Positive even-odd effects in the maximal kinetic energy and negative
even-odd effects in the minimal excitation energy of fragments
from thermal neutron induced fission of $^{235}\text{U}$

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Based on the Coulomb effect hypothesis it is shown that positive even-odd effects of the maximal total kinetic energy ($K_{\text{max}}$) and negative even-odd effects of the total minimal excitation energy ($X_{\text{max}}$), as a function of charge ($Z$) and neutron number ($A$) of fragments, respectively, are not in contradiction. According to the Coulomb effect hypothesis, $K_{\text{max}}$ is equal to the maximal Coulomb interaction energy ($C_{\text{max}}$) reached by the most compact scission configuration. The fragmentation corresponding to $Z = 41$ and $A = 103$ is an exceptional case for which scission configuration is formed by complementary fragments in their corresponding ground states. However, more symmetrical or more asymmetrical fragmentations than that need to be out of their ground states, which implies that $K_{\text{max}} = C_{\text{max}} < Q$.

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1. Introduction

Positive even-odd effects, in proton and neutron numbers distributions, in higher windows of kinetic energy of fragments from thermal neutron induced fission of actinides are well established [1].

The even-odd effect of charge distribution is defined by the relation

$$\delta Z = \frac{Y_e^Z - Y_o^Z}{Y_e^Z + Y_o^Z}$$

where $Y_e^Z$ y $Y_o^Z$ are the yield of fragments with even and odd proton numbers, respectively. Similarly are defined the even-odd effect in the distribution neutron number ($\delta N$) and nucleon numbers ($\delta A$), respectively.

However, when C. Signarbieux et al. [2] found the evidence of the existence of cold fission, corresponding to highest kinetic energy windows, for which the excitation energy is not enough for fragments to emit neutrons, they did not find a significant even-odd effect in the distribution of the mass numbers. This set a controversy in those authors that, based on the even-odd effects in proton and neutron number distributions, respectively, observed in light fragment kinetic energy, supported the hypothesis that the fission process is superfluid [2]. However, in 1981, M. Montoya [3,4] deduced the relation

$$\delta A = \delta Z + \delta N - 1,$$

which is confirmed by H. Nifenecker [5]. After this relation there is no contradiction between null even-odd effects in mass distribution and positive even-odd effects in proton and neutron number distributions, respectively.

In 1991 F. Gönnenwein and B. Börsig [6] show that the minimum excitation energy is lower for the odd than for the even $Z$. In 1993 the positive even-odd in cold fission of actinides is questioned by F.-J. Hambsch [7]. In 2013, F. Gönnenwein confirms the hypothesis of negative even-odd effects on total excitation energy in cold fission [8]. In 2016, M. Mirea proposes a microscopic model in order to explain negative odd-even effects in excitation energy in cold fission [9].

In this work we review experimental data to show that there is no contradiction between positive even-odd effects in total maximal kinetic energy and negative effects in minimal excitation energy of fragments.

2. Formalism for even-odd effects in cold fission

In order describe even-odd effects in cold fission is useful to recall some definitions related to them. See Ref. 6. Let be a fissile nucleus with charge $Z_f$ and mass $A_f$ that splits in a light fragment with $Z_L$ protons, $N_L$ neutrons (number of nucleons $A_L = Z_L + N_L$) and a heavy fragment with $Z_H$ protons, $N_H$ neutrons (number of nucleons $A_H = Z_H + N_H$). These numbers obey the following relations:

$$Z_f = Z_L + Z_H$$

and

$$A_f = A_L + A_H$$

In order to simplify notations, $Z_L$, $N_L$, and $A_L$ will be renamed $Z$, $N$, and $A$, respectively.
After scission, light and heavy fragments acquire kinetic energies $K_L$, $K_H$, and excitation energies $X_L$, $X_H$, respectively. Thus, the total kinetic energy ($K$) and the total excitation energy ($X$) are

$$K = K_L + K_H$$

and

$$X = X_L + X_H$$

respectively. These quantities are limited by the energy balance equation:

$$Q = K + X,$$

where $Q$ is the available energy of the reaction.

At the scission point, the available energy is spent into deformation energy ($D$), Coulomb interaction energy ($C$) and free energy ($F$), according to relation

$$Q = C + D + F.$$

The free energy is partitioned into intrinsic energy ($X^*$) and total pre-scission energy of fragments ($K_{sc}$):

$$F = X^* + K_{sc}$$

One assumes that, for a given fragmentation corresponding to proton and mass numbers $Z$ and $A$, respectively, the maximum total kinetic energy ($K_{max}$) is reached by a configuration with $X^* = 0$, maximum Coulomb interaction energy ($C_{max}$), and a minimum total deformation energy ($D_{min}$), limited by the equation

$$Q = C_{max} + D_{max}$$

Because Coulomb repulsion between fragments is the unique force after scission, Coulomb interaction potential energy at scission becomes the final total kinetic energy, so that:

$$K_{max} = C_{max} = Q - D_{min}.$$  

Let be $A$ an odd nucleon number of the light fragment, the local even-odd effect in the maximum $Q$-value ($Q_{A_{max}}$) as a function of mass, is defined as

$$\delta_A Q_{max} = \frac{Q_{A_{max}}^{A-1} + Q_{A_{max}}^{A+1}}{2} - Q_{max}^A.$$  

In general $\delta_A Q_{max}$ is positive. Similarly there are local positive even-odd effects in $Q_{max}$ as a function of proton number ($\delta_Z Q_{max}$) and as a function of neutron number ($\delta_N Q_{max}$), respectively.

Because the even-odd effect of charge and mass distribution, respectively, increases with the fragment kinetic energy [1], a positive local even-odd effect in the maximum total kinetic energy as a function of mass

$$\delta_A K_{max} = \frac{K_{max}^{A-1} + K_{max}^{A+1}}{2} - K_{max}^A,$$

and positive values of $\delta_Z K_{max}$ and $\delta_N K_{max}$, which correspond to even-odd effects in the maximum total kinetic energy as a function of $Z$ and $N$, respectively, are expected.

Figure 1. Thermal neutron induced fission of 235U. Curve of the maximum total kinetic energy ($K_{max}$) as a function of the light fragment mass number is presented. The measured odd charges that maximize $K_{max}$ for several mass fragmentations are indicated. The other cases correspond to neighboring even charge fragmentations. Taken from Ref. 10.

3. Even-odd effects in the maximum total kinetic energy

In 1986 J. Trochon et al. [10] present the curve of the maximum total kinetic energy as a function of light fragment mass from the reaction $^{235}$U($n_{th}$, f). For each $A$ they identify the charge $Z$ that maximizes the total kinetic energy $K$. See Fig. 1.

The $K_{max}$ value corresponding to an odd mass are generally below the average of the values corresponding to two neighboring even masses, which means that $\delta_A K_{max}$ is positive.

In general, for each $A$, an even charge maximizes $K$, except in transitions between two neighboring even charges. These cases are the following:

$$(Z, A) = (35, 87); (37, 93); (39, 99); (41, 103).$$

Applying the definition of $\delta_Z K_{max}$ and $\delta_N K_{max}$ to these cases, one obtains results presented in Figs. 2 and 3, respect-
The existence of positive even-odd effects on the maximal total kinetic energy as a function of $Z$ and $N$, respectively, is confirmed. $\delta Z K_{\text{max}}$ values are approximately 0.8 MeV, except for $(Z, A) = (37, 93)$, in which case is 0.2 MeV. The explanation of this may be in the fact that $Q$-values corresponding to $(Z, A) = (36, 92), (37, 93), (38, 93)$ are approximately the same (189.3 MeV). See Fig. 4.

Taking regions with even $Z$, one also observes positive $\delta_N K_{\text{max}}$ values. For masses 95, 97, 101 and 107, $\delta_N K_{\text{max}}$ is approximately 1.2 MeV. The lower values, corresponding to $A = 91$ and 105, are approximately 0.5 MeV.

One must notice that $\delta_N K_{\text{max}}$ is negative (near null) for $Z = 36$ and $A = 89$. This result may be explained by the fact that the $Q$-value corresponding to $A = 88 (Q = 186.3 \text{ MeV})$ is very close to the corresponding to $A = 89 (Q = 185.8 \text{ MeV})$.

From experimental result one can observe that

$$\delta Z K_{\text{max}} < \delta Z Q_{K_{\text{max}}}$$

and

$$\delta_N K_{\text{max}} < \delta_N Q_{K_{\text{max}}}$$

To interpret this result one must take into account that

$$Q_{K_{\text{max}}} < C_{\text{max}} + D_{\text{min}},$$

and

$$\delta_A C_{\text{max}} = \delta_A K_{\text{max}} = \delta_A Q_{k_{\text{max}}} - \delta_A D_{\text{min}},$$

regarding which it follows that

$$\delta_Z D_{\text{min}} > 0$$

which suggests that the even-even fragments are harder than odd $A$ fragments, they need higher deformation energy to get the most compact configuration that obeys the relation

$$C_{\text{max}} = Q_{K_{\text{max}}} - D_{\text{min}}.$$
4. Discussion

In this work positive even-odd effects on maximal kinetic energy as a function of $Z$, $N$ and $A$, respectively, of light fragments from the reaction $^{235}\text{U}(n_{th}, f)$, were put in evidence.

One must notice that, in 2013, F. Gönnenwein and B. Börsig [10] show that, for isobaric fragmentations 104/132, the kinetic energy associated to the charge fragmentation 41/51 reach the $Q$-value of the reaction, while the corresponding to the fragmentation 42/50 reaches a total maximum kinetic energy below 3 MeV the corresponding $Q$-value. These authors suggest that this is due to the fact the charge split 41/51 corresponds to odd fragment charges. However, we should note that charge fragmentation 41/51 is more asymmetric than the 42/50 fragmentation. Therefore that result is also consistent with the Coulomb effect after which for neighboring masses with similar values of energy available, the more asymmetric fragmentation reaches the higher values of total kinetic energy [12,13].

The fact that the charge split 41/51 reaches the $Q$-value means that both fragments are in their respective ground states ($X = 0$) whose corresponding scission configuration is so that $C_{\text{max}} = Q$.

With a same configuration the charge split 42/50 with fragments in their corresponding ground states have not necessarily a similar relation. It means that at least one fragment must be deformed out of its ground state, then

\[ X_{\text{min}} > 0 \]

and

\[ K_{\text{max}} = Q - X_{\text{min}} < Q . \]

Moreover, the isobaric mass fragmentation 104/132 corresponds to a pronounced turning point in the $Q$-values. See Fig. 4. The highest $Q$-value and the highest $K_{\text{max}}$ correspond to the transitional deformed nucleus $^{104}_{42}\text{Mo}$ [14], and the double magic spherical nucleus $^{152}_{50}\text{Sn}$ [15]. These conditions makes difficult to distinguish even-odd from transitional effects.

5. Conclusion

Based on the Coulomb effect hypothesis [12,13] we have demonstrated that, in thermal neutron induced fission of $^{235}\text{U}$, there is no contradiction between the positive even-odd effects in the maximal kinetic energy (measured by J. Trochon et al. [10]), and the negative even-odd effects in the minimal excitation energy of fragments (shown by F. Gönnenwein [8]) as a function of fragment neutron and proton numbers, respectively. Assuming that the maximum Coulomb energy configuration corresponds to the minimum excitation energy one, the deformation energy explains both seemingly contradictory mentioned results.