \quad (26)$$

where α is the fine structure constant. The partial wave expansion of the spinless MNK equation (25) is expressed as

$$f(M_{nl},p)\phi_{nl}(p)=\frac{1}{(2\pi)^3}\int_0^\infty V^l(p,p')\phi_{nl}(p')p'^2dp', \quad (27)$$

where n is the principal quantum number, l is the orbital angular quantum number. $f(M,p)$ reads

$$f(M,p)=$$
$$\frac{W\left[(\eta_1 M+p_l^+)^2-\omega_1^2+(\eta_2 M-p_l^+)^2-\omega_2^2\right]}{(\eta_1 M+p_l^++\omega_1)(\eta_2 M-p_l^++\omega_2)}. \quad (28)$$

$V^l(p,p')$ is the partial wave expanded Coulomb potential,

$$V^l(p,p')=-8\pi^2\alpha\frac{Q_l(z)}{pp'},\quad z\equiv\frac{p^2+p'^2}{2p'p}, \quad (29)$$

where $Q_l(z)$ is the Legendre polynomial of the second kind,

$$Q_l(z)=P_l(z)Q_0(z)-w_{l-1}(z),\quad Q_0(z)=\frac{1}{2}\ln\frac{z+1}{z-1},$$
$$w_{l-1}(z)=\sum_{m=1}^{l}\frac{1}{m}P_{l-m}(z)P_{m-1}(z). \quad (30)$$

The Coulomb potential has the logarithmic singularity at point $p'=p$, and the singularity comes from $Q_0(z)$.

Applying the Landé subtraction method [17–21] to cancel out the singularity, the singular equation (27) becomes

$$f(M_{nl},p)\phi_{nl}(p)=-\frac{\alpha p}{\pi}\frac{\pi^2}{2}P_l(1)\phi_{nl}(p)-\frac{\alpha}{\pi p}\int_0^\infty P_l(z)$$
$$\times\frac{Q_0(z)}{p'}\left[p'^2\phi_{nl}(p')-\frac{P_l(z')}{P_l(z)}p^2\phi_{nl}(p)\right]dp'$$
$$+\frac{\alpha}{\pi p}\int_0^\infty w_{l-1}(z)\phi_{nl}(p')p'dp', \quad (31)$$

where $z'=1$, $P_l(1)=1$. In the above calculation, we have used the identity

$$\int_0^\infty\frac{1}{p'}Q_0(z)\,dp'=\frac{\pi^2}{2}. \quad (32)$$

3. Numerical results and discussions

In this section, the spinless MNK equation with the Coulomb potential is solved numerically by employing the Gauss-Legendre quadrature rule. The positronium, muonium and true muonium are discussed.

3.1. Eigenvalue integral equation

The eigenvalue integral equation (31) can be written formally as

$$g(M,p)\psi(p)=\int_0^\infty K(p,p')\psi(p')dp'. \quad (33)$$

Due to the complicated form of $g(M,p)$, Eq. (33) cannot be solved directly. Rewrite the above equation as [22]

$$\varepsilon\psi(p)=-g(\tilde{M},p)\psi(p)+\int_0^\infty K(p,p')\psi(p')dp', \quad (34)$$

where $\tilde{M}$ is a trial value. If $\tilde{M}=M$, ε will be equal to zero. The eigenvalue equation (34) can be solved by standard method.

3.2. Gauss-Legendre quadrature rule

Rewrite the subtracted integral equation (31) in the form of Eq. (34), then apply the Gauss-Legendre quadrature rule to the regular integral [21]. Finally, a matrix equation can be obtained from Eq. (31) by employing the Nyström method and it can be solved easily.

At first, we map the semi-infinite interval $[0,\infty)$ onto some standard finite interval $[a,b]$ which we take to be

$[-1, 1)$. In this paper, we may take the rational transformation,

$$p = \xi \frac{1+s}{1-s}, \quad p' = \xi \frac{1+t}{1-t}, \tag{35}$$

where ξ is a numerical parameter providing additional control of the rate of convergence. Then we have

$$dp' = \frac{2\xi}{(1-t)^2} dt. \tag{36}$$

The Gauss-Legendre quadrature formula for regular integral reads

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=0}^{N} w_i f(x_i), \tag{37}$$

where

$$w_i = \frac{2}{(1-x_i^2)\left[P'_{N+1}(x_i)\right]^2}. \tag{38}$$

In Eq. (38), prime stands for the derivative.

3.3. Numerical results and discussions

By employing the methods discussed above, the spinless MNK equation [Eq. (25)] with the Coulomb potential is solved numerically and the numerical results are listed in Tables I, II and III. The input parameters are $N = 180$, $\xi = m_i\alpha$ where m_i is the mass of the lighter constituent. For comparison of the eigenvalues obtained in this paper with the eigenvalues obtained in Ref. 17, the electron mass $m_e = 0.51099906$ MeV/c^2, the muon mass $m_\mu = 105.658389$ MeV/c^2 and fine structure constant $\alpha = 1/137.0359895$ are used [23].

In the spinless MNK equation (25), the virtuality parameter ι is expressed in Eq. (15). As $-1 < \iota < 1$, constituents are virtual. As $\iota = \pm 1$, regardless whether the constituents are light or heavy, the spinless MNK equation (25) reduces to the spinless Salpeter equation [17] in which both of constituents are on their mass shell. The spinless Salpeter equation is a well-defined standard approximation to the Bethe-Salpeter equation and a relativistic extension of the nonrelativistic Schrödinger equation. By comparing the binding energies of the Schrödinger equation and that of the spinless Salpeter equation, we can obtain the relativistic effects because the constituents are also put on-mass-shell in the Schrödinger equation as in the spinless Salpeter equation. From Tables I, II and III, we can see that the relativistic corrections are the differences in energies which occur after a few decimal places.

The spinless MNK equation includes not only the relativistic effects but also the virtuality effects. By comparing the eigenvalues of the spinless MNK equation and that of the spinless Salpeter equation, we can obtain the virtuality effects. The eigenvalues of the spinless MNK equation are smaller than that of the spinless Salpeter equation and the Schrödinger equation, see Tables I, II and III. It means that the virtuality effect of constituents results in stronger binding. For the positronium, the virtuality effect is about of the same order as the relativistic effects. For the muonium, the virtuality effect is smaller than the relativistic effect. The data show that the virtuality effect varies with the virtuality parameter ι. For more general cases, the relation between the virtuality effect and ι will become complex [22].

The spinless MNK equation (25) describes the bound states composed of the spinless virtual constituents. By employing the approaches applied in Refs. [6] and [24], the spin-independent terms and spin-dependent terms can be obtained from Eq. (24). Then spin effects can be included according to the discussed problems.

TABLE I. Binding energies $\epsilon_{nl} = M_{nl} - m_1 - m_2$ (in eV) for a spinless electron-positron bound state (positronium) calculated by solving the spinless MNK equation [Eq. (31)], which are compared with the eigenvalues of the spinless Salpeter equation (SSE) and the Schrödinger equation (SCH). n is the principal quantum number, l is the orbital angular quantum number. A negative sign before the energy has been omitted everywhere.

nl	MNK	SSE [17]	SCH
10	6.803 149 6	6.802 961 6	6.802 849 0
20	1.700 768 7	1.700 730 6	1.700 712 2
21	1.700 726 4	1.700 715 5	1.700 715 2
30	0.755 916 2	0.755 878 0	0.755 872 1
31	0.755 893 8	0.755 873 5	0.755 872 1
32	0.755 869 0	0.755 872 6	0.755 872 1

TABLE II. Same as Table I, except for a spinless muon-antimuon bound state (true muonium).

nl	MNK	SCH
10	1406.675 4	1406.613 3
20	351.664 99	351.653 32
21	351.656 24	351.653 32
30	156.299 49	156.290 36
31	156.294 84	156.290 36
32	156.289 73	156.290 36

TABLE III. Same as Table I, except for a spinless muon-electron bound state (muonium).

nl	MNK	SSE [17]	SCH
10	13.541 104	13.541 092	13.540 213
20	3.385 213 6	3.385 196,3	3.385 053,2
21	3.385 082 5	3.385 078,7	3.385 053,2
30	1.504 544 4	1.504 513,5	1.504 468,1
31	1.504 496 9	1.504 478,5	1.504 468,1
32	1.504 466 5	1.504 471,6	1.504 468,1

4. Conclusion

In this paper, the spinless Maung-Norbury-Kahana equation is derived and is solved numerically. Taken as examples, the positronium, muonium and true muonium are studied by employing the spinless MNK equation with the Coulomb potential. The MNK equation allows the constituents of bound states to go off-mass-shell proportionally to their masses. The numerical results show that the binding of virtual constituents will be stronger than that of the on-mass-shell constituents and the virtuality effect varies with different virtuality parameter ι and different mass ratio m_1/m_2.

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1. E.E. Salpeter and H.A. Bethe, *Phys. Rev.* **84** (1951) 1232; E.E. Salpeter, *Phys. Rev.* **87** (1952) 328.
2. J. Carbonell, T. Frederico and V.A. Karmanov, *Eur. Phys. J.C* **77** (2017) 58; J. Carbonell and V.A. Karmanov, *Phys. Rev. D* **90** (2014) 056002; V.A. Karmanov, J. Carbonell and M. Mangin-Brinet, *Nucl. Phys. A* **790** (2007) 598c; S.M. Dorkin, L.P. Kaptari, C.C. degli Atti and B. Kampfer, *Few Body Syst.* **49** (2011) 233.
3. W. de Paula, T. Frederico, G. Salmè and M. Viviani, *Phys. Rev. D* **94** (2016) 071901; T. Frederico, G. Salmè, M. Viviani, *Phys. Rev. D* **85** (2012) 036009; K. Kusaka, A.G. Williams, *Phys. Rev. D* **51** (1995) 7026; V.A. Karmanov, J. Carbonell, *Eur. Phys. J.A* **27** (2006) 1.
4. N. Nakanishi, *Prog. Theor. Phys. Suppl.* **43** (1969) 1; *Prog. Theor. Phys. Suppl.* **95** (1988) 1.
5. J. Resag, C.R. Munz, B.C. Metsch and H.R. Petry, *Nucl. Phys. A* **578** (1994) 397; W. Lucha and F.F. Schöberl, *Phys. Rev. D* **87** (2013) 016009.
6. J.K. Chen, Z.X. Tang and Q.D. Chen, *Mod. Phys. Lett. A* **22** (2007) 2979; J.K. Chen and Y. Liu, *IL Nuovo Cimento B* **122** (2007) 1.
7. F. Gross, *Phys. Rev. C* **26** (1982) 2203; F. Gross, *Phys. Rev.* **186** (1969) 1448.
8. K.M. Maung, J.W. Norbury, and D.E. Kahana, *J. PHys. G: Nucl. Part. Phys.* **22** (1996) 315.
9. E.D. Cooper and B.K. Jennings, *Nucl. Phys. A* **500** (1989) 553; V.G. Kadyshevsky, *Nucl. Phys. B* **6** (1968) 125; K. Erkelenz, *Phys. Rept.* **13** (1974) 191; R.H. Thompson, *Phys. Rev. D* **1** (1970) 110.
10. T. Kopaleishvili, *Phys. Part. Nucl.* **32** (2001) 560; [*Fiz. Elem. Chast. Atom. Yadra* **32** (2001) 1061].
11. A. Klein and T.S.H. Lee, *Phys. Rev. D* **10** (1974) 4308.
12. R.J. Yaes, *Phys. Rev. D* **3** (1971) 3086.
13. A.J. MacFarlane, *Rev. Mod. Phys.* **34** (1962) 41; P.C. Tiemeijer and J.A. Tjon, *Phys. Rev. C* **49** (1994) 494.
14. Y.B. Dai, C.S. Huang and H.Y. Jin, *Phys. Rev. D* **51** (1995) 2347.
15. C.H. Chang and Y.Q. Chen, *Commun. Theor. Phys.* **23** (1995) 451; C.H. Chang, J.K. Chen and G.L. Wang, *Commun. Theor. Phys.* **46** (2006) 467.
16. J.K. Chen, *Rev. Mex. Fis.* **62** (2016) 235; J.K. Chen, *Chin. J. Phys.* **53** (2015) 100201; C. Chen and J.K. Chen, Pramana **86** (2016) 809.
17. J.W. Norbury, K.M. Maung, and D.E. Kahana, *Phys. Rev. A* **50** (1994) 3609.
18. R.H. Landau, *Phys. Rev. C* **28** (1983) 1324; R.H. Landau, *Phys. Rev. C* **27** (1983) 2191; Y.R. Kwon and F. Tabakin, *Phys. Rev. C* **18** (1978) 932.
19. J.W. Norbury, K.M. Maung, and D.E. Kahana, *Phys. Rev. A* **50** (1994) 2075. and references therein; A. Tang and J.W. Norbury, *Phys. Rev. E* **63** (2001) 066703; K.M. Maung, D.E. Kahana, and J.W. Norbury, *Phys. Rev. D* **47** (1993) 1182; J.W. Norbury, D.E. Kahana, and K.M. Maung, *Can. J. Phys.* **70** (1992) 86.
20. J.K. Chen, *Phys. Rev. D* **86** (2012) 036013; *Phys. Rev. D* **89** (2014) 099904(E); J.K. Chen, *Few-Body Syst.* **54** (2013) 2081.
21. J.K. Chen, *Phys. Rev. D* **88** (2013) 076006.
22. Jiao-Kai Chen, submitted.
23. L. Montanet *et al.* [Particle Data Group], *Phys. Rev. D* **50** (1994) 1173; K. Hikasa *et al.* [Particle Data Group], *Phys. Rev. D* **45** (1992) S1; *Phys. Rev. D* **46** (1992) 5210(E).
24. D. Gromes, *Nucl. Phys. B* **131** (1977) 80; A. Gara, B. Durand, L. Durand and L.J. Nickisch, *Phys. Rev. D* **40** (1989) 843; A. Gara, B. Durand and L. Durand, *Phys. Rev. D* **42** (1990) 1651; *Phys. Rev. D* **43** (1991) 2447(E).