

On polarization of the fluorescence in erbium-doped fibers

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Theoretical evaluations of polarization degree of the fluorescence excited in erbium doped fibers (EDF) dependence on the excitation light power are presented. In calculations we use the model of randomly oriented and not interacting Er^{3+} ions. Assumption of the reduced optical anisotropy of the individual ions (with the absorption/emission cross-sections described with the ellipsoids) is shown to be necessary to get a reasonable agreement with the existing experimental data. Similar values of the anisotropy parameters (0.6-0.7) are also utilized to explain the experimental data on polarization hole burning in EDF earlier.

Keywords: Erbium doped fibers; polarization hole burning; fluorescence.

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1. Introduction

The erbium-doped fibers (EDF) and the devices based on them are of a major importance for the modern optoelectronics and for the optical communications in particular. The EDFs form the basis for the fiber optical amplifiers operating in the main spectral transmission window (centered around 1550 nm) of the modern low-loss optical fibers [1,2] and for the fiber lasers operating in this important spectral region [3]. Still, very important fundamental problems directly associated with the optical pumping and irradiation of the light by Er^{3+} ions in silica optical fibers are not clarified completely. In particular, the problems of a spatial migration of the excited state among the active erbium ions, the processes of the resonance frequency up-conversion, and closely associated and practically very important effect of a concentration quenching of the fluorescence/gain are still widely discussed in the current literature (see *e.g.* [4-7]).

Below in this paper we present an original analysis of the polarization of the fluorescence in EDF and its dependence on the exciting light power. This effect (of the spontaneous emission) is, obviously, closely related to the optical gain (*i.e.* the stimulated emission) observed in this material, which is utilized in optical amplifiers and lasers [2,3]. For this reason, the experimental investigation of the fluorescence is considered as one of the most important general techniques for characterization of the laser materials.

In particular, this method can be utilized for characterization of a saturation of the optical absorption/gain in the erbium-doped fibers under consideration. Besides being very important for operation of the erbium fiber optical amplifiers and lasers, these saturation effects are utilized for recording of the dynamic population gratings in the rare-earth doped fibers and in EDF in particular [8]. Such gratings form the basis for another class of original optoelectronic devices such as narrow band optical filters [9,10], single-frequency fiber lasers [11,12], adaptive interferometric sensors [13,14], and

are used in experiments with slow/fast light propagation [15]. In its turn, the polarization (or as it, probably, better to say, the depolarization) of the fluorescence in EDF can be also very informative in investigation of the excited state migration among the active erbium ions in EDF. Indeed, the excitation migration among the randomly oriented erbium ions can be considered as one of the mechanisms leading to the loss of memory on the exciting light polarization and, finally, to the depolarization of the fluorescence.

While for the bulk glasses doped with materials erbium there are some experimental publications [16,17], as far as we know, there are, practically, no experimental data published on the depolarization of the fluorescence in EDF. Only recently we have published in Ref. 18 some preliminary experimental results on this topic. To analyze and to evaluate the obtained experimental results some theoretical analysis is, obviously, needed, but, as far as we know, there is also no such analysis published until now. Below in this paper we fill this gap and present the theoretical analysis of the fluorescence polarization in EDF.

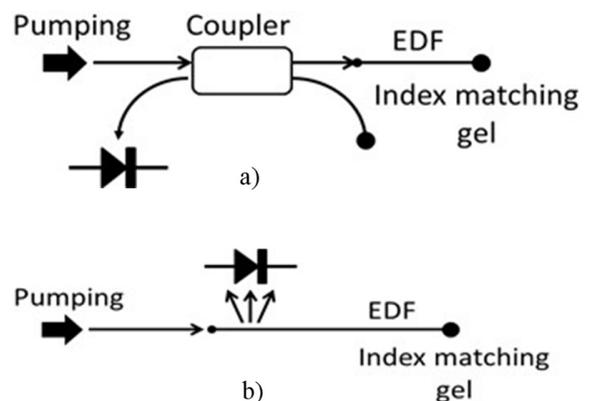


FIGURE 1. Experimental configurations for observation of fluorescence from EDF: the longitudinal (a) and the transversal one (b).

2. Experimental configurations for observation of the fluorescence in the doped fibers

Basically, the fluorescence in the doped fibers (and in EDF in particular) is easily observed in the longitudinal configuration presented in Fig. 1a. Here the fiber is excited (*i.e.* optically pumped) through the fiber end which is also utilized to pick up the fluorescence signal. If the level of the excitation by pumping light is low and the inversion of the system is insignificant one can neglect possible amplified stimulated emission [2,3] and consider that the detected signal represents the original fluorescence signal.

In this configuration the fluorescence light power can be collected from a significant length of the fiber and can be rather strong. On the other hand, this can be considered as a drawback of this configuration since the detected signal is emitted from different cross-sections of the fiber where the pumping power (because of the fiber saturated absorption) can be rather different and also varying with the pumping level.

At least partially, this problem can be overcome in the transverse configuration presented in Fig. 1b. In this case, the fluorescence power is collected from a short segment of the fiber, where conditions of the pumping can be considered as uniform. Also, if the point of observation is located closely to the entrance to the doped fiber, the incident pumping power can be considered as a fixed and equal of the incident power.

The sensitivity of the longitudinal configuration is higher than that of the transverse one and, for this reason, it can be utilized for spectral measurements. The transverse detection configuration is, however, more suitable for the amplitude and for the temporal measurements. Such measurements can be performed using the intensity modulated pumping wave, which allows us in the same experiment to obtain both the amplitude and the temporal data. Typical fluorescence signal observed in response to the rectangular modulation of the incident pumping power is shown in Fig. 2a. The main parameters of interest in this experiment is the maximal (steady-state) fluorescence amplitude, and the characteristic relaxation rates (*i.e.* inverse relaxation times) at two fronts of the detected fluorescence response signal—see Fig. 2b,c [19].

In a simplified model of saturation of a two energy level system [20] all the participating ions are usually considered to be as simple dipoles oriented collinearly to the polarization of the excitation light. In EDF the fluorescence and the optical gain in the fundamental absorption/emission band (centered around 1550 nm) is usually excited by the pumping wavelength 980 nm through the third excited energy level $^4I_{11/2}$ (Fig. 3). Because of the short (around $\approx 15 \mu\text{s}$ [2,3]) time of the radiation-less transition from this level to the second meta-stable level $^4I_{13/2}$ one can neglect the stay of the ion at this energy level and consider the excitation as passing directly to the meta-stable level. If the fiber is illuminated by the wavelength from the fundamental absorption band (1480–1570 nm) the meta-stable level is excited, obviously, in the direct way. The fluorescence under consideration in this pa-

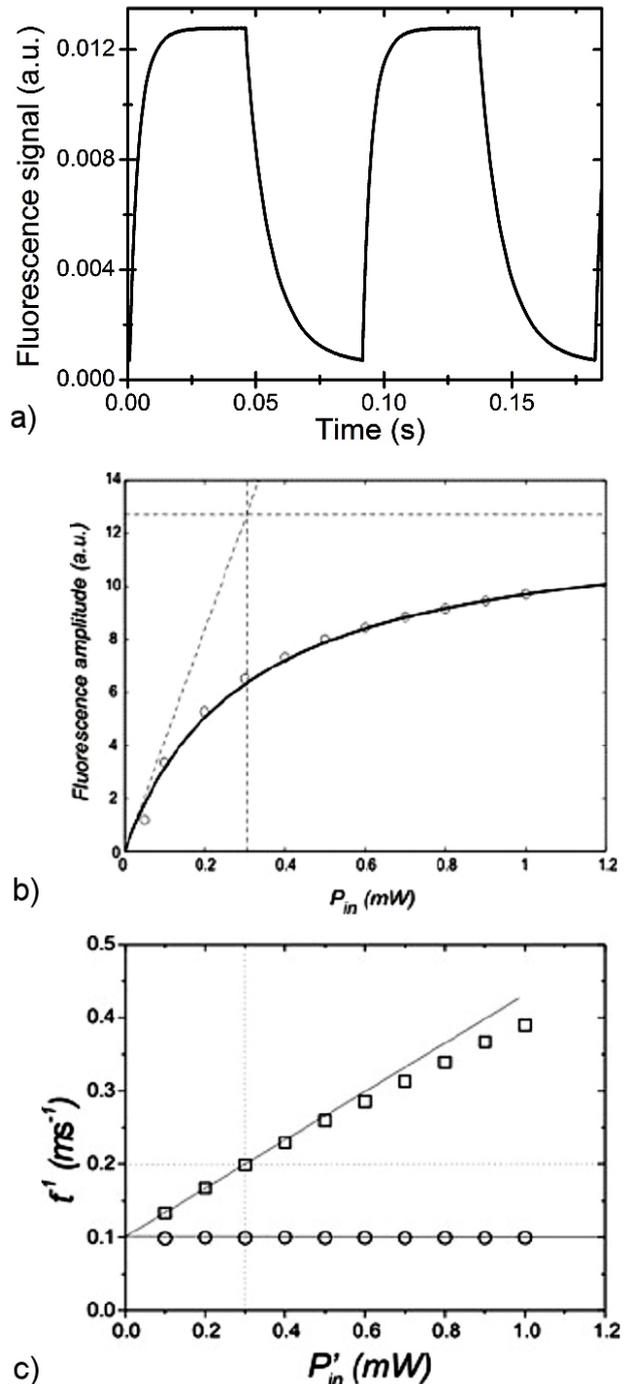


FIGURE 2. Typical fluorescence response observed for periodically modulated incident light (a), and the incident power dependences of the steady-state fluorescence level (b) and of the fluorescence growth (squares) and decay (circles) rates (c) (EDF Er107 of INO, $\lambda = 1526 \text{ nm}$ [19]).

per appears as a result of a spontaneous transition (with the characteristic time $\tau_0 \approx 10 \text{ ms}$ [2,3]) from the meta-stable level to the fundamental level $^4I_{15/2}$.

In this simplified model the absorption and emission properties of the erbium ions are characterized by the absorption and by the emission cross-sections (σ_a, σ_e), both of

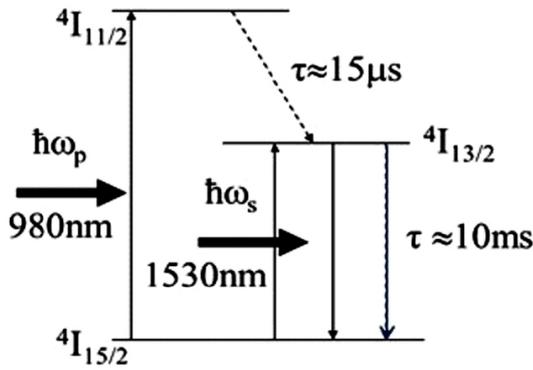


FIGURE 3. Energy level structure of Er³⁺ and the main photo-induced and radiation-less transitions.

which, however, are assumed to be wavelength dependent values. If all ions are considered to be oriented parallel to the polarization of the saturating/pumping light (see Fig. 4a) all of them are excited in the same way, and the population of the meta-stable level can be expressed by the following simple relation [20]:

$$N_2 = N_0 \frac{\frac{I}{I_{sat}} \frac{\sigma_a}{\sigma_e + \sigma_a}}{1 + \frac{I}{I_{sat}}} \quad (1)$$

where I is the intensity of the incident pumping wave and the saturation intensity I_{sat} equals:

$$I_{sat} = \frac{\hbar\omega}{(\sigma_e + \sigma_a)\tau_0} \quad (2)$$

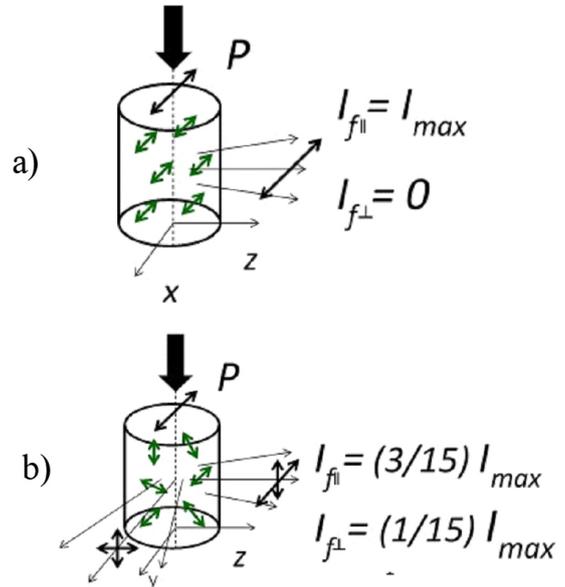
The fluorescence intensity depends on the population of the meta-stable level only, for this reason, it can be expressed by the following very simple expression:

$$I_f \propto \frac{\frac{I}{I_{sat}}}{1 + \frac{I}{I_{sat}}} = \frac{\frac{P}{P_{sat}}}{1 + \frac{P}{P_{sat}}} = \frac{P'}{1 + P'} \quad (3)$$

In the above equation instead of the light intensities we have utilized the light powers, which in the optical fiber are equal to the corresponding intensities (the incident and the saturation one) multiplied by the effective fiber mode area.

The relaxation of the fluorescence signal follows the relaxation of the meta-stable level population. For this reason, in the dark (*i.e.* between two subsequent excitation light pulses) the fluorescence relaxes with the spontaneous relaxation rate τ_0^{-1} . However, under illumination with the light the fluorescence grows to its nonzero steady-state level with the rate [20]:

$$\begin{aligned} \tau_f^{-1} &= \tau_0^{-1} \left(1 + \frac{I}{I_{sat}} \right) \\ &= \tau_0^{-1} \left(1 + \frac{P}{P_{sat}} \right) = \tau_0^{-1} (1 + P') \end{aligned} \quad (4)$$



$$\begin{aligned} I_{f_y} &= I_{f_z} = (1/15) I_{max} \\ I_{f_x} &= I_{f^{\perp}} = (1/15) I_{max} \end{aligned}$$

FIGURE 4. Polarization of fluorescence from collinearly oriented dipoles (a), and that from randomly oriented dipoles (b).

From the above Eqs. (3,4) one can easily see how the incident light power dependencies of the fluorescence intensity and of the fluorescence growth rate can be used for evaluation of the saturation power of the doped fiber. Note that this parameter is a critical one for the effects associated with the recording of the population gratings in the rare-earth doped fibers [8].

3. Depolarization of the fluorescence from the randomly oriented non-interacting dipoles

In the ideal case considered above, when all the active dipoles are oriented collinearly and are excited by the collinearly polarized light, the fluorescence is to be also completely polarized as the irradiation of one particular dipole (Fig. 4a). However, in the real case of the erbium ions in fused silica, because of the amorphous nature of the host material, the erbium ions are randomly oriented (Fig. 4b). Even if they do not interact (*i.e.* when no migration of the excitation occurs among the ions [21]) the observed fluorescence is to be partially depolarized because the misaligned ions also make their contributions to the total fluorescence signal with polarizations corresponding to their own orientations.

In the evaluations below we consider every ion as a dipole and follow the standard averaging procedure presented, in particular, in Ref. 22. In this consideration, the effective excitation light intensity for this particular ion is equal to $I \cos^2 \theta$ where θ is the angle between the light polarization vector and the dipole axis (Fig. 5). The fluorescence intensity irradiated

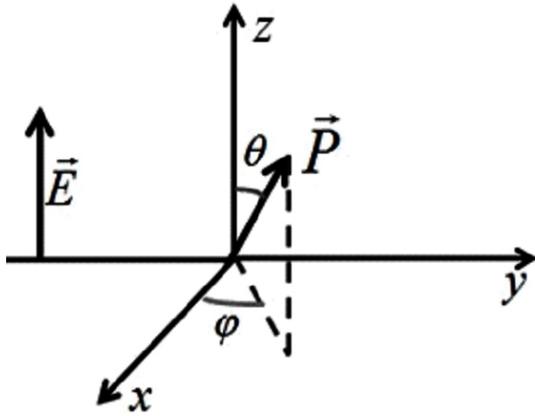


FIGURE 5. Mutual orientation of the excitation light polarization (\vec{E}) and of the effective dipole (\vec{P}) of the erbium ion.

by this dipole in the plane perpendicular to the excitation light polarization is also proportional to $\cos^2 \theta$. In order to obtain the total fluorescence intensity it is necessary to integrate over all possible orientations of the ions which have the normalized density:

$$\sin \theta d\theta d\varphi / 4\pi \tag{5}$$

For the component of the fluorescence with the polarization parallel to that of the incident (pump) light this integration gives us:

$$\begin{aligned} I'_{f\parallel} &= \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^\pi d\theta \frac{P' \sin \theta \cos^4 \theta}{1 + P' \cos^2 \theta} \\ &= \frac{P'}{2} \int_0^\pi d\theta \frac{\sin \theta \cos^4 \theta}{1 + P' \cos^2 \theta} \end{aligned} \tag{6}$$

The above equation presents the fluorescence intensity normalized to the maximal (saturated) fluorescence intensity emitted by the same total number of the dipoles with the same collinear orientation. Similarly, the fluorescence detected along the same direction but polarized in a perpendicular way to the pump wave has the following normalized intensity:

$$\begin{aligned} I'_{f\perp} &= \frac{1}{4\pi} \int_0^{2\pi} d\varphi \cos^2 \varphi \int_0^\pi d\theta \frac{P' \sin^3 \theta \cos^2 \theta}{1 + P' \cos^2 \theta} \\ &= \frac{P'}{4} \int_0^\pi d\theta \frac{\sin^3 \theta \cos^2 \theta}{1 + P' \cos^2 \theta} \end{aligned} \tag{7}$$

Obtained in this way, the normalized intensities of two orthogonally polarized fluorescence components, together with the result for the reference case of completely collinear ions (Eq. 3), are presented in Fig. 6.

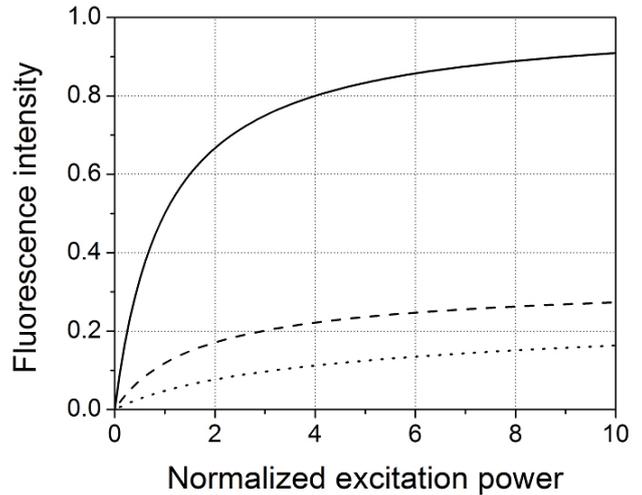


FIGURE 6. Normalized fluorescence intensity from a set of non-interacting dipoles as a function of the normalized excitation power: collinear dipoles (solid line), parallel (dashed line) and orthogonal component (dotted line) from the randomly oriented dipoles.

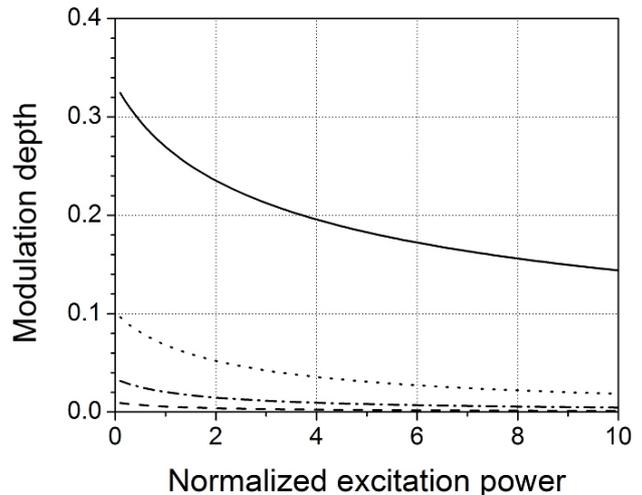


FIGURE 7. The modulation depth (normalized amplitude of the periodic part) in the fluorescence signal observed for periodic modulation of the pumping light polarization calculated: for randomly oriented dipoles (solid line) and for the ions with reduced anisotropy for α : 0.2 (dotted line), 0.4 (dot-dashed line), and 0.6 (dashed line).

The fluorescence polarization degree is usually defined as [22]:

$$P = \frac{I'_{f\parallel} - I'_{f\perp}}{I'_{f\parallel} + I'_{f\perp}} \tag{8}$$

One can see from Fig. 6 that this value can be rather high, about 50%, along the direction perpendicular to the polarization of the excitation light (Fig. 4b). However, for obvious symmetry reasons, the fluorescence should be completely depolarized along the parallel direction of observation. Figure 7 shows the modulation depth in the detected fluorescence sig-

nal if we periodically switch the polarization of the excitation light between two orthogonal directions:

$$m = \frac{(I'_{f\parallel} + I'_{f\perp}) - 2I'_{f\perp}}{(I'_{f\parallel} + I'_{f\perp}) + 2I'_{f\perp}} = \frac{I'_{f\parallel} - I'_{f\perp}}{I'_{f\parallel} + 3I'_{f\perp}} \quad (9)$$

as we did in our recent experiments described in Ref. 18. Note only that in this case during one semi-period of modulation we detect the total fluorescence signal along the direction of the incident light polarization, and along the orthogonal direction during the second modulation semi-period. From the presented curve one can see that, theoretically, the normalized effective amplitude of the obtained in this way fluorescence polarization signal can reach 1/3 for low excitation power.

In conclusion to this section we summarize again the main reason of polarization of the fluorescence from the set of the randomly oriented dipoles. Within the accepted model it appears not because the individual ions remember in some way the polarization of the excitation light. On the opposite, every individual dipole irradiates the fluorescence light in accordance with its own orientation only. The total fluorescence signal proves to be polarized predominantly along the polarization of the excitation light only because the dipoles with orientation of the incident light polarization prove to be excited in the most efficient way. In this sense, it, clearly, does not matter whether the ions are excited directly to the meta-stable level (${}^4I_{13/2}$ in case of Er^{3+} ions-see Fig. 3), or through the third energy level (${}^4I_{11/2}$) with the following relaxation to the meta-stable level via radiation-less transitions.

4. Active ions with the reduced optical anisotropy

In Ref. 18 we have presented preliminary results on measurements of polarization degree of the total fluorescence excited in EDF by some selected laser lines in the frequency band 1490-1570 nm. It proved that experimentally the fluorescence was depolarized significantly stronger than it is predicted by the above analysis (see Fig. 6,7). Similar results were also presented earlier for the fluorescence observed from Er^{3+} ions in bulk glasses [16,17]. What is important, earlier attempts [23,24] to explain the existing experimental data on polarization hole burning (PHB) observed in EDF using a similar model of the randomly oriented non-interacting dipoles also failed. Experimentally the PHB, as the polarization degree of the fluorescence under discussion, proved to be significantly lower than it is predicted by this simple model.

Without giving a detailed microscopic explanation for such a strong discrepancy of the experimental data on PHB with the prediction of the randomly oriented dipoles model, the authors of [23,24] proposed a rather phenomenological approach of essentially reduced anisotropy of optical properties of the erbium ions. They assumed that every ion is characterized by ellipsoids of the absorption and emission cross-

sections with three main eigen-values corresponding to the three orthogonally oriented proper axes.

In general, this model can include quite a number of the fitting parameters (three main values for every cross-section and additional three angles describing orientations of these two ellipsoids). To simplify the problem, the authors assumed, however, that the two ellipsoids are of cylindrical symmetry, have the same ratio of two semi-axes $\alpha = \sigma_{a\perp}/\sigma_{a\parallel} = \sigma_{e\perp}/\sigma_{e\parallel}$, and are collinearly oriented. Under such strong assumptions they have managed to explain the reduced value of the PHB effect observed in EDF using only one fitting anisotropy parameter $\alpha \approx 0.5-0.7$. Below, we apply the same approach to our consideration of polarization of the fluorescence. Obviously, the above-considered model of the randomly oriented dipoles can be considered as some special case (with $\alpha = 0$) of this more general ‘‘ellipsoid’’ approach.

Within this model it is assumed that the incident light power is to be divided among the three polarizations oriented along the proper semi-axes of every particular ion with its random orientation. The total absorbed light power by the ion is considered as an incoherent sum of these three components. After this it is supposed that the ion irradiates the fluorescence power with three independent mutually incoherent and orthogonally polarized components. Note that mutual incoherence of the orthogonally polarized proper processes of the light absorption and illumination is a very important feature of this model. Indeed, three-dimensional (even anisotropic) dipoles give even higher degree of the fluorescence polarization than the randomly oriented one-dimensional dipoles considered above.

Omitting simple geometrical considerations, we give here directly the results for two orthogonally polarized components of the normalized fluorescence intensity:

$$I'_{f\parallel} = \frac{P'}{2} \int_0^\pi d\theta \frac{\sin \theta (\cos^2 \theta + \alpha \sin^2 \theta)^2}{1 + P'(\cos^2 \theta + \alpha \sin^2 \theta)} \quad (10)$$

$$I'_{f\perp} = \frac{P'}{4} \int_0^\pi d\theta \times \frac{\sin \theta (\cos^2 \theta + \alpha \sin^2 \theta)(\alpha + \sin^2 \theta + \alpha \cos^2 \theta)}{1 + P'(\cos^2 \theta + \alpha \sin^2 \theta)} \quad (11)$$

As in Fig. 6, in Fig. 8 we present the normalized intensities of two orthogonally polarized components of the fluorescence as functions of the incident light power calculated using Eq. (10,11) for several values of the anisotropy parameter α . Fig. 7 also shows the modulation contrast in the detected total fluorescence signal for a fast periodic change of the excitation light polarization (Eq. 9). These evaluations were performed for the same discrete set of the anisotropy values. Finally, Fig. 9 shows the dependence of the contrast of this signal as a function of the anisotropy parameter α .

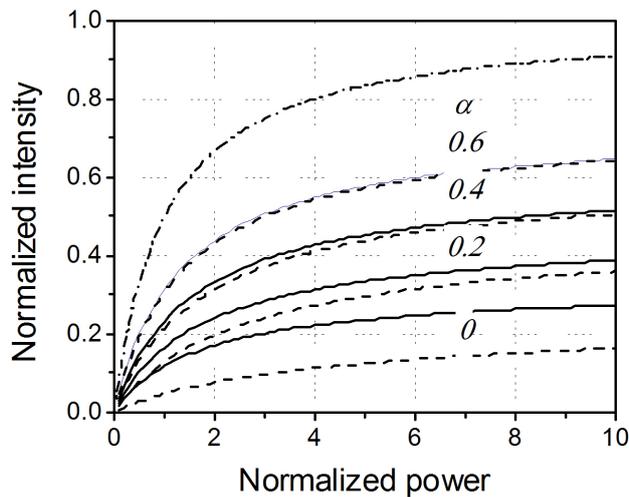


FIGURE 8. Normalized intensities of the orthogonally polarized fluorescence components: parallel (solid lines) and perpendicular (dashed lines) calculated for different values of α . For reference, the dash-dot line shows the normalized intensity of the fluorescence from the set of collinearly oriented dipoles.

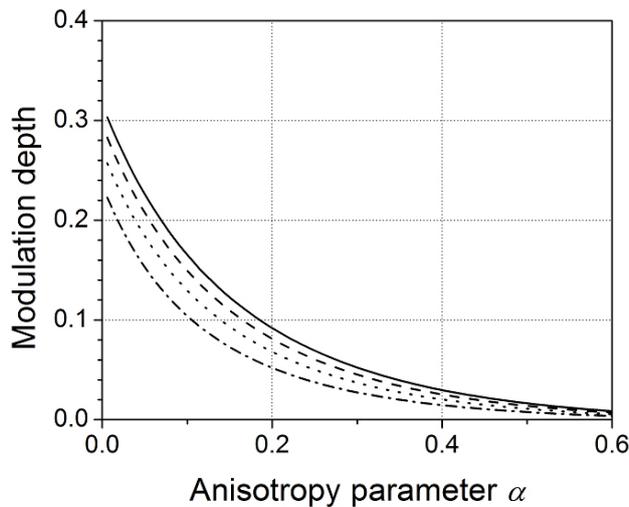


FIGURE 9. Modulation depth of the fluorescence signal as a function of the anisotropy parameter α evaluated for normalized excitation power P/P_{sat} : 0.2 (solid line), 0.5 (dashed line), 1 (dotted line), and 2 (dash-dotted line).

From these figures one can see that for $\alpha = 0.6$ the expected maximum modulation depth in the fluorescence polarization signal reduces to $\approx 1\%$ which is rather close to the experimental values reported in Ref. 18. This value of α is, in fact, rather close to the anisotropy parameter = 0.535 which was utilized in Ref. 23 to explain the experimentally observed PHB at 1558 nm, or to 0.67 evaluated in Ref. 24 for 1559 nm. Clearly, more precise fitting needs more detailed and precise fluorescence experimental data.

Without giving the details, we mention that our recently reported fluorescence data [18] indicate that this fitting parameter proves to be significantly dependent on the excitation light wavelength in EDF. As we have also reported in Ref. 18,

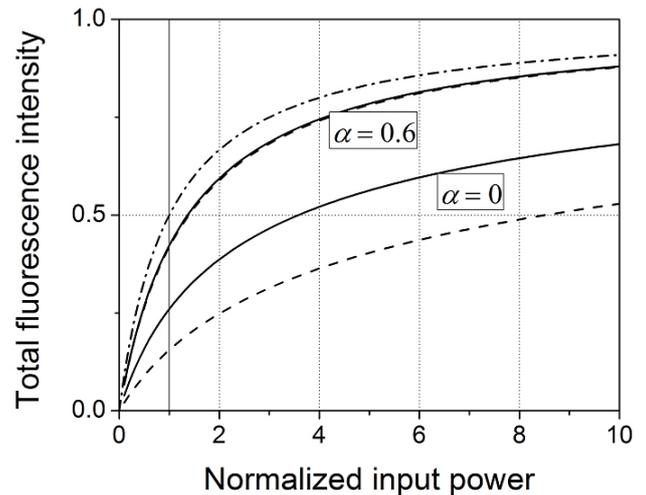


FIGURE 10. Normalized to their maximum (saturated) values of total fluorescence intensities as functions of the linearly polarized excitation power. The solid lines correspond to detection along the direction perpendicular to polarization of the incident light, and the dashed lines—along the parallel direction. The dash-dot line presents a reference curve corresponding to the ideal case of a collinear orientation of all dipoles (with $\alpha = 0$).

in the fibers (and also in bulk erbium doped glasses [17]) with high erbium concentration the depolarization of the fluorescence is significantly stronger. Explication of these results will probably need consideration of an interaction among the erbium ions and excitation migration process which will clearly make the model much more complicated.

The above-presented analysis, while justifying the conventionally accepted procedure of evaluation of the saturation power from the experimental data on the incident power dependence of the fluorescence intensity (Fig. 2b), also introduces some corrections in it. The theoretical curves presented in Fig. 10 (calculated for $\alpha = 0$ and 0.6) show how the normalized to their own maximum (saturated) values total (*i.e.* of all polarizations) fluorescence intensities depend on the linearly polarized incident excitation light power. The solid lines correspond to the case when the total fluorescence signal is detected along the direction orthogonal to the polarization of the incident light, and the dashed lines—to the parallel direction of observation. For a reference, the curve corresponding to an ideal case of a collinear orientation of all dipoles is shown with the dash-dotted line. One can see that for the case of relatively large anisotropy parameters α the standard evaluation procedure (see Fig. 2b) is quite applicable, but gives somewhat increased (for about 30% for $\alpha = 0.6$) value of the maximal individual ion saturation power. In case of low anisotropy parameter, a more detailed analysis of the experimental data is obviously needed.

5. Conclusion

Summarizing, we have presented the results of a theoretical consideration of the depolarization of the fluorescence ex-

cited by the linearly polarized light in EDF. As a general approach to the description of the erbium ions in the silica fiber core we have accepted the model of the non-interacting randomly oriented ions. Comparison with the available experimental data for EDF leads to the conclusion that only the assumption of a reduced anisotropy of the individual ions de-

scribed by the ellipsoids of the optical absorption/emission cross-sections, which was used earlier to explain the PHB effect, can result in a reasonable agreement. This approach cannot, obviously, explain any possible dependence of the fluorescence depolarization on the dopant concentration.

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