Optimal ratios of the piston speeds for a finite speed endoreversible Carnot heat engine cycle

Huijun Feng, Lingen Chen*, and Fengrui Sun
Postgraduate School, Naval University of Engineering, Wuhan 430033, P.R. China.

Recibido el 22 de junio de 2009; aceptado el 11 de enero de 2010

The performance of an endoreversible Carnot heat engine cycle is analyzed and optimized using the theory of finite time thermodynamics based on Agrawal and Menon's[i] model of finite speed of the piston on the four branches and Curzon and Ahlborn's[ii] model of finite rate of heat transfer. The finite speeds of the piston on the four branches are further assumed to be different, which is unlike the model of constant-speed of the piston on the four branches. The analytical formula between power and efficiency of the cycle is derived for a fixed cycle period. There exist optimal ratios of the finite piston speeds on the four branches. The effects of the temperature ratio of the heat reservoirs on the dimensionless power versus efficiency of the cycle and isothermal expansion ratio are obtained by numerical examples.

Keywords: Finite time thermodynamics; endoreversible Carnot heat engine; finite speed of the piston; finite rate of heat transfer; power; efficiency.

Se analiza y optimiza el funcionamiento cíclico de un motor endoreversible de Carnot, utilizando la teoría termodinámica de tiempo finito basada en el modelo de Agrawal y Menon[i] de velocidad finita del pistón en los cuatro cilindros, y en el modelo de rapidez finita de transporte de calor de Curzon y Ahlborn[ii]. También se supone que las velocidades del pistón en los cuatro cilindros son diferentes. Se deduce la fórmula analítica de la potencia y la eficiencia para un período del ciclo. Resultan cocientes óptimos para las velocidades finitas del pistón en los cuatro cilindros. Se obtienen, mediante ejemplos numéricos, los efectos del cociente de temperatura de los focos térmicos sobre la potencia versus la eficiencia del ciclo y el coeficiente de dilatación isotérmica.

Descriptores: Termodinámica de tiempo finito; motor endoreversible de Carnot; velocidad finita del pistón; potencia; eficiencia.

PACS: 05.70.-a; 05.30-d

1. Introduction

Since the new efficiency limit, characterized by finite rate, finite duration and finite size, was first derived by Novikov [1] and Chambdal [2] simultaneously and rediscovered by Curzon and Ahlborn [3], the analysis and optimization of all kinds of thermodynamic cycles has made tremendous progress by using finite time thermodynamics [4-12]. In the research field of reciprocating heat engine cycles, adiabatic processes are always considered to be instantaneous compared with the isothermal processes. This may be applicable for some problems, but sometimes it will degrade the overall cycle performance in other problems. Petrescu *et al.* analyzed the problem, and two fundamental and powerful tools, named the First Law of Thermodynamics for Processes with Finite Speed and the Direct Method, were introduced in their work. Meanwhile, a series of work had been performed in Refs. 13 to 18, and the well-established fundamentals of Thermodynamics with Finite Speed (TFS) were shown in their analyses. The idea of TFS is going ahead, and Agrawal and Menon [19] calculated the cycle time of each branch of a finite speed Carnot engine on the assumption that the finite speeds of the piston on the four branches are equal. Based on this cycle model, they investigated the relation between power and efficiency of a finite speed Carnot engine, and reached the conclusion that the temperature ratio and the isothermal expansion ratio at maximum dimensionless power were not affected by the finite speed of the piston. Furthermore, Agrawal [20] combined the two models of achieving finite power output in a Carnot engine, *i.e.*, the model of finite rate of heat transfer established by Curzon and Ahlborn [3] and the model of finite speed of piston established by Agrawal and Menon [19], to study the behavior of power output in terms of isothermal expansion ratio and the temperature differences presented at the hot- and cold-reservoir branches.

Obviously, Agrawal and Menon's [19] and Agrawal's [20] work greatly enriched the theory of finite time thermodynamics. Their cycle models can be extended further. The finite speeds of the piston on adiabatic branches are much larger than those in heating and cooling branches in practical heat engines. Therefore, Agrawal and Menon's assumption that the finite speeds of the piston on the four branches are equal can be improved. On the basis of Refs. 3, 19, and 20, this paper will fix the cycle period and assume that the finite speeds of the piston on the four branches are unequal. Then, the analytical formula relating power and efficiency of the cycle is derived. The focus of this paper is to search the optimal ratios of the finite piston speeds on the four branches for a fixed cycle period. Moreover, the effects of the temperature ratio of the heat reservoirs on the dimensionless power versus efficiency of the cycle and isothermal expansion ratio are obtained by numerical examples.

2. Cycle model

A finite speed endoreversible Carnot heat engine cycle model between an infinite heat source at temperature T_H and an in-

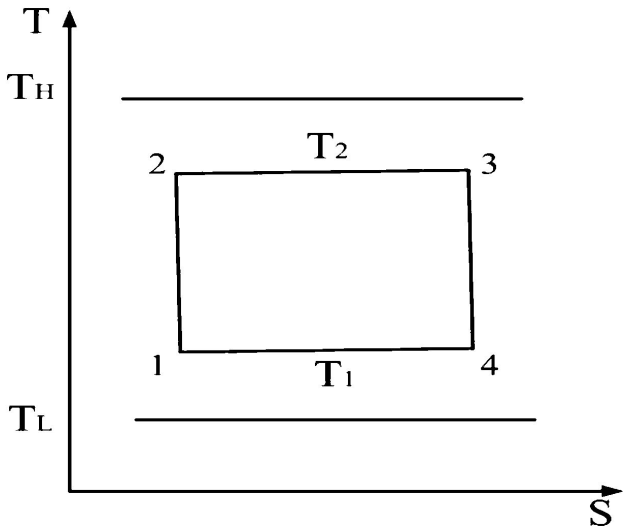

FIGURE 1. Cycle model.

finite heat sink at temperature T_L is shown in Fig. 1 [3,20]. In this T-s diagram, the processes between 1 and 2, as well as between 3 and 4, are two adiabatic branches; the process between 2 and 3 is one isothermal heating branch, and the process between 4 and 1 is one isothermal cooling branch. The volume V and temperature T of the gas at the four corners of the cycle are labelled by the corresponding suffixes. The temperature ratio of the heat reservoirs is defined as $\tau = T_H/T_L$. The following relations are well-known:

$$\frac{T_3}{T_4} = \frac{T_2}{T_1} = T_3^* \tag{1}$$

$$\frac{V_1}{V_2} = \frac{V_4}{V_3} = \left(\frac{T_3}{T_4}\right)^{\frac{1}{k-1}} = (T_3^*)^{\frac{1}{k-1}} \tag{2}$$

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} = V_3^* \tag{3}$$

where T_3^* is the isothermal temperature ratio, V_3^* is the isothermal expansion ratio, and k is the ratio of the specific heats.

Unlike the cycle model in Refs. 19 and 20, one can assume that the finite speeds of the piston on the four branches are constants but unequal. The change rates of the volume on the four branches $1 \rightarrow 2$, $2 \rightarrow 3$, $3 \rightarrow 4$ and $4 \rightarrow 1$, that is, the product of the finite speed of the piston and the cross-sectional area of the piston, are defined as u_1, u_2, u_3 and u_4, respectively. Furthermore, the change rates of the volume on the two adiabatic branches are assumed to be equal, that is, $u_1 = u_3$. Also, two ratios $x = u_1/u_2$ and $y = u_1/u_4$are defined. x and y are the distributions of the finite speeds of the piston on the four branches. The times taken by the piston to move on the four branches $1 \rightarrow 2$,$2 \rightarrow 3$, $3 \rightarrow 4$ and $4 \rightarrow 1$ are defined as t_1, t_2, t_3 and t_4, respectively. They are

$$t_1 = \frac{V_1 - V_2}{u_1} = \frac{V_2}{u_1}(\frac{V_1}{V_2} - 1) = \frac{V_2}{u_1}\left[(T_3^*)^{\frac{1}{k-1}} - 1\right] \tag{4}$$

$$t_2 = \frac{V_3 - V_2}{u_2} = \frac{V_2}{u_1}\frac{u_1}{u_2}(\frac{V_3}{V_2} - 1) = \frac{xV_2}{u_1}(V_3^* - 1) \tag{5}$$

$$t_3 = \frac{V_4 - V_3}{u_3} = \frac{V_2}{u_1}\frac{V_3}{V_2}(\frac{V_4}{V_3} - 1) = \frac{V_2}{u_1}V_3^*\left[(T_3^*)^{\frac{1}{k-1}} - 1\right] \tag{6}$$

$$t_4 = \frac{V_4 - V_1}{u_4} = \frac{V_2}{u_1}\frac{u_1}{u_4}\frac{V_4}{V_3}(\frac{V_3}{V_2} - 1)$$
$$= \frac{yV_2}{u_1}(T_3^*)^{\frac{1}{k-1}}(V_3^* - 1) \tag{7}$$

3. Performance analysis

According to the heat transfer between the working fluid and heat reservoirs and the properties of working fluid, the rate of heat transfer (Q_H) supplied by the heat source, and the rate of heat transfer (Q_L) released to the heat sink are, respectively, given by

$$Q_H = \alpha F_1 (T_H - T_2) t_2 = mRT_2 \ln V_3^* \tag{8}$$

$$Q_L = \beta F_2 (T_1 - T_L) t_4 = mRT_1 \ln V_3^* \tag{9}$$

where α and β $(kW/m^2 \cdot K)$ are the heat transfer coefficients between the working fluid and heat reservoirs, F_1 and F_2 (m^2) are the heat transfer surface areas of the heat reservoirs, m (kg) is the mass of the working fluid, and R $[kJ/(kg \cdot K)]$ is the gas constant.

Combining Eqs. (1) with (8) and (9) gives

$$W = Q_H - Q_L = mR(T_2 - T_1)\ln V_3^*$$
$$= \frac{T_3^* - 1}{T_3^*} mRT_2 \ln V_3^* \tag{10}$$

Consider the endoreversible cycle $1-2-3-4-1$. Applying the second law of thermodynamics gives

$$\Delta S = Q_H/T_2 - Q_L/T_1 = 0 \tag{11}$$

From Eqs. (1), (10) and (11), one has

$$T_3^* = T_2/T_1 = 1/(1 - W/Q_H) = 1/(1-\eta) \tag{12}$$

where η is the thermal efficiency of the cycle. Combining Eqs. (5), (8) with (12) gives

$$T_1 = \frac{x\alpha F_1 T_H (V_3^* - 1)(1-\eta)}{(u_1 mR \ln V_3^*)/V_2 + x(V_3^* - 1)\alpha F_1} \tag{13}$$

$$T_2 = \frac{x\alpha F_1 T_H (V_3^* - 1)}{(u_1 mR \ln V_3^*)/V_2 + x(V_3^* - 1)\alpha F_1} \tag{14}$$

From Eqs. (7), (9), (12) and (13), one has

$$\frac{mR\ln V_3^*}{\beta F_2 y(1-\eta)^{\frac{-1}{k-1}}(V_3^* - 1)V_2/u_1}$$
$$= 1 - \frac{(u_1 mR \ln V_3^*)/V_2 + x(V_3^* - 1)\alpha F_1}{\alpha F_1 x\tau (V_3^* - 1)(1-\eta)} \tag{15}$$

Setting the cycle period t as a constant, from Eqs. (4)-(7), one has

$$t = t_1 + t_2 + t_3 + t_4 = \frac{V_2}{u_1}\left[(1-\eta)^{\frac{-1}{k-1}} - 1 + x(V_3^* - 1) + V_3^*(1-\eta)^{\frac{-1}{k-1}} - V_3^* + y(1-\eta)^{\frac{-1}{k-1}}(V_3^* - 1)\right] \tag{16}$$

From Eqs. (15) and (16), one has

$$x = \frac{\alpha F_1\left[\tau - (1-\eta)^{-1}\right]\left[\frac{u_1 t - (V_3^*+1)(1-\eta)^{\frac{-1}{k-1}}V_2}{u_1} + \frac{(V_3^*+1)V_2}{u_1}\right] + \frac{(mR\ln V_3^*)}{(1-\eta)} - \frac{(\alpha F_1 \tau mR\ln V_3^*)}{(\beta F_2)} + \sqrt{H}}{2\alpha F_1\left[\tau - (1-\eta)^{-1}\right]\frac{(V_3^*-1)V_2}{u_1}} \tag{17}$$

$$y = \frac{\frac{t(1-\eta)^{\frac{1}{k-1}}u_1}{V_2} - (V_3^*+1)\left[1 - (1-\eta)^{\frac{1}{k-1}}\right]}{V_3^* - 1} - \frac{\alpha F_1\left[\tau - (1-\eta)^{-1}\right]\left[\frac{u_1 t - (V_3^*+1)(1-\eta)^{\frac{-1}{k-1}}V_2}{u_1} + \frac{(V_3^*+1)V_2}{u_1}\right] + \frac{(mR\ln V_3^*)}{(1-\eta)} - \frac{(\alpha F_1 \tau mR\ln V_3^*)}{(\beta F_2)} + \sqrt{H}}{2\alpha F_1\left[\tau - (1-\eta)^{-1}\right]\frac{(V_3^*-1)(1-\eta)^{\frac{-1}{k-1}}V_2}{u_1}} \tag{18}$$

where

$$\begin{aligned} H = &\left\{\alpha F_1\left[\tau - (1-\eta)^{-1}\right]\left[t - (V_3^*+1)(1-\eta)^{\frac{-1}{k-1}}V_2/u_1 + (V_3^*+1)V_2/u_1\right] + (mR\ln V_3^*)/(1-\eta)\right.\\ &\left. - (\alpha F_1 \tau mR\ln V_3^*)/(\beta F_2)\right\}^2 - 4\alpha F_1\left[\tau - (1-\eta)^{-1}\right]\\ &\times\left[t - (V_3^*+1)(1-\eta)^{\frac{-1}{k-1}}V_2/u_1 + (V_3^*+1)V_2/u_1\right](mR\ln V_3^*)/(1-\eta) \end{aligned} \tag{19}$$

Combining Eqs. (10), (12), (14) with (17) gives the power output

$$P_W = \frac{W}{t} = \frac{T_H I\eta\ln V_3^*}{t\left[2\left[\tau - (1-\eta)^{-1}\right]\ln V_3^* + I/(mR)\right]} \tag{20}$$

where

$$\begin{aligned} I = &\alpha F_1\left[\tau - (1-\eta)^{-1}\right]\left[t - (V_3^*+1)(1-\eta)^{\frac{-1}{k-1}}V_2/u_1 + (V_3^*+1)V_2/u_1\right]\\ &+ (mR\ln V_3^*)/(1-\eta) - (\alpha F_1 \tau mR\ln V_3^*)/(\beta F_2) + \sqrt{H} \end{aligned} \tag{21}$$

From Eq. (20), one has the dimensionless power output

$$\begin{aligned} p = &\frac{P_W}{\alpha F_1 T_H}\\ = &\frac{I\eta\ln V_3^*}{\alpha}F_1 t\left[2\left[\tau - (1-\eta)^{-1}\right]\ln V_3^* + I/(mR)\right] \end{aligned} \tag{22}$$

From Eq. (22), one can see that the dimensionless power (p) is related to the efficiency (η) of the cycle, the isothermal expansion ratio (V_3^*), the starting value (V_2/u_1), the cycle period (t), the temperature ratio (τ) of the heat reservoirs, and the products (αF_1 and βF_2) of heat transfer coefficient and heat transfer surface area. When $t_1 > 0$, $t_2 > 0$, $t_3 > 0$, $t_4 > 0$, $x > 0$, $y > 0$,$V_3^* > 1$,and $H \geq 0$, one may find that for a fixed V_2/u_1, t, τ, αF_1 and βF_2, there exist an efficiency (η_p) and an isothermal expansion ratio (V_{3p}^*) corresponding to the maximum power output. Substituting η_p and V_{3p}^* into Eqs. (17) and (18) yields the corresponding x_p andy_p at maximum power output point, which are also the optimal distributions of the finite piston speeds on the four branches.

4. Numerical example

To illustrate the preceding analysis, a numerical example is provided. In the calculations, it is set that T_L=320K, τ=2.5, m=4.553 × 10^{-4}kg[21], t=100ms, V_2/u_1=15ms, k=1.4, R=0.287kJ/(kg·K), and αF_1=βF_2=1.2 × 10^{-3}kW / K. Moreover, when one discusses the effects of τ and V_2/u_1 on the dimensionless power output versus efficiency of the cycle and isothermal expansion ratio, τ is set as 2.4, 2.5, and 2.6, respectively; V_2/u_1 is set as 14 ms,15ms, and 16 ms, respectively.

Figure 2 shows the effect of the temperature ratio of the heat reservoirs on the dimensionless power output versus the isothermal expansion ratio with the efficiency $\eta = 0.24$ of the cycle and the starting value $V_2/u_1 = 15ms$. One can see that there exists an optimal isothermal expansion ratio V_{3p}^* corresponding to the maximum power output point. Moreover, for a fixed V_3^*, the dimensionless power output p increases monotonically when τ increases.

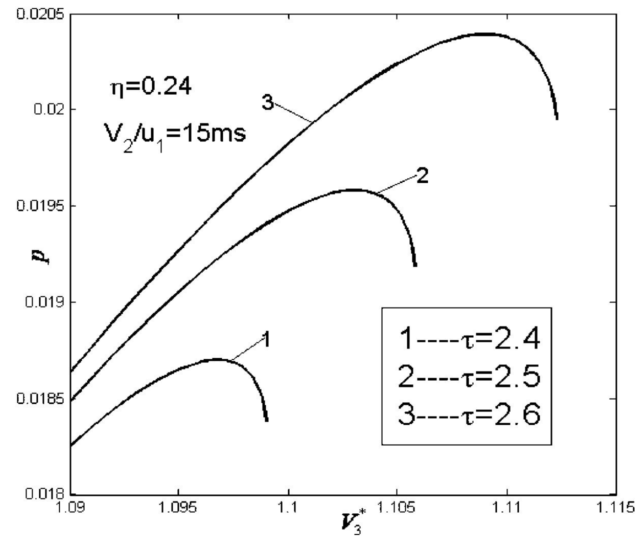

FIGURE 2. Effect of τ on the dimensionless power output versus the isothermal expansion ratio.

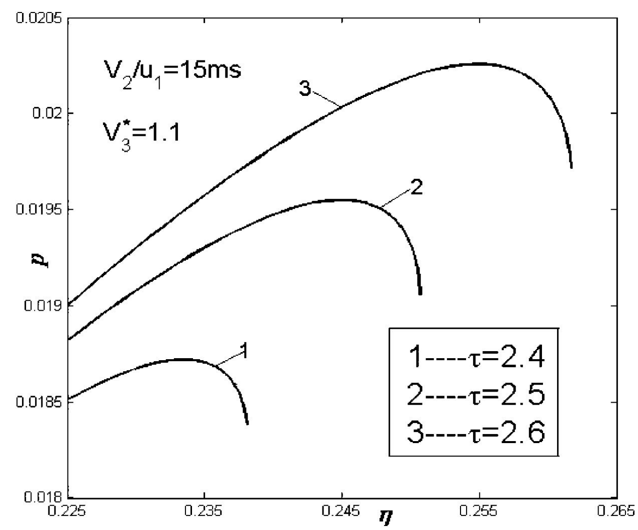

FIGURE 3. Effect of τ on the dimensionless power output versus efficiency of the cycle.

Figure 3 shows the effect of the temperature ratio of the heat reservoirs on the dimensionless power output versus the efficiency of the cycle with the isothermal expansion ratio $V_3^* = 1.1$ and the starting value $V_2/u_1 = 15ms$. One can see that there exists an optimal efficiency η_p corresponding to the maximum power output. For a fixed η, the dimensionless power output p increases monotonically when τ increases. Furthermore, numerical calculations show that, with the temperature ratio of the heat reservoirs $\tau = 2.5$ and the starting value $V_2/u_1 = 15ms$, the maximum dimensionless power output is $p_{\max} = 0.0195$, and the corresponding optimal isothermal heating and cooling temperatures of working fluid are $T_2 = 609.18K$ and $T_1 = 459.93K$, the corresponding optimal efficiency of the cycle is $\eta = 0.245$, the corresponding optimal isothermal expansion ratio is $V_3^* = 1.101$. Substituting the corresponding optimal efficiency and optimal isothermal expansion ratio into Eqs. (17) and (18), the corresponding optimal ratios of the finite piston speeds on the four branches become $x = 22.07$ and $y = 11.26$, respectively. The proportions of the change rates of the volume on the four branches are: $u_1 : u_2 : u_3 : u_4 = 22.07 : 1 : 22.07 : 1.96$. This is the major result different from that of Ref. 20.

Figure 4 shows the effect of V_2/u_1 on the dimensionless power output versus the isothermal expansion ratio with $\tau = 2.5$ and $\eta = 0.24$. One can see that, for a fixed V_3^*, the dimensionless power output p decreases monotonically when V_2/u_1 increases. Figure 5 shows the effect of V_2/u_1 on the dimensionless power output versus efficiency of the cycle with $\tau = 2.5$ and $V_3^* = 1.1$. One can see that, for a fixed efficiency η, the dimensionless power output p decreases monotonically when V_2/u_1 increases.

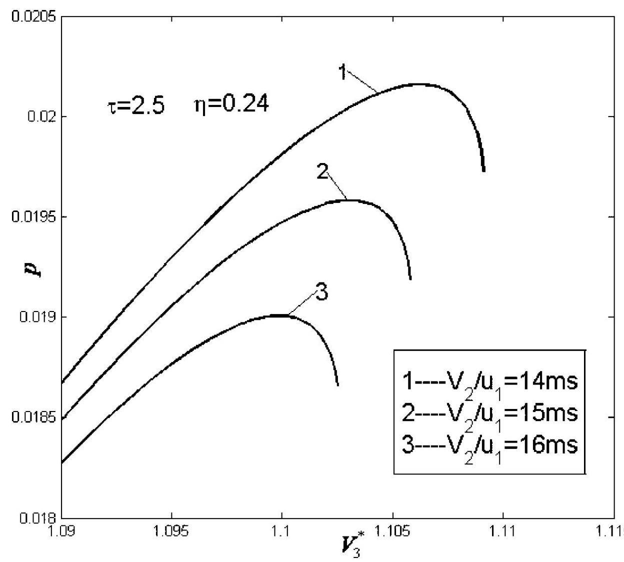

FIGURE 4. Effect of V_2/u_1 on the dimensionless power output versus the isothermal expansion ratio.

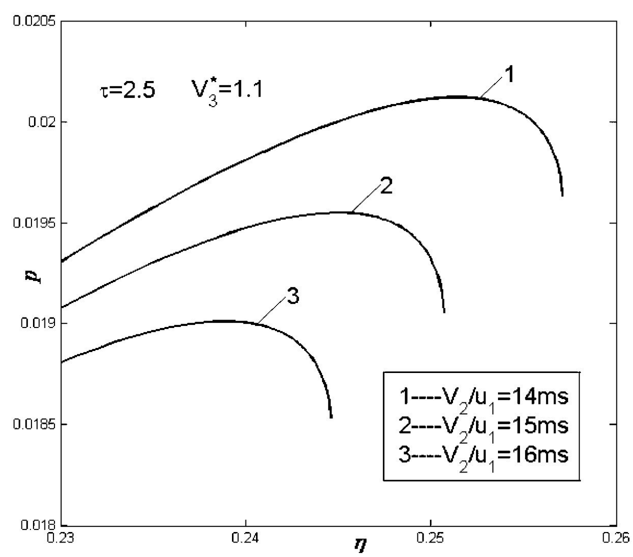

FIGURE 5. Effect of V_2/u_1 on the dimensionless power output versus efficiency of the cycle.

Nomenclature	
F	heat transfer surface area (m^2)
k	ratio of the specific heats
m	mass of the working fluid (kg)
W	work output of the cycle (kJ)
P_W	power output of the cycle (kW)
P	dimensionless power output
Q	rate of heat transfer (kJ)
Δ_S	entropy generation of the cycle (kJK^{-1})
t	cycle period (s)
$t_i(i = 1, 2, 3, 4)$	time taken by the piston to move on each branche (s)
T	temperature (K)
T_3^*	isothermal temperature ratio
u	product of the finite speed of the piston and the cross-sectional area of the piston (m^3/s)
V	volume (m^3)
V_3^*	isothermal expansion ratio
x, y	distributions of the finite speeds of the piston
Greek symbols	
α	heat-transfer coefficient (kW/(K$\cdot$ m^2))
η	efficiency
τ	temperature ratio of the heat reservoirs
Subscripts	
H	hot side/heat source
L	cold side/heat sink
P	power
1, 2, 3, 4	state points of the model cycle

5. Conclusion

On the basis of the model of finite piston speed on the four branches of Carnot engine in Refs. 19 and 20 and the model of finite rate heat transfer of endoreversible Carnot heat engine in Refs. 3 and 20, this paper extends cycle model by assuming that the finite speeds of the piston on the four branches are unequal and combining two approaches of calculating heat absorbed and heat rejected together. The analytical formula between power output and efficiency of the cycle model are derived for a fixed cycle period. The analysis and optimization of the cycle model are carried out in order to investigate the optimal ratios of the finite piston speeds on the four branches. The effects of the temperature ratio of the heat reservoirs and the starting value V_2/u_1 on the dimensionless power output versus efficiency of the cycle and isothermal expansion ratio are obtained by numerical examples. One can see that the work of optimizing the ratios of the finite piston speeds of the cycle model is necessary, and can provide some theoretical guidelines for the design of practical heat engines.

Acknowledgments

This paper is supported by Program for New Century Excellent Talents in University of P.R. China (Project No: NCET-04-1006) and the Foundation for the Author of National Excellent Doctoral Dissertation of P.R. China (Project No: 200136). The authors wish to thank the reviewers for their careful, unbiased and constructive suggestions, which led to this revised manuscript.

$*$. To whom all correspondence should be addressed: e-mail address: lgchenna@yahoo.com, lingenchen@hotmail.com; Fax: 0086-27-83638709 Tel: 0086-27-83615046.

i. D.C. Agrawal and V.J. Menon, *Eur. J. Phys.* **30** (2009) 2951.

ii. Curzon and Ahlborn, *Am. J. Phys.* **43** (1975) 22.

1. I.I. Novikov, *Atommaya Energiya* **3** (1957) 409.
2. P. Chambdal, *Les Centrales Nucleases* (Paris: Armand Colin, 1957) p. 41.
3. F.L. Curzon and B. Ahlborn, *Am. J. Phys.* **43** (1975) 22.
4. B. Andresen, *Finite Time Thermodynamics. Physics Laboratory II* (University of Copenhagen, 1983).
5. A. De Vos, *Endoreversible Thermodynamics of Solar Energy Conversion* (Oxford: Oxford University Press, 1992).
6. A. Bejan, *J. Appl. Phys.* **79** (1996) 1191.
7. M. Feidt, *Thermodynamique et Optimisation Energetique des Systems et Procedes*, 2nd Ed. (Paris: Technique et Documentation, Lavoisier, 1996).

8. R.S. Berry *et al.*, *Thermodynamic Optimization of Finite Time Processes* (Chichester: Wiley, 1999).
9. L. Chen, C. Wu, and F. Sun, *J. Non-Equilib. Thermodyn.* **24** (1999) 327.
10. L. Chen and F. Sun, *Advances in Finite Time Thermodynamics: Analysis and optimization* (New York: Nova Science Publishers, 2004).
11. W. Muschik and K.H. Hoffmann, *J. Non-Equilib. Thermody.* **31** (2006) 293.
12. M. Feidt, *Int. J. Exergy* **5** (2008) 500.
13. S. Petrescu, M. Costea, M. Feidt, and O. Malancioiu, *Proc. Birac, Bucuresti, Romania* (2000) 22.
14. S. Petrescu, M. Costea, and M. Feidt, *Optimization of a Carnot cycle engine by using finite speed thermodynamics and the direct method* Proc. ECOS'2001, Istanbul, Turkey, July, 2001, Vol. I, pp. 151-161.
15. S. Petrescu, M. Feidt, and M. Costea, *Optimization of Carnot cycle engine with internal and external irreversibilities by using thermodynamics with finite speed* Proc. of the 8th Francophone Congress Recent Progress on Process Design, CFGP' 2001, Nancy, A. Storck, J. Boudrant, T. Tondeur, eds., Tech&Doc, Paris, France, Vol. 15-2001, No. 83, pp. 149-156, 2001.
16. S. Petrescu, M. Costea, C. Harman, and M. Costea, *Optimization of the irreversible Carnot cycle engine for maximum efficiency and maximum power through use of finite speed thermodynamic analysis* Proc. ECOS'2002, Berlin, Germany, July 3-5, 2002, Vol. II, pp. 1361-1368.
17. S. Petrescu, C. Harman, M. Costea, and M. Feidt, *Thermodynamics with finite speed versus thermodynamics in finite time in the optimization of Carnot cycle* Proc. of the 6th ASME-JSME Thermal Engineering Joint Conference, Hawaii, March16-20, 2003.
18. M. Feidt, M. Costea, C. Petre, and S.Petrescu *Appl. Thermal Engineering* **27** (2007) 829.
19. D.C. Agrawal and V.J. Menon, *Eur. J. Phys.* **30** (2009) 295.
20. D.C. Agrawal, *Eur. J. Phys.* **30** (2009) 587.
21. M. Mozurkewich and R.S. Berry *J. Appl. Phys. 53* (1982) 34.