Signatures for shape-phase transitions in the rare-earth nuclei, in the evolution of single-particle spectra and two-particle transfer-intensities

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The rare-earth Nd, Sm, Gd and Dy nuclei are well known to undergo a shape-phase transition around $N \sim 90$ from vibrational to rotational behaviour - or, correspondingly - from a spherical nuclear shape to an axial-symmetric deformed shape. This can experimentally be verified by, for example, the evolution of the $R_{4/2} = E(4_1^+)/E(2_1^+)$ ratio, or the evolution of the quadrupole moment, Q_2. Recently, the study of nuclear phase-shape transitions has gained considerable interest, since the introduction of exact algebraic solutions for the critical points of many of the different phase-shape transitions that are possible in the atomic nucleus. In this contribution, in the first part, we investigate the microscopic underlying mechanism that drives the rare-earth isotopes towards deformation, studying the evolution of their proton and neutron single-particle spectra within the Relativistic Hartree-Bogoliubov model. In the second part, within the Interacting Boson Model and the framework with boson coherent states, and in the light of renewed interest in experiments on two-particle transfer-reactions, we study the evolution of the transfer spectroscopic intensities as a possible signature of shape-phase transitions.

Keywords: Nuclear structure models and methods; quantum phase transitions.

Los núcleos de las tierras raras Nd, Sm, Gd, y Dy son bien conocidos por poseer una transición de fase cerca de $N \approx 90$ de un comportamiento vibracional a otro rotacional, o correspondientemente, de una forma esférica a aquella de forma deformada axialmente. Esto puede ser verificado experimentalmente por la evolución de la razón $R_{4/2} = E(4_1^+)/E(2_1^+)$, o la evolución del mometo cuadrupolar Q_2. Recientemente, el estudio de las transiciones de fase han ganado considerable interés desde la introducción de soluciones algebraicas para los puntos críticos de las diferentes transiciones de fase entre las formas posibles del núcleo. En la primera parte de esta contribución investigamos el mecanismo microscópico subyacente que conduce a los isótopos de las tierras raras hacia la deformación, estudiando la evolución de los espectros de partícula independiente de protones y neutrones a través del modelo Hartree-Bogoliubov relativista. En la segunda parte, utilizando el modelo de bosones interactuantes en el marco de los estados coherentes bosónicos, y considerando el renovado interés en los experimentos sobre reacciones de tranferencia de dos partículas, estudiamos la evolución de la tranferencia de intensidades espectroscópicas como una posible señal de transiciones de fase en la forma del núcleo.

Descriptores: Modelos y métodos de la estructura nuclear; transiciones de fase cuántica.

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1. Introduction

One of the most important notions in physics is the concept of "benchmark". In nuclear physics in particular, models have been conceived (and rewarded with the Nobel prize!) that give analytical solutions for the structure and the excitation spectrum of some ideal nuclear cases: the perfect vibrator and the perfect rotator (for collective nuclei), and the perfect independent-particle nucleus (for magic and near-magic nuclei). Real nuclei, of course, can deviate in important ways from these ideal cases. Nuclei that always have been the hardest to understand, are the so-called "transitional nuclei", on the way from one benchmark to another. The rare-earth Nd, Sm, Gd and Dy isotope series are examples of transitional nuclei: near $N \sim 90$ abruptly going from a spherical or near-spherical nuclear shape to a axial-symmetric deformed one (an effect which is coined "shape transition") - or, correspondingly - from vibrational behaviour to rotational behaviour (coined "phase transition", in analogy with the more familiar phase transitions of macroscopic systems). This sudden change in nuclear structure can be seen experimentally, *e.g.* by the evolution of the $R_{4/2} = E(4_1^+)/E(2_1^+)$ ratio (see Fig. 3 of Ref. 1), or the evolution of the quadrupole moment, Q_2.

Phase-shape transitions have gained much interest lately, because of the recent introduction of some new benchmarks: for the critical points of some of the most common phase-shape transitions that are possible in the atomic nucleus, giving analytical solutions for the excitation spectrum of the critical-point nucleus [2]. The $N \sim 90$ isotones of the Nd, Sm, Gd and Dy nuclei have been proposed as good candidates of the new X(5) benchmark [2].

In Ref. 3, we have studied the evolution of the nuclear shape through the Nd, Sm, Gd and Dy isotope series, constructing Potential Energy Surfaces (PES), based on β_2-constrained calculations within the Relativistic Hartree-Bogoliubov model (RHB). In Sec. 2, we present the evolution of the quadrupole moment through these series of isotopes, based on the calculations of Ref. 3. We reproduce the results for the PES surfaces of Ref. 3 for the Dy series, for comparison with the evolution of the proton and neutron single-particle spectra of the same isotope series, in which we look for the microscopical driving mechanism behind the nuclear shape/phase transition in this region [5].

In Sec. 3, we study the evolution of the two-particle transfer intensity through the series of the Nd, Sm, Gd and Dy isotopes, within the Interacting Boson Model (IBM) and the framework with boson coherent states, to find experimental fingerprints for the nuclear phase/shape transition around $N \sim 90$ (see also Ref. 11).

In Sec. 4, the conclusions of this contribution are presented.

2. Evolution of the proton and neutron single-particle spectra through the rare-earth isotope series

R. Casten, in his Nature Review on nuclear phase/shape transitions, suggested the Nd, Sm, Gd and Dy isotope series as a good example of a first-order transition, going from a spherical nuclear shape towards an axial-symmetric deformed nuclear shape. He presented the evolution of the $R_{4/2} = E(4_1^+)/E(2_1^+)$ ratio to identify the $N{\sim}90$ isotones as the critical point in all 4 series. In Fig. 1, we show the evolution of the quadrupole moment, Q_2 (a direct measure of the nuclear deformation), based on our RHB calculations of Ref. 3, for the same series. A steady increase in the nuclear deformation can indeed be observed from the neutron closed shell at $N{=}82$ towards higher masses. The predicted nuclear deformation is always larger than the experimental data (where available), but is in accordance with other theoretical results within relativistic mean-field models (see, *e.g*. Refs. 6 and 7) and non-relativistic mean field (see, *e.g*. Ref. 8). In Fig. 2, we reproduce the PES surfaces for the Dy isotope series from

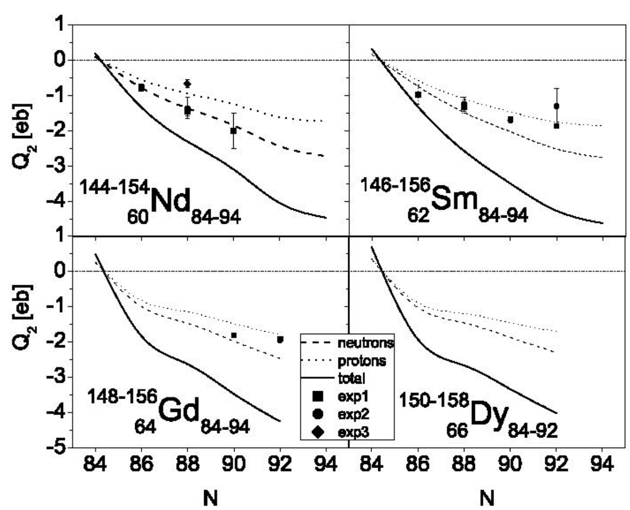

FIGURE 1. The evolution of the quadrupole moment Q_2 in units eb for the Nd, Sm, Gd and Dy isotope series. Proton, neutron and total quadrupole moments are shown, derived from β_2-constrained calculations in the RHB framework from Ref. 3. Experimental results come from [4].

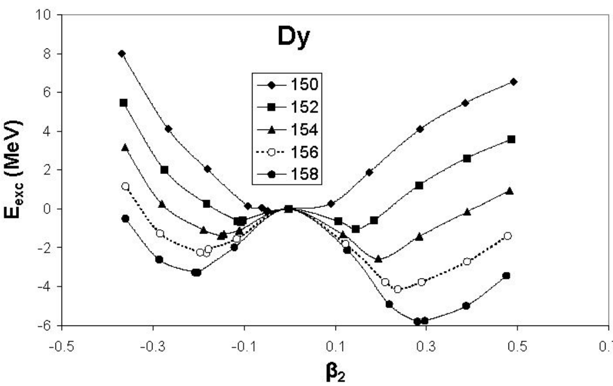

FIGURE 2. Evolution of the PES of the Dy isotope series. The critical point nucleus ^{156}Dy is highlighted. Taken from RHB β_2-constrained calculations of [3].

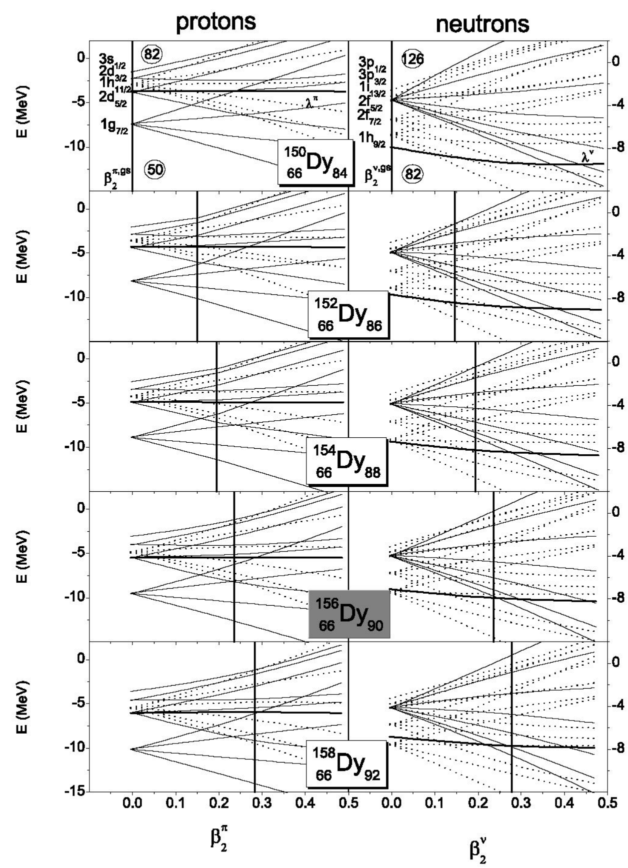

FIGURE 3. Evolution of the proton (left-hand panels) and neutron (right-hand panels) single-particle spectra with mass number for the Dy isotope series, from β_2-constrained calculations in the RHB model in correspondence with the PES surfaces shown in Fig. 2. As the Dy isotopes of interest have spherical or prolate ground states, only results for $\beta_2 \geq 0$ are shown. The ground-state deformation for protons (or neutrons), $\beta_2^{\pi,gs}$ (or $\beta_2^{\nu,gs}$), corresponding to the absolute minima in the PES surfaces of Fig. 2, is indicated by the thick vertical line. The proton (neutron) fermi level, λ^π (λ^ν), is indicated by the thick horizontal line. Results for the proton major shell, $Z = 50 - 82$, and the neutron major shell, $N = 82 - 126$ are shown. The critical-point nucleus ^{156}Dy is highlighted.

Ref. 3. ^{150}Dy is spherical, whereas the isotopes with $A \geq 152$ have two minima in their PES, an oblate and a prolate one. The ground states in all cases for $A \geq 152$ have a prolate deformation. The critical-point nucleus ^{156}Dy is highlighted in Fig. 2.

R. Casten contributes the increase in nuclear deformation to the obliteration of the $Z=64$ subshell gap, that separates the proton $\pi 1h^{-}_{11/2}$ intruder single-particle orbital from the rest of the proton space, for neutron numbers $N \geq 90$, making available a much larger proton single-particle space that drives the nuclear deformation. Starting from $N=90$, the neutrons would be starting to fill up the neutron $\nu 1h^{-}_{9/2}$ single-particle orbital, that overlaps well with the proton $\pi 1h^{-}_{11/2}$ orbital, allowing the latter to come down in the single-particle spectrum (Federman-Pittel mechanism).

The above arguments are however based on the spherical nuclear shell-model, and lose their strict validity for deformed nuclei. More in particular, the specific proton and neutron single-particle levels that play a role, and the order in which they come into play, will change when deformation becomes increasingly more important. In Fig. 3, the variation in energy of the proton and neutron single-particle orbitals with the quadrupole deformation β_2 is given for the whole Dy isotope series. The thick vertical lines labeled with $\beta_2^{\pi,gs}$ ($\beta_2^{\nu,gs}$) indicate the ground-state deformation for the protons (neutrons) for each isotope, corresponding to the minima in the PES surfaces of Fig. 2. The thick horizontal lines are the fermi surface for the protons (or neutrons), λ^{π} (λ^{ν}).

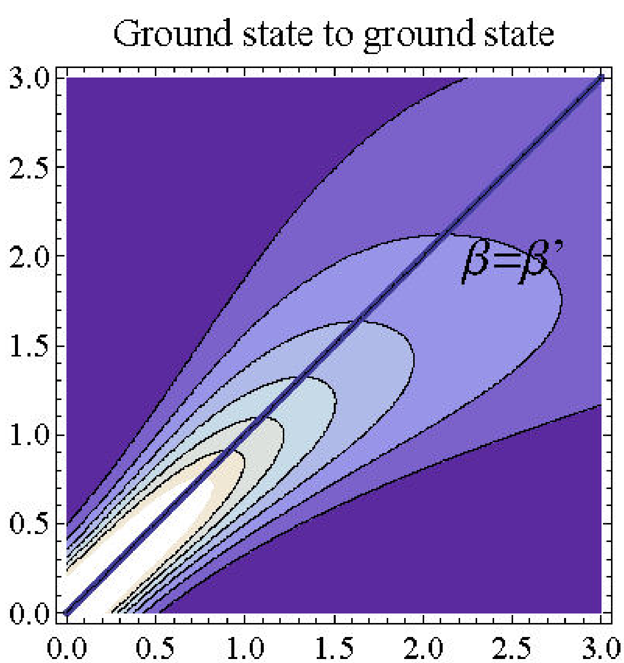

FIGURE 4. Contour plot of the dependence of the intensity $I(\beta, \beta', N)$ for two-particle (s-boson) transfer between the ground states of a nucleus with N and with $N+1$ bosons (square of matrix element 1) on the quadrupole deformation of the initial (β, horizontal axis) and final (β', vertical axis) nucleus. A line is drawn for $\beta = \beta'$. Results are for $N = 10$.

The intersection of both lines gives the position of the highest occupied single-particle orbital (for protons or neutrons) in the nuclear ground state. The results for the Nd, Gd and Sm isotope series will be given elsewhere [5], but are very similar to the results for the Dy series. The proton single-particle orbitals that play a role in the Dy isotopes mentioned (left-hand panels in Fig. 3) are the $\pi 1g^{+}_{7/2}$, $\pi 2d^{+}_{5/2}$ and $\pi 1h^{-}_{11/2}$ orbitals. For the neutrons (right-hand panels), up to $N<90$, the $\nu 1h^{-}_{9/2}$, $\nu 2f^{-}_{7/2}$ and $\nu 2f^{-}_{5/2}$ single-particle levels come into play subsequently. They interact, according to the Federman-Pittel mechanism, with especially the former two of the proton single-particle levels. Starting from $N = 90$, corresponding to the critical-point isotope ^{150}Dy (highlighted in Fig. 3), neutrons start to occupy the $1i^{+}_{13/2}$ intruder single-particle orbital. The interplay of this neutron single-particle orbital with the proton $\pi 1h^{-}_{11/2}$ orbital gives the deformation a boost. As we get very similar results also for the Nd, Sm and Gd isotopes series, it would seem that it is the interaction between this specific proton and neutron single-particle orbitals that is responsible for the phase/shape transition in this region of the nuclear chart.

3. Evolution of the intensity of two-particle transfer through the rare-earth isotope series

In the light of renewed interest in experiments on two-particle transfer-reactions, especially with radioactive beams [9], we want to study the evolution of the transfer spectroscopic intensities as a possible signature of shape/phase transitions in atomic nuclei. The two-particle transfer process can be described by pair creation and annihilation operatores that have a straightforward expression within the Interacting Boson Model (IBM). More in particular, up to first order, the transfer of a nucleon pair with $L = 0$ can be described as the transfer of an s-boson. Transfer intensities from ground state to ground state and from the ground state to excited states can be calculated analytically for three special cases or "limits" of the IBM model: U(5) (vibrational), SU(3) (rotational) and O(6) ($\gamma-$unstable) [10]. In between the limits, for the transitional nuclei we are interested in, calculations should be done numerically. The transfer of a nucleon pair with $L = 2$ in good approximation can be described as the transfer of a d-boson, but in this case transfer intensities can be calculated analytically only in a very limited number of cases [10].

It is well known that - in a macroscopic picture - in the transfer process of two nucleons with $L = 0$, it is predominantly the ground state that is populated. Considerable strength only goes to excited states in the case of a change in the shape of the nuclear ground state (see the first of the papers cited in [9]). It is thus instructive to study two-particle transfer also in the framework with boson coherent states, that gives the geometrical interpretation of the IBM: U(5) (spherical), SU(3) (axial-symmetric deformed) and O(6) ($\gamma-$ unstable). In the case of s-boson transfer,

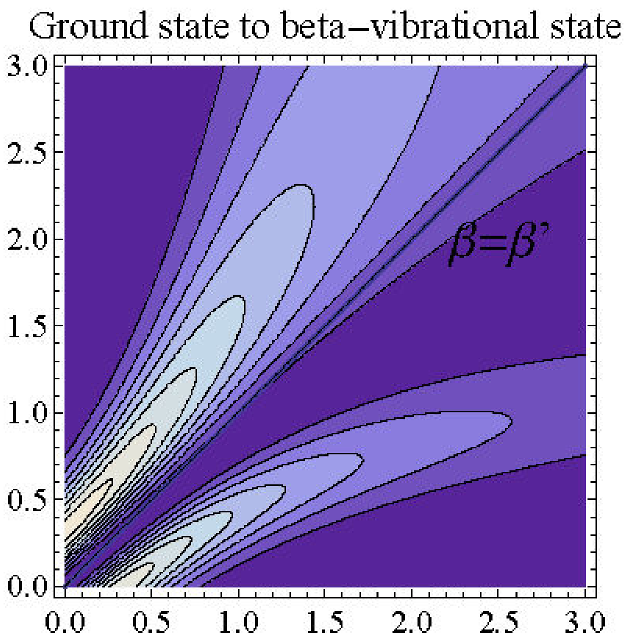

FIGURE 5. Contour plot of the dependence of the intensity $I(\beta, \beta', N)$ for two-particle (s-boson) transfer between the ground state of a nucleus with N and the beta-vibrational state with $N+1$ bosons (square of matrix element 2) on the quadrupole deformation of the initial (β, horizontal axis) and final (β', vertical axis) nucleus. A line is drawn for $\beta = \beta'$. Results are for $N = 10$.

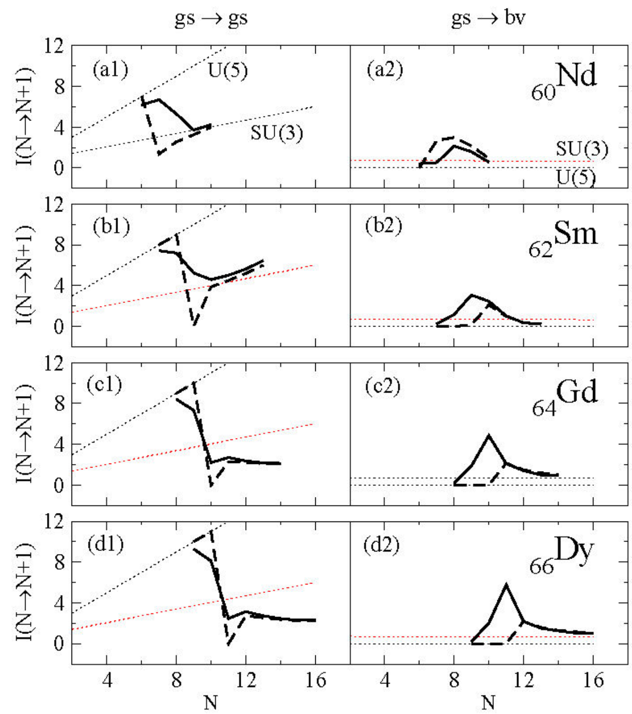

FIGURE 6. Two-particle transfer-intensities within the IBM (full lines) and the framework with boson coherent states (broken lines) for the $^{144-154}$Nd (with $N_\pi = 5$ proton bosons and $N_\nu = 1-6$ neutron bosons), $^{146-160}$Sm ($N_\pi = 6$ and $N_\nu = 1$–8), $^{148-162}$Gd ($N_\pi = 7$ and $N_\nu = 1-8$), and $^{150-166}$Dy ($N_\pi = 8$ and $N_\nu = 1-9$) isotope chains, displayed versus the total number of bosons N (with $N = N_\pi + N_\nu$). In the panels on the left-hand side (panels (a1), (b1), (c1) and (d1)) results for the gs → gs transfer are shown, whereas in the panels on the right-hand side (panels (a2), (b2), (c2) and (d2)) results for the gs → bv transfer are shown. Results for the Nd, Sm, Gd and Dy series are compared with predictions for the U(5) and SU(3) IBM-limits (dotted lines).

we were able to derive simple algebraic expressions for the matrix elements of two-particle transfer in function of the quadrupole deformation of the initial and final nucleus, *i.e.* (β, γ) and (β', γ') (the transfer intensity is then given by the square of the matrix element) [11]. For a transfer between the ground-state bands of a nucleus with N bosons and a nucleus with $N+1$ bosons we get,

$$\left\langle N+1; gs(\beta', \gamma') || s^\dagger || N; gs(\beta, \gamma) \right\rangle = \sqrt{N+1} \frac{(1 + \beta\beta' \cos(\gamma - \gamma'))^N}{\sqrt{(1+\beta'^2)^{N+1}(1+\beta^2)^N}}, \tag{1}$$

and for the transfer between the ground-state band of a nucleus with N bosons and the beta-vibrational band of a nucleus with $N+1$ bosons,

$$\left\langle N+1; bv(\beta', \gamma') || s^\dagger || N; gs(\beta, \gamma) \right\rangle = \frac{(1 + \beta\beta' \cos(\gamma - \gamma'))^{N-1}}{\sqrt{(1+\beta'^2)^{N+1}(1+\beta^2)^N}} \Big(N(\beta \cos(\gamma - \gamma') - \beta') - \beta'(1 + \beta\beta' \cos(\gamma - \gamma')) \Big). \tag{2}$$

The study of d-boson transfer is in preparation [12]. Contour plots of the above expressions for the transfer intensity in function of the quadrupole deformation of the initial and final nucleus, β and β' respectively, give exactly what is expected: pair transfer between the ground states band of two nuclei is only possible for similar ground-state deformations, $\beta \approx \beta'$ (see Fig. 4), whereas the transfer strengh will necessarily go to excited states when the ground-state shape of the initial and final nucleus differ (see Fig. 5 for the case of transfer between a ground-state band and a beta-vibrational band).

In Ref. 13, a systematic study of the spectroscopy of the Nd, Sm, Gd and Dy isotope series was performed in the IBM. In Fig. 6, the same parameters are used to calculate the intensity for two-particle transfer with $L = 0$ to the ground state (left-hand panels) and to the band-head of the beta-vibrational band (right-hand panels) for the isotope series of interest, within the IBM model, and within the framework with boson coherent-states, using the above algebraic formulae (see Eqs. (1) and (2) - for details see Ref. 11). Results for the isotope series are compared with predictions for two of the limits of the IBM, the U(5) limit (spherical vibrating nucleus) and the SU(3) limit (axial-symmetric deformed rotor). It can be seen that the Nd, Sm, Gd and Dy series perform a transition from the U(5) to the SU(3) limit, showing a discontinuity around neutron number $N \sim 90$. We suggest the

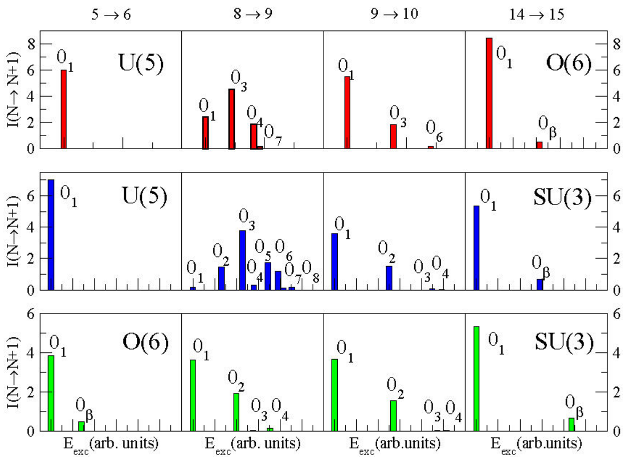

FIGURE 7. The fragmentation of the 2-particle transfer intensity in IBM numerical calculations is shown for the U(5)-O(6) transition (top row), the U(5)-SU(3) transition (the transition performed by the Nd, Sm, Gd and Dy series of fig. **??**) (middle row), and the U(5)-SU(3) cross-over (bottom row). Whereas near the dynamical limits U(5), O(6) and SU(3) (for N=5→6 and 14→15) transfer only occurs between ground states or the beta-vibrational excited 0^+_β, near the critical or cross-over point $N=9$, the transfer intensity is fragmented over a large number of excited 0^+ states and it is not obvious to identify the beta-vibrational or double-beta vibrational state.

appreciable population of excited excited 0^+ states in the $L=0$ two-particle process, in correspondence with a loss of intensity in the transfer to the ground state, as a signature of the occurence of a shape transition.

Typical for the limits in the IBM is that in s-boson ($L=0$) transfer only the ground state can be populated (in the U(5) limit), or the ground state and the first (or second) excited state (SU(3) and O(6) limit) [10]. In Ref. 11, we found a considerable fragmentation of the transfer strength to a large number of excited states near the critical point of the phase transition in the rare-earth region (see Fig. 7).

In some series of isotopes, there is some disagreement about whether the evolution in the nuclear ground-state shape that is observed experimentally corresponds to a shape transition (the gradual or sudden change of the nuclear ground-state shape through a series of isotopes), or a phenomenon of shape coexistence (the coexistence of two or more distinct nuclear shapes in the low-energy spectrum through the whole series of isotopes). What the Pt isotopes is concerned, the authors of Ref. 14 support the coexistence interpretation, whereas the authors of Ref. 15 suggest an explanation with shape transition. We believe that studying the evolution of the two-particle transfer intensity through the Pt isotope series might point out the correct interpretation. More in particular, in the case of a shape transition in the series of Pt isotopes, probably a large fragmentation of the transfer strength near the critical point would be observed (compare with the fragmentation of the transfer strength near the critical point for the Nd, Sm, Gd and Dy series in Fig. 7). In the case of a coexistence of two distinct shapes in the low-energy spectrum throughout the whole Pt isotope series, we would expect a population of only the ground state, except for those isotopes where the ground-state shape and the low-energy excited second shape switch order, there we would expect a population of the ground state and an excited state. We are planning to study the two-particle transfer strength through the series of Pt isotopes, within the IBM model and the framework with boson coherent states, in a forthcoming article [16].

4. Conclusions

It is well known that the Nd, Sm, Gd and Dy isotope series of the rare-earth region perform a phase/shape transition from a spherical nuclear shape to an axial-symmetric deformed one, which can be seen in, e.g., the evolution of the $R_{4/2} = E(4_1^+)/E(2_1^+)$ ratio (see Fig. 3 of Ref. 1), the evolution of the quadrupole moment and the evolution of the minima in PES surfaces in mean-field calculations (see Figs. 1 and 2) of Sec. 2, or the evolution of the two-particle transfer intensity (see Fig. 6 of Sec. 3). We believe that the underlying microscopical mechanism of the phase transition in this region is the interplay of the proton $\pi 1h^-_{11/2}$ single-particle orbital with the neutron $1i^+_{13/2}$ single-particle orbital (Federman-Pittel mechanism), the latter orbital coming into play starting from $N \geq 90$. In fact, the $N=90$ isotones in this region have been suggested as good examples of the X(5) critical-point solution.

Acknowledgments

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