Lepton flavour violating processes in an S_3-symmetric model

A. Mondragón, M. Mondragón, and E. Peinado
Instituto de Física, Universidad Nacional Autónoma de México,
Apartado Postal 20-364, México, D.F. 01000 México.

Recibido el 18 de marzo de 2008; aceptado el 15 de junio de 2008

A variety of lepton flavour violating effects related to the recent discovery of neutrino oscillations and mixings is here systematically discussed in terms of an S_3-flavour permutational symmetry. After presenting some relevant results on lepton masses and mixings, previously derived in a minimal S_3-invariant extension of the Standard Model, we compute the branching ratios of some selected flavour-changing neutral current processes (FCNC) as well as the contribution of the exchange of neutral flavour-changing scalar to the anomaly of the magnetic moment of the muon. We found that the minimal S_3-invariant extension of the Standard Model describes successfully masses and mixings, as well as, all flavour changing neutral current processes in excellent agreement with experiment.

Keywords: Flavour symmetries; quark and lepton masses and mixings; neutrino masses and mixings; flavour changing neutral currents; muon anomalous magnetic moment.

Una multiplicidad de efectos de violacion del sabor leptónico relacionados con el reciente descubrimiento de oscilaciones y mezclas de los neutrinos son discutidos aquí en términos de una simetría permutacional S_3 del sabor. Después de presentar algunos resultados relevantes acerca de masas y mezclas de leptones, derivados anteriormente en una extension mínima S_3-invariante del Modelo Estándar, calculamos las tasas de ramificación de algunos procesos que cambian el sabor leptónico (FCNC) así como la contribución debida al intercambio de escalares neutros que cambian el sabor a la anomalía del momento magnético del muón. Encontramos que la extension mínima S_3-invariante del Modelo Estándar describe con éxito masas y mezclas, los procesos que ocurren por intercambio de corrientes neutras que violan el sabor en excelente acuerdo con los experimentos.

Descriptores: Simetrías del sabor; masas y mezclas de quarks y leptones; masas y mezclas de neutrinos; corrientes neutras que violan el sabor; momento magnético anómalo del muón.

PACS: 11.30.Hv; 12.15.Ff; 14.60.Pq

1. Introduction

In the past nine years, the experiments and observations of flavour oscillations of solar [1-4], atmospheric [5, 6] and reactor [7, 8] neutrinos, established beyond reasonable doubt that neutrinos have non-vanishing masses and mix among themselves much like the quarks do. This is also the first experimental evidence that lepton flavour is not conserved in Nature [9, 10].

On the theoretical side, the discovery of neutrino masses and mixings is the first conclusive evidence of the incompleteness of the Standard Model, expected on theoretical grounds since long ago [11-16]. Hence, the need of extending the Standard Model in a logically consistent and physically coherent way to allow for lepton flavour violation and a unified and systematic treatment of the observed hierarchies of masses and mixings of all fermions. At the same time, it would be highly desirable to reduce drastically the number of free parameters in the theory. A flavour symmetry that generates the observed pattern of fermion mixings and masses could easily satisfy these two apparently contradictory requirements.

In a recent paper [17], we proposed a minimal extension of the Standard Model in which the permutational symmetry, S_3, is assumed to be an exact flavour symmetry at the weak scale. In a serie of subsequent papers [18-21], we made a detailed analysis of masses and mixings in the leptonic sector of the S_3-invariant extended model and also analyzed the flavour changing neutral current (FCNC) processes in the leptonic sector of the theory [19] as well as the contribution of the exchange of flavour changing scalars to the anomaly of the magnetic moment of the muon [20].

In this paper, we review, extend and update the results of our previous analysis of lepton masses and mixings in Sec. 2, flavour changing neutral currents are discussed in Sec. 3 and, in Sec. 4, the contribution of the exchange of neutral scalars to the anomaly of the magnetic moment of the muon is explained in some detail. We end our paper with a short summary of results and some conclusions.

2. A Minimal S_3-invariant Extension of the Standard Model

In the Standard Model, the Higgs and Yukawa sectors which are responsible for the generation of masses of quarks and charged leptons do not give mass to the neutrinos. Furthermore, the Yukawa sector of the Standard Model already has too many parameters whose values can only be determined from experiment. These two facts point to the necessity and convenience of extending the Standard Model in order to make a unified and systematic treatment of the observed hierarchies of masses and mixings of all fermions, as well as the presence or absence of CP violating phases in the mixing matrices. At the same time, we would also like to reduce drastically the number of free parameters in the theory. These two seemingly contradictory demands can be met by means of a flavour symmetry under which the families transform in a non-trivial fashion.

Recently, we introduced a minimal S_3-invariant Extension of the Standard Model [17] in which we argued that such a flavour symmetry unbroken at the Fermi scale, is the permutational symmetry of three objects S_3. In this model, we imposed S_3 as a fundamental symmetry in the matter sector. This assumption led us necessarily to extend the concept of flavour and generations to the Higgs sector. Hence, going to the irreducible representations of S_3, we added to the Higgs $SU(2)_L$ doublet in the S_3-singlet representation two more Higgs $SU(2)_L$ doublets, which can only belong to the two components of the S_3-doublet representation. In this way, all the matter fields in the Minimal S_3-invariant Extension of the Standard Model - Higgs, quark and lepton fields, including the right handed neutrino fields- belong to the three dimensional representation $\mathbf{1} \oplus \mathbf{2}$ of the permutational group S_3. The quark, lepton and Higgs fields are

$$\begin{array}{l} Q^T = (u_L, d_L)\ ,\ u_R\ ,\ d_R\ , \\ L^T = (\nu_L, e_L)\ ,\ e_R\ ,\ \nu_R \ \text{ and } \ H, \end{array} \tag{1}$$

in an obvious notation. All of these fields have three species, and, as explained above, we assume that each one form a reducible representation $\mathbf{1}_S \oplus \mathbf{2}$. The doublets carry capital indices I and J, which run from 1 to 2, and the singlets are denoted by $Q_3,\ u_{3R},\ d_{3R},\ L_3,\ e_{3R},\ \nu_{3R}$ and H_S. Note that the subscript 3 denotes the singlet representation and not the third generation.

Due to the presence of three Higgs field, the Higgs potential $V_H(H_S, H_D)$ is more complicate than that of the Standard Model

$$V_H(H_S, H_D) = V_1 + V_2 \tag{2}$$

where

$$\begin{aligned} V_1 &= \mu^2 \left[(\bar{H}_{D1} H_{D1}) + (\bar{H}_{D2} H_{D2}) + (\bar{H}_3 H_3) \right] \\ &+ \frac{1}{2}\lambda_1 \left[(\bar{H}_{D1} H_{D1}) + (\bar{H}_{D2} H_{D2}) + (\bar{H}_3 H_3) \right]^2 \end{aligned} \tag{3}$$

and

$$V_2 = \eta_1 (\bar{H}_3 H_3) \left[(\bar{H}_{D1} H_{D1}) + (\bar{H}_{D2} H_{D2}) \right]. \tag{4}$$

A Higgs potential invariant under S_3 was first proposed by Pakvasa and Sugawara [22], who assumed an additional reflection symmetry $R:\ H_s \to\ -H_s$. These authors found that in addition to the S_3 symmetry, their Higgs potential has an accidental symmetry S_2':$H_1 \leftrightarrow H_2$. The accidental S_2' symmetry is also present in our $V_H(H_S, H_D)$, therefore, $\langle H_1 \rangle = \langle H_2 \rangle$.

The most general renormalizable Yukawa interactions of this model are given by

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \tag{5}$$

where

$$\begin{aligned} \mathcal{L}_{Y_D} &= -Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} \\ &- Y_2^d [\, \overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR} \,] \\ &- Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + \text{ h.c.}, \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{L}_{Y_U} &= -Y_1^u \overline{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (i\sigma_2) H_S^* u_{3R} \\ &-Y_2^u [\overline{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \overline{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR} \,] \\ &- Y_4^u \overline{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (i\sigma_2) H_I^* u_{3R} + \text{h.c.}, \end{aligned} \tag{7}$$

$$\begin{aligned} \mathcal{L}_{Y_E} &= -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} \\ &- Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] \\ &- Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{h.c.}, \end{aligned} \tag{8}$$

$$\begin{aligned} \mathcal{L}_{Y_\nu} &= -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\ &- Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] \\ &- Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{h.c.}, \end{aligned} \tag{9}$$

and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{10}$$

Furthermore, we add to the Lagrangian the Majorana mass terms for the right-handed neutrinos

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R}. \tag{11}$$

With these assumptions, the Yukawa interactions, Eqs. (6) to (9) yield mass matrices, for all fermions in the theory, of the general form [17]

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}. \tag{12}$$

The Majorana mass for the left handed neutrinos ν_L is generated from (11) by the see-saw mechanism,

$$\mathbf{M}_\nu = \mathbf{M}_{\nu_\mathbf{D}} \tilde{\mathbf{M}}^{-1} (\mathbf{M}_{\nu_\mathbf{D}})^T, \tag{13}$$

where

$$\tilde{\mathbf{M}} = \text{diag}(M_1, M_1, M_3).$$

In principle, all entries in the mass matrices can be complex since there is no restriction coming from the flavour symmetry S_3. The mass matrices are diagonalized by bi-unitary transformations as

$$\begin{aligned} U_{d(u,e)L}^\dagger \mathbf{M}_{d(u,e)} U_{d(u,e)R} &= \text{diag}(m_{d(u,e)}, m_{s(c,\mu)}, m_{b(t,\tau)}), \\ U_\nu^T \mathbf{M}_\nu U_\nu &= \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \end{aligned} \tag{14}$$

TABLE I. Z_2 assignment in the leptonic sector.

$-$	$+$
$H_S,\ \nu_{3R}$	$H_I,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

The entries in the diagonal matrices may be complex, so the physical masses are their absolute values.

The mixing matrices are, by definition,

$$V_{CKM} = U_{uL}^{\dagger} U_{dL}, \quad V_{PMNS} = U_{eL}^{\dagger} U_{\nu} K. \qquad (15)$$

where K is the diagonal matrix of the Majorana phase factors.

3. The mass matrices in the leptonic sector and Z_2 symmetry

The number of free parameters in the leptonic sector may be further reduced by means of an Abelian Z_2 symmetry. A possible set of charge assignments of Z_2, compatible with the experimental data on masses and mixings in the leptonic sector is given in Table I. These Z_2 assignments forbid the following Yukawa couplings

$$Y_1^e = Y_3^e = Y_1^{\nu} = Y_5^{\nu} = 0. \qquad (16)$$

Hence, the corresponding entries in the mass matrices vanish, *i.e.*,

$$\mu_1^e = \mu_3^e = 0,$$

and

$$\mu_1^{\nu} = \mu_5^{\nu} = 0.$$

The mass matrix of the neutrinos

According to the Z_2 selection rule, Eq (16), the mass matrix of the Dirac neutrinos takes the form

$$\mathbf{M}_{\nu_\mathbf{D}} = \begin{pmatrix} \mu_2^{\nu} & \mu_2^{\nu} & 0 \\ \mu_2^{\nu} & -\mu_2^{\nu} & 0 \\ \mu_4^{\nu} & \mu_4^{\nu} & \mu_3^{\nu} \end{pmatrix}. \qquad (17)$$

Then, the mass matrix for the left-handed Majorana neutrinos, $\mathbf{M}_{\nu}$, obtained from the see-saw mechanism, $\mathbf{M}_{\nu}{=}\mathbf{M}_{\nu_\mathbf{D}}\tilde{\mathbf{M}}^{-1}(\mathbf{M}_{\nu_\mathbf{D}})^T$, is

$$\mathbf{M}_{\nu} = \begin{pmatrix} 2(\rho_2^{\nu})^2 & 0 & 2\rho_2^{\nu}\rho_4^{\nu} \\ 0 & 2(\rho_2^{\nu})^2 & 0 \\ 2\rho_2^{\nu}\rho_4^{\nu} & 0 & 2(\rho_4^{\nu})^2 + (\rho_3^{\nu})^2 \end{pmatrix}, \qquad (18)$$

where $\rho_2^{\nu}{=}(\mu_2^{\nu})/M_1^{1/2}$, $\rho_4^{\nu}{=}(\mu_4^{\nu})/M_1^{1/2}$ and $\rho_3^{\nu}{=}(\mu_3^{\nu})/M_3^{1/2}$; M_1 and M_3 are the masses of the right handed neutrinos appearing in Ref. 11.

The non-Hermitian, complex, symmetric neutrino mass matrix M_{ν} may be brought to a diagonal form by a unitary transformation, as

$$U_{\nu}^T M_{\nu} U_{\nu} = \mathrm{diag}\left(|m_{\nu_1}|e^{i\phi_1}, |m_{\nu_2}|e^{i\phi_2}, |m_{\nu_3}|e^{i\phi_{\nu}}\right), \qquad (19)$$

where U_{ν} is the matrix that diagonalizes the matrix $M_{\nu}^{\dagger}M_{\nu}$.

Written in polar form, the matrix U_{ν} takes the form

$$U_{\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_{\nu}} \end{pmatrix} \begin{pmatrix} \cos\eta & \sin\eta & 0 \\ 0 & 0 & 1 \\ -\sin\eta & \cos\eta & 0 \end{pmatrix}, \qquad (20)$$

if we require that the defining Eq. (19) be satisfied as an identity, we may solve the resulting equations for $\sin\eta$ and $\cos\eta$ in terms of the neutrino masses. This allows us to reparametrize the matrices M_{ν} and U_{ν} in terms of the complex neutrino masses,

$$M_{\nu} = \begin{pmatrix} m_{\nu_3} & 0 & \sqrt{(m_{\nu_3}-m_{\nu_1})(m_{\nu_2}-m_{\nu_3})}e^{-i\delta_{\nu}} \\ 0 & m_{\nu_3} & 0 \\ \sqrt{(m_{\nu_3}-m_{\nu_1})(m_{\nu_2}-m_{\nu_3})}e^{-i\delta_{\nu}} & 0 & (m_{\nu_1}+m_{\nu_2}-m_{\nu_3})e^{-2i\delta_{\nu}} \end{pmatrix} \qquad (21)$$

and

$$U_{\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_{\nu}} \end{pmatrix} \times \begin{pmatrix} \sqrt{\frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_2}-m_{\nu_1}}} & \sqrt{\frac{m_{\nu_3}-m_{\nu_1}}{m_{\nu_2}-m_{\nu_1}}} & 0 \\ 0 & 0 & 1 \\ -\sqrt{\frac{m_{\nu_3}-m_{\nu_1}}{m_{\nu_2}-m_{\nu_1}}} & \sqrt{\frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_2}-m_{\nu_1}}} & 0 \end{pmatrix}, \qquad (22)$$

The unitarity of U_{ν} constrains $\sin\eta$ to be real and thus $|\sin\eta| \leq 1$, this condition fixes the phases ϕ_1 and ϕ_2 as

$$|m_{\nu_1}|\sin\phi_1 = |m_{\nu_2}|\sin\phi_2 = |m_{\nu_3}|\sin\phi_{\nu}. \qquad (23)$$

The only free parameters in the matrices M_{ν} and U_{ν}, are the phase ϕ_{ν}, implicit in m_{ν_1}, m_{ν_2} and m_{ν_3}, and the Dirac phase δ_{ν}.

The mass matrix of the charged leptons

The mass matrix of the charged leptons takes the form

$$M_e = m_{\tau} \begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix}. \qquad (24)$$

The unitary matrix U_{eL} that enters in the definition of the mixing matrix, V_{PMNS}, is calculated from

$$U_{eL}^{\dagger} M_e M_e^{\dagger} U_{eL} = \mathrm{diag}(m_e^2, m_{\mu}^2, m_{\tau}^2), \qquad (25)$$

where m_e, m_{μ} and m_{τ} are the masses of the charged leptons.

The parameters $|\tilde{\mu}_2|$, $|\tilde{\mu}_4|$ and $|\tilde{\mu}_5|$ may readily be expressed in terms of the charged lepton masses. The resulting expression for M_e, written to order $\left(m_\mu m_e/m_\tau^2\right)^2$ and $x^4 = (m_e/m_\mu)^4$ is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}. \tag{26}$$

This approximation is numerically exact up to order 10^{-9} in units of the τ mass. Notice that this matrix has no free parameters other than the Dirac phase δ_e.

The unitary matrix U_{eL} that diagonalizes $M_e M_e^\dagger$ and enters in the definition of the neutrino mixing matrix V_{PMNS} may be written in polar form as

$$U_{eL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_e} \end{pmatrix} \begin{pmatrix} O_{11} & -O_{12} & O_{13} \\ -O_{21} & O_{22} & O_{23} \\ -O_{31} & -O_{32} & O_{33} \end{pmatrix}, \tag{27}$$

where the orthogonal matrix $\mathbf{O}_{eL}$ in the right-hand side of Eq. (27), written to the same order of magnitude as M_e, is

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}}x\frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}}\frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}x\frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}}\frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x\frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix}, \tag{28}$$

where, as before, $\tilde{m}_\mu = m_\mu/m_\tau$, $\tilde{m}_e = m_e/m_\tau$ and $x = m_e/m_\mu$.

The neutrino mixing matrix

The neutrino mixing matrix V_{PMNS}, is the product $U_{eL}^\dagger U_\nu K$, where K is the diagonal matrix of the Majorana phase factors, defined by

$$\text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = K^\dagger \text{diag}(|m_{\nu_1}|, |m_{\nu_2}|, |m_{\nu_3}|)K^\dagger. \tag{29}$$

Except for an overall phase factor $e^{i\phi_1}$, which can be ignored, K is

$$K = \text{diag}(1, e^{i\alpha}, e^{i\beta}), \tag{30}$$

where $\alpha = 1/2(\phi_1 - \phi_2)$ and $\beta = 1/2(\phi_1 - \phi_\nu)$ are the Majorana phases.

Therefore, the theoretical mixing matrix V_{PMNS}^{th}, is given by

$$V_{PMNS}^{th} = \begin{pmatrix} O_{11}\cos\eta + O_{31}\sin\eta e^{i\delta} & O_{11}\sin\eta - O_{31}\cos\eta e^{i\delta} & -O_{21} \\ -O_{12}\cos\eta + O_{32}\sin\eta e^{i\delta} & -O_{12}\sin\eta - O_{32}\cos\eta e^{i\delta} & O_{22} \\ O_{13}\cos\eta - O_{33}\sin\eta e^{i\delta} & O_{13}\sin\eta + O_{33}\cos\eta e^{i\delta} & O_{23} \end{pmatrix} \times K, \tag{31}$$

where $\cos\eta$ and $\sin\eta$ are given in Eqs. (20) and (22) , O_{ij} are given in Eqs. (27) and (28), and $\delta = \delta_\nu - \delta_e$.

To find the relation of our results with the neutrino mixing angles we make use of the equality of the absolute values of the elements of V_{PMNS}^{th} and V_{PMNS}^{PDG} [23], that is

$$|V_{PMNS}^{th}| = |V_{PMNS}^{PDG}|. \tag{32}$$

This relation allows us to derive expressions for the mixing angles in terms of the charged lepton and neutrino masses.

The magnitudes of the reactor and atmospheric mixing angles, θ_{13} and θ_{23}, are determined by the masses of the charged leptons only. Keeping only terms of order (m_e^2/m_μ^2) and $(m_\mu/m_\tau)^4$, we get

$$\sin\theta_{13} \approx \frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4}}, \qquad \sin\theta_{23} \approx \frac{1}{\sqrt{2}} \frac{1+\frac{1}{4}x^2-2\tilde{m}_\mu^2+\tilde{m}_\mu^4}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4}}. \tag{33}$$

The magnitude of the solar angle depends on charged lepton and neutrino masses, as well as, the Dirac and Majorana phases.

$$|\tan\theta_{12}|^2 = \frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_3}-m_{\nu_1}} \left(\frac{1-2\frac{O_{11}}{O_{31}}\cos\delta\sqrt{\frac{m_{\nu_3}-m_{\nu_1}}{m_{\nu_2}-m_{\nu_3}}}+\left(\frac{O_{11}}{O_{31}}\right)^2\frac{m_{\nu_3}-m_{\nu_1}}{m_{\nu_2}-m_{\nu_3}}}{1+2\frac{O_{11}}{O_{31}}\cos\delta\sqrt{\frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_3}-m_{\nu_1}}}+\left(\frac{O_{11}}{O_{31}}\right)^2\frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_3}-m_{\nu_1}}} \right). \tag{34}$$

The dependence of $\tan\theta_{12}$ on the Dirac phase δ, see (34), is very weak, since $O_{31} \sim 1$ but $O_{11} \sim 1/\sqrt{2}(m_e/m_\mu)$. Hence, we may neglect it when comparing (34) with the data on neutrino mixings.

The dependence of $\tan\theta_{12}$ on the phase ϕ_ν and the physical masses of the neutrinos enters through the ratio of the neutrino mass differences, it can be made explicit with the help of the unitarity constraint on U_ν, Eq. (23),

$$\frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_3}-m_{\nu_1}} = \frac{(|m_{\nu_2}|^2-|m_{\nu_3}|^2\sin^2\phi_\nu)^{1/2}-|m_{\nu_3}||\cos\phi_\nu|}{(|m_{\nu_1}|^2-|m_{\nu_3}|^2\sin^2\phi_\nu)^{1/2}+|m_{\nu_3}||\cos\phi_\nu|}. \tag{35}$$

Similarly, the Majorana phases are given by

$$\sin 2\alpha = \sin(\phi_1-\phi_2) = \frac{|m_{\nu_3}|\sin\phi_\nu}{|m_{\nu_1}||m_{\nu_2}|}\left(\sqrt{|m_{\nu_2}|^2-|m_{\nu_3}|^2\sin^2\phi_\nu}+\sqrt{|m_{\nu_1}|^2-|m_{\nu_3}|^2\sin^2\phi_\nu}\right), \tag{36}$$

$$\sin 2\beta = \sin(\phi_1-\phi_\nu) = \frac{\sin\phi_\nu}{|m_{\nu_1}|}\left(|m_{\nu_3}|\sqrt{1-\sin^2\phi_\nu}+\sqrt{|m_{\nu_1}|^2-|m_{\nu_3}|^2\sin^2\phi_\nu}\right). \tag{37}$$

A more complete and detailed discussion of the Majorana phases in the neutrino mixing matrix V_{PMNS} obtained in our model is given by J. Kubo [24].

4. Neutrino masses and mixings

The determination of neutrino oscillation parameters [25–29] has finally entered the high precision age, with many experiments underway and a new generation coming. The results of two global analysis of neutrino oscillations, updated to september 2007, are summarized in Table II taken from [25]. In this section, numerical values of the neutrino masses and mixing angles will be obtained from the theoretical expressions derived in the previous section and the numerical values of the neutrino oscillation parameters given in Table II. In the present S_3-invariant extension of the Standard Model, the experimental restriction $|\Delta m_{12}^2| < |\Delta m_{13}^2|$ implies an inverted neutrino mass spectrum

$$|m_{\nu_3}| < |m_{\nu_1}| < |m_{\nu_2}| \qquad [17].$$

In this model, the reactor and atmospheric mixing angles, θ_{13} and θ_{23}, are determined by the masses of the charged leptons only, in very good agreement with the experimental values

$$(\sin^2\theta_{13})^{th} = 1.1\times 10^{-5},$$
$$(\sin^2\theta_{13})^{\exp} \le 0.028 \tag{38}$$

and

$$\begin{aligned}(\sin^2\theta_{23})^{th} &= 0.5,\\ (\sin^2\theta_{23})^{\exp} &= 0.5^{+0.06}_{-0.05}.\end{aligned} \tag{39}$$

As can be seen from Eqs. (34) and (35), the solar angle is sensitive to the differences of the squared neutrino masses and the phase ϕ_ν but is only weakly sensitive to the charged lepton masses. If the small terms proportional to O_{11} and O_{11}^2 are neglected in Ref. 34, we obtain

TABLE II. Best-fit values, 2σ and 3σ intervals (1 d.o.f) for the three-flavour neutrino oscillation parameters from global data including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K and MINOS) experiments.

parameter	best fit	2σ	3σ
$\Delta m_{21}^2[10^{-5}$ eV$]$	7.6	7.3–8.1	7.1–8.3
$\Delta m_{31}^2[10^{-3}$ eV$]$	2.4	2.1–2.7	2.0–2.8
$\sin^2\theta_{12}$	0.32	0.28–0.37	0.26–0.40
$\sin^2\theta_{23}$	0.50	0.38–0.63	0.34–0.67
$\sin^2\theta_{13}$	0.007	≤ 0.033	≤ 0.050

$$\tan^2\theta_{12} = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2\cos^2\phi_\nu)^{1/2} - |m_{\nu_3}|\cos\phi_\nu}{(\Delta m_{13}^2 + |m_{\nu_3|}^2\cos^2\phi_\nu)^{1/2} + |m_{\nu_3}|\cos\phi_\nu} \tag{40}$$

From this equation, we may readily derive expressions for the neutrino masses in terms of $\tan\theta_{12}$, $\cos\phi_\nu$ and the differences of the squared neutrino masses

$$|m_{\nu_3}| = \frac{\sqrt{\Delta m_{13}^2}}{2\tan\theta_{12}\cos\phi_\nu}\frac{1-\tan^4\theta_{12}+r^2}{\sqrt{1+\tan^2\theta_{12}}\sqrt{1+\tan^2\theta_{12}+r^2}} \tag{41}$$

and

$$|m_{\nu_1}| = \sqrt{|m_{\nu_3}|^2 + \Delta m_{13}^2}, \qquad |m_{\nu_2}| = \sqrt{|m_{\nu_3}|^2 + \Delta m_{13}^2(1+r^2)} \tag{42}$$

where $r^2 = \Delta m_{12}^2/\Delta m_{13}^2 \approx 3\times 10^{-2}$.

As $r^2 \ll 1$, the sum of the neutrino masses is

$$\sum_{i=1}^{3}|m_{\nu_i}| \approx \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu\tan\theta_{12}}\left(1+2\sqrt{1+2\tan^2\theta_{12}\,(2\cos^2\phi_\nu-1)+\tan^4\theta_{12}}-\tan^2\theta_{12}\right). \tag{43}$$

The most restrictive cosmological upper bound for this sum is [30]

$$\sum|m_\nu| \leq 0.17\text{eV}. \tag{44}$$

This upper bound and the experimentally determined values of $\tan\theta_{12}$ and Δm_{ij}^2, give a lower bound for $\cos\phi_\nu$,

$$\cos\phi_\nu \geq 0.55 \tag{45}$$

or $0 \leq \phi_\nu \leq 57°$.

The neutrino masses $|m_{\nu_i}|$ assume their minimal values when $\cos\phi_\nu = 1$. When $\cos\phi_\nu$ takes values in the range $0.55 \leq \cos\phi \leq 1$, the neutrino masses change very slowly with $\cos\phi_\nu$. In the absence of experimental information we will assume that ϕ_ν vanishes. Hence, setting $\phi_\nu = 0$ in our formula, we find

$$\begin{aligned} m_{\nu_1} &= 0.052 \text{ eV}\\ m_{\nu_2} &= 0.053 \text{ eV}\\ m_{\nu_3} &= 0.019 \text{ eV}. \end{aligned} \tag{46}$$

Hence, the computed sum of the neutrino masses is

$$\left(\sum_{i=1}^{3}|m_{\nu_i}|\right)^{th} = 0.13 \text{ eV} \tag{47}$$

below the cosmological upper bound given in Eq. (44), as expected, since we used the cosmological bound to fix the bound on $\cos\phi_\nu$.

Now, we may compare our results with other bounds on the neutrino masses.

The effective Majorana mass in neutrinoless double beta decay $\langle m_{2\beta}\rangle$, is defined as [31]

$$\langle m_{2\beta}\rangle = |\sum_{i=1}^{3} V_{ei}^2 m_{\nu_i}|. \tag{48}$$

The most stringent bound on $\langle m_{2\beta}\rangle$, obtained from the analysis of the data collected by the Heidelberg-Moscow experiment on neutrinoless double beta decay in enriched Ge [32], is

$$\langle m_{2\beta}\rangle < 0.3 \text{ eV}. \tag{49}$$

In our model, and assuming that the Majorana phases vanish we get

$$\langle m_{2\beta}\rangle^{th} = 0.053 \text{ eV} \tag{50}$$

well below the experimental upper bound.

The most restrictive direct neutrino measurement involving electron type neutrinos, is based on fitting the shape of the beta spectrum [29]. In such measurement, the quantity

$$\bar{m}_{\nu_e} = \sqrt{\sum_i |V_{ei}|^2 m_{\nu_i}} \tag{51}$$

is determined or constrained. A very restrictive upper bound for this sum is obtained from nucleosynthesis processes [33,34]

$$(\bar{m}_{\nu_e})^{\text{exp}} < 0.37 \text{ eV}. \tag{52}$$

From Eqs. (38) and (39), we obtain

$$(\bar{m}_{\nu_e})^{th} = 0.053 \text{ eV}. \tag{53}$$

again, well below the experimental upper bound given in Eq. (52).

5. Lepton flavour violating processes

It is well known that models with more than one Higgs $SU(2)$ doublet may in general, have tree-level flavour changing neutral currents (FCNC) [35, 36]. Lepton flavour violating couplings, due to FCNC, naturally appear in the minimal S_3-invariant extension of the Standard Model, since in this extended model there are three Higgs $SU(2)$ doublets, one per generation, coupled to all fermions. In a flavour labeled, symmetry adapted weak basis, the flavour changing Yukawa couplings may be written as in this expression, the entries in

$$\begin{aligned}\mathcal{L}_Y^{\rm FCNC} &= \left(\overline{E}_{aL} Y_{ab}^{ES} E_{bR} + \overline{U}_{aL} Y_{ab}^{US} U_{bR} + \overline{D}_{aL} Y_{ab}^{DS} D_{bR}\right) H_S^0 \\ &+ \left(\overline{E}_{aL} Y_{ab}^{E1} E_{bR} + \overline{U}_{aL} Y_{ab}^{U1} U_{bR} + \overline{D}_{aL} Y_{ab}^{D1} D_{bR}\right) H_1^0 \\ &+ \left(\overline{E}_{aL} Y_{ab}^{E2} E_{bR} + \overline{U}_{aL} Y_{ab}^{U2} U_{bR} + \overline{D}_{aL} Y_{ab}^{D2} D_{bR}\right) H_2^0 + \text{h.c.}\end{aligned} \tag{54}$$

the column matrices $E's$, $U's$ and $D's$ are the left and right fermion fields and $Y_{ab}^{(E,U,D)S}$, $Y_{ab}^{(E,U,D)1,2}$ are 3×3 matrices of the Yukawa couplings of the fermion fields to the neutral Higgs fields H_s^0 and $H_{1,2}^0$ in the S_3-singlet and doublet representations respectively.

In this basis, the Yukawa couplings of the Higgs fields to each family of fermions may be written in terms of matrices $\mathcal{M}_Y^{(e,u,d)}$, which give rise to the corresponding mass matrices $M^{(e,u,d)}$ when the gauge symmetry is spontaneously broken. From this relation we may calculate the flavour changing Yukawa couplings in terms of the fermion masses and the vacuum expectation values of the neutral Higgs fields.

The matrix $\mathcal{M}_Y^e$ of the charged leptons is written in terms of the matrices of the Yukawa couplings of the charged leptons as

$$\mathcal{M}_Y^e = Y_w^{E1} H_1^0 + Y_w^{E2} H_2^0, \tag{55}$$

where, the index w means that the Yukawa matrices are defined in the weak basis,

$$Y_w^{E1} = \frac{m_\tau}{v_1}\begin{pmatrix} 0 & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 & 0 \end{pmatrix} \tag{56}$$

and

$$Y_w^{E2} = \frac{m_\tau}{v_2}\begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ 0 & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}. \tag{57}$$

In the computation of the flavour-changing neutral couplings, the Yukawa couplings are defined in the mass basis which are obtained from Y_w^{EI}, given in Refs. 56 and 57, according to

$$\tilde{Y}_m^{EI} = U_{eL}^\dagger Y_w^{EI} U_{eR}$$

where U_{eL} and U_{eR} are the matrices that diagonalize the charged lepton mass matrix defined in Eqs. (14) and (27).

We get,

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1}\begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m, \tag{58}$$

and

$$\tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m , \tag{59}$$

where $\tilde{m}_\mu = 5.94 \times 10^{-2}$, $\tilde{m}_e = 2.876 \times 10^{-4}$ and $x = m_e/m_\mu = 4.84 \times 10^{-3}$.

All the nondiagonal elements in $\tilde{Y}_m^{EI}$ are responsible for tree-level FCNC processes. The actual values of the Yukawa couplings in Eqs. (58) and (59) still depend on the VEV's of the Higgs fields v_1 and v_2, and, hence, on the Higgs potential. If the S_2' accidental symmetry in the Higgs sector is preserved [22], $\langle H_1^0 \rangle = \langle H_2^0 \rangle = v$. With the purpose of making an order of magnitude estimate of the coefficient m_τ/v multiplying the Yukawa matrices, we may further assume that the VEV's for all the Higgs fields are comparable, that is,

$$\langle H_s^0 \rangle = \langle H_1^0 \rangle = \langle H_2^0 \rangle = \frac{\sqrt{2}}{\sqrt{3}} \frac{M_W}{g_2}$$

then

$$m_\tau/v = \sqrt{3}/\sqrt{2} g_2 m_\tau / M_W$$

and we may estimate the numerical values of the Yukawa couplings from the numerical values of the lepton masses.

Let us consider, first, the flavour violating process $\tau^- \to \mu^- e^+ e^-$, the amplitude of this process is proportional to $\tilde{Y}_{\tau\mu}^{EI} \tilde{Y}_{ee}^{EI}$ [37]. Then, the leptonic branching ratio for this process is

$$Br(\tau \to \mu e^+ e^-) = \frac{\Gamma(\tau \to \mu e^+ e^-)}{\Gamma(\tau \to e\nu\bar{\nu}) + \Gamma(\tau \to \mu\nu\bar{\nu})} \tag{60}$$

where

$$\Gamma(\tau \to \mu e^+ e^-) \approx \frac{m_\tau^5}{3 \times 2^{10} \pi^3} \frac{\left(\tilde{Y}_{\tau\mu}^{EI} \tilde{Y}_{ee}^{EI}\right)^2}{\left(M_{H_I}\right)^4} \tag{61}$$

which is the dominant term, the index I denotes the Higgs boson in the S_3-doublet with the smallest mass. This equation, and the well-known expression for $\Gamma(\tau \to e\nu\bar{\nu})$ and $\Gamma(\tau \to \mu\nu\bar{\nu})$ [23], give

$$Br(\tau \to \mu e^+ e^-) \approx \frac{9}{4} \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_{H_{1,2}}}\right)^4, \tag{62}$$

if we take for $M_{H_{1,2}} \sim 120\, GeV$, we obtain

$$Br(\tau \to \mu e^+ e^-) \approx 3.15 \times 10^{-17}, \tag{63}$$

well below the experimental upper bound for this process, which is 2.7×10^{-8} [38]. Similar computations give the following estimates

$$Br(\tau \to e\gamma) \approx \frac{3\alpha}{8\pi} \left(\frac{m_\mu}{M_H}\right)^4, \tag{64}$$

$$Br(\tau \to \mu\gamma) \approx \frac{3\alpha}{128\pi} \left(\frac{m_\mu}{m_\tau}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4, \tag{65}$$

$$Br(\tau \to 3\mu) \approx \frac{9}{64} \left(\frac{m_\mu}{M_H}\right)^4, \tag{66}$$

$$Br(\mu \to 3e) \approx 18 \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4, \tag{67}$$

and

$$Br(\mu \to e\gamma) \approx \frac{27\alpha}{64\pi} \left(\frac{m_e}{m_\mu}\right)^4 \left(\frac{m_\tau}{M_H}\right)^4. \tag{68}$$

From these equations, we see that FCNC processes in the leptonic sector are strongly suppressed by the small values of the mass ratios m_e/m_τ, m_μ/m_τ and m_τ/M_H. The numerical estimates of the branching ratios and the corresponding experimental upper bounds are shown in Table III. It may be seen that, in all cases considered, the numerical values for the branching ratios of the FCNC in the leptonic sector are well below the corresponding experimental upper bounds.

6. The anomalous magnetic moment of the muon

In models with more than one Higgs $SU(2)$ doublet, the exchange of flavour changing scalars may contribute to the anomalous magnetic moment of the muon. In the minimal S_3-invariant extension of the Standard Model we are considering here, we have three Higgs $SU(2)$ doublets, one in the singlet and the other two in the doublet representations of the S_3 flavour group. The Z_2 symmetry decouples the charged leptons from the Higgs boson in the S_3 singlet representation. Therefore, in the theory there are two neutral scalars and two neutral pseudoscalars whose exchange will contribute to the anomalous magnetic moment of the muon, in the leading order of magnitude. Since the heavier generations have larger flavour-changing couplings, the largest contribution comes from the heaviest charged leptons coupled to the lightest of the neutral Higgs bosons, $\mu - \tau - H$, as shown in Fig. 1. The contribution, $\delta a_\mu^{(H)}$, to the magnetic moment of the muon from the exchange of the lightest neutral

TABLE III. Leptonic FCNC processes, calculated with $M_{H_{1,2}} \sim 120$ GeV.

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \to 3\mu$	8.43×10^{-14}	3.2×10^{-8}	Y. Miyazaki *et al.* [38]
		5.3×10^{-8}	B. Aubert *et. al.* [39]
$\tau \to \mu e^+ e^-$	3.15×10^{-17}	2.7×10^{-8}	Y. Miyazaki *et al.* [38]
		8×10^{-8}	B. Aubert *et. al.* [39]
$\tau \to \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}	B. Aubert *et. al.* [40]
$\tau \to e\gamma$	5.22×10^{-16}	1.1×10^{-11}	B. Aubert *et. al.* [41]
$\mu \to 3e$	2.53×10^{-16}	1×10^{-12}	U. Bellgardt *et al.* [42]
$\mu \to e\gamma$	2.42×10^{-20}	1.2×10^{-11}	M. L. Brooks *et al.* [43]

Higgs boson computed in the leading order of magnitude is

$$\delta a_\mu^{(H)} = \frac{\tilde{Y}_{\mu\tau}\tilde{Y}_{\tau\mu}}{16\pi^2}\frac{m_\mu m_\tau}{M_H^2}\left(\log\left(\frac{M_H^2}{m_\tau^2}\right) - \frac{3}{2}\right). \quad (69)$$

the Yukawa couplings appearing in this expression are given in Refs. 58 and 59. Hence, we obtain

$$\delta a_\mu^{(H)} = \frac{m_\tau^2}{(246 \text{ GeV})^2}\frac{(2+\tan^2\beta)}{32\pi^2}\frac{m_\mu^2}{M_H^2} \times \left(\log\left(\frac{M_H^2}{m_\tau^2}\right) - \frac{3}{2}\right), \quad (70)$$

in this expression, $\tan\beta = v_s/v_1$, is the ratio of the vacuum expectation values of the Higgs scalars in the singlet representation, v_s, and in the doublet representation, v_1, of the S_3 flavour group. The most restrictive upper bound on $\tan\beta$ may be obtained from the experimental upper bound on $Br(\mu \to 3e)$ given in Ref 67, and in Table III, we obtain

$$\tan\beta \leq 14 \quad (71)$$

substitution of this value in Ref. 70 and taking for the Higgs mass the value $M_H = 120$ GeV gives an estimate of the largest possible contribution of the FCNC to the anomaly of the magnetic moment of the muon

$$\delta a_\mu^{(H)} \approx 1.7 \times 10^{-10}. \quad (72)$$

This number has to be compared with the difference between the experimental value and the Standard Model prediction for the anomaly of the magnetic moment of the muon [44]

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (28.7 \pm 9.1) \times 10^{-10}, \quad (73)$$

which means

$$\frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx 0.06. \quad (74)$$

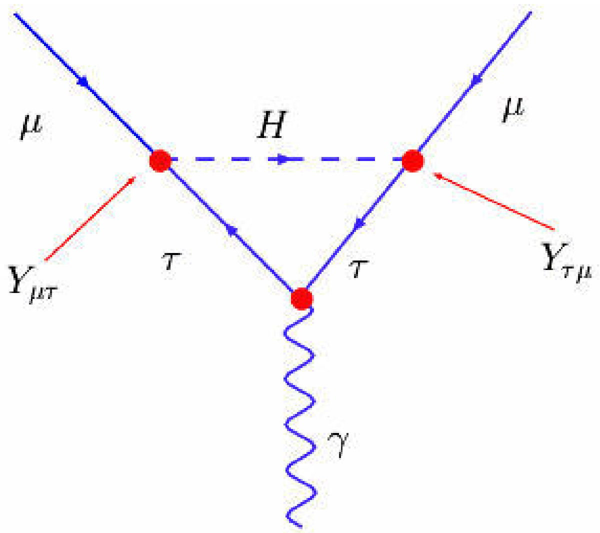

FIGURE 1. The contribution, $\delta a_\mu^{(H)}$, to the anomalous magnetic moment of the muon from the exchange of flavour changing scalars. The neutral Higgs boson can be a scalar or a pseudoscalar.

Hence, the contribution of the flavour changing neutral currents to the anomaly of the magnetic moment of the muon is smaller than or of the order of 6% of the discrepancy between the experimental value and the Standard Model prediction. This discrepancy is of the order of three standard deviations and quite important, but its interpretation is compromised by uncertainties in the computation of higher order hadronic effects mainly from three-loop vacuum polarization effects, $a_\mu^{VP}(3, had) \approx -1.82 \times 10^{-9}$ [45], and from three-loop contributions of hadronic light by light type, $a_\mu^{LBL}(3, had) \approx 1.59 \times 10^{-9}$ [45]. As explained above, the contribution to the anomaly from flavour changing neutral currents in the minimal S_3-invariant extension of the Standard Model, computed in this work is, at most, 6% of the discrepancy between the experimental value and the Standard Model prediction for the anomaly, and is of the same order of magnitude as the uncertainties in the higher order hadronic contributions, but still it is not negligible and is certainly compatible with the best, state of the art, experimental measurements and theoretical computations.

7. Conclusions

A variety of flavour violating effects related to the recent discovery of neutrino oscillations and mixings was discussed in the framework of the minimal S_3-invariant extension of the Standard Model that we proposed recently [17]. After a brief review of some relevant results on lepton masses and mixings that had been derived in this minimal S_3-invariant extension of the SM, we extended and updated our previous results on the branching ratios of some selected flavour-changing neutral currents processes (FCNC) [19] as well as the contribution to the magnetic moment of the muon [20]. Some intersting results are the following:

- The magnitudes of the three neutrino mixing angles, θ_{12}, θ_{23} and θ_{13}, are determined by an interplay of the $S_3 \times Z_2$ symmetry, the see-saw mechanism and the lepton mass hierarchy.

- The neutrino mixing angles, θ_{23} and θ_{13}, depend only on the masses of the charged leptons and their predicted numerical values are in excellent agreement with the best experimental values.

- The solar mixing angle, θ_{12}, fixes the scale and origin of the neutrino mass spectrum which has an inverted mass hierarchy with values

$$m_{\nu_1} = 0.052 \text{ eV},$$
$$m_{\nu_2} = 0.053 \text{ eV},$$
$$m_{\nu_3} = 0.019 \text{ eV}.$$

- The branching ratios of all flavour changing neutral current processes in the leptonic sector are strongly suppressed by the $S_3 \times Z_2$ symmetry and powers of the small mass ratios m_e/m_τ, m_μ/m_τ, and $\left(m_\tau/M_{H_{1,2}}\right)^4$, but could be important in astrophysical processes [46, 47].

- The anomalous magnetic moment of the muon gets a small but non-negligible contribution from the exchange of flavour changing scalar fields.

In conclusion, we may say that the minimal S_3-invariant extension of the Standard Model describes successfully masses and mixings in the quark [17] (not discussed here) and leptonic sectors with a small number of free parameters. It predicts the numerical values of the θ_{23} and θ_{13} neutrino mixing angles, as well as, all flavour changing neutral current processes in the leptonic sector, in excellent agreement with experiment. In this model, the exchange of flavour changing scalars gives a small but non-negligible contribution to the anomaly of the magnetic moment of the muon.

Acknowledgements

We thank Prof. Jens Erler and Dr. Genaro Toledo-Sanchez for helpful discussions about $g-2$. This work was partially supported by CONACYT México under contract No 51554-F and by DGAPA-UNAM under contract PAPIIT-IN115207-2.

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