Neutron emission effects on final fragments mass and kinetic energy distribution from low energy fission of ^{234}U

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The standard deviation of the final kinetic energy distribution (σ_e) as a function of mass of final fragments (m) from low energy fission of ^{234}U, measured with the Lohengrin spectrometer by Belhafaf *et al.*, presents a peak around $m = 109$ and another around $m = 122$. The authors attribute the first peak to the evaporation of a large number of neutrons around the corresponding mass number, *i.e.* there is no peak on the standard deviation of the primary kinetic energy distribution (σ_E) as a function of primary fragment mass (A). The second peak is attributed to a real peak on $\sigma_E(A)$. However, theoretical calculations related to primary distributions made by H.R. Faust and Z. Bao do not suggest any peak on $\sigma_E(A)$. In order to clarify this apparent controversy, we have made a numerical experiment in which the masses and the kinetic energy of final fragments are calculated, assuming an initial distribution of the kinetic energy without structures on the standard deviation as function of fragment mass. As a result we obtain a pronounced peak on $\sigma_e(m)$ curve around $m = 109$, a depletion from $m = 121$ to $m = 129$, and an small peak around $m = 122$, which is not as great as that measured by Belhafaf *et al.*Our simulation also reproduces the experimental results on the yield of the final mass $Y(m)$, the average number of emitted neutrons as a function of the provisional mass (calculated from the values of the final kinetic energy of the complementary fragments) and the average value of fragment kinetic energy as a function of the final mass ($\bar{e}$). From our results we conclude that there are no peaks on the $\sigma_E(A)$ curve, and the observed peaks on $\sigma_e(m)$ are due to the emitted neutron multiplicity and the variation of the average fragment kinetic energy as a function of primary fragment mass.

Keywords: Monte-Carlo; low energy fission; ^{234}U; fragment kinetic energy; standard deviation.

Las mediciones sobre la desviación estándar de la distribución de energía cinética final (σ_e) en función de la masa final (m) de los fragmentos de la fisión de baja energía del ^{234}U, hechas por Belhafaf *et al.*, presentan un pico alrededor de $m = 109$ y otro alrededor de $m = 122$. Los autores atribuyen el primer pico a la evaporación de un elevado número de neutrones alrededor del correspondiente número másico, es decir que no hay un pico en la desviación estándar de la distribución de energía cinética primaria en función de la masa primaria ($\sigma_E(A)$). El segundo pico es atribuida a un pico real en $\sigma_E(A)$. Sin embargo, cálculos teóricos relacionados con la distribución primaria, hechos por H.R. Faust and Z. Bao, no sugieren ningún pico en $\sigma_E(A)$. Para clarificar esta aparente controversia, hemos hecho un experimento numérico en el que la distribución de masa y energía cinética final es calculada suponiendo una distribución inicial de energía cinética sin estructuras en su desviación estándar en función de la masa inicial de los fragmentos. Como resultado obtenemos un pico pronunciado en la curva $\sigma_e(m)$ alrededor de $m = 109$, una depresión desde $m = 121$ hasta $m = 129$, un pequeño pico alrededor de $m = 122$, el que no es tan grande como el medido por Belhafaf *et al.*Nuestra simulación también reproduce los resultados experimentales del rendimiento de la masa final ($Y(m)$, el promedio del número de neutrones emitidos en función de la masa provisional (calculada a partir de los valores de la energía cinética de los fragmentos complementarios) y del valor promedio de la energía cinética como función de la masa final ($\bar{e}(m)$). De nuestros resultados concluimos que no hay picos en $\sigma_E(A)$ y los picos en $\sigma_e(m)$ son debidos a la multiplicidad de neutrones emitidos y a la variación de la energía cinética promedio en función de la masa primaria.

Descriptores: Monte-Carlo; fisión de baja energía; ^{234}U; energía cinética de fragmentos; desviación estándar.

PACS: 21.10.Gv; 25.85.Ec; 24.10.Lx

1. Introduction

One of the most studied quantities to understand the fission process is the fission fragment mass and kinetic energy distribution, which is very closely related to the topological features in the multi-dimensional potential energy surface [1]. Structures on the distribution of primary (before neutron emission) mass and kinetic energy may be interpreted by shell effects on potential energy of the fissioning system, determined by the Strutinsky prescription and discussed by Dickmann *et al.* [2] and Wilkins *et al.* [3].

One expression of the above mentioned primary kinetic energy distribution is constituted by the average value ($\bar{E}$) and the standard deviation (σ_E) as a function of primary mass (A). The difficulty is that only final fragments -after neutron emission- are accessible to experimental instruments.

Considering that reality, the distribution of final fragment kinetic energy (e) as a function of final fragment mass (m), from thermal neutron induced fission of ^{233}U, was measured by Belhafaf *et al.* [4], using the Lohengrin spectrometer. This distribution was represented by the average value of kinetic energy ($\bar{e}$) and the standard deviation (SD) σ_e as a function of m. The results present a first peak on $\sigma_e(m)$ around $m = 108$ and a second one around $m = 122$ (see Fig. 1).

The authors attribute the first peak to a large number of evaporated neutrons (ν) around the corresponding primary mass(A), *i.e.* there is no peak on $\sigma_E(A)$.

Based on the small number of emitted neutrons measured around $A = 122$, the second peak is attributed to the distribution of the primary fragment kinetic energy (E). In other words Belhafaf *et al.* assume that the standard deviation of primary fragment kinetic energy σ_E as a function of primary mass (A) have a peak around $A = 122$.

Belhafaf *et al.* take the supposed peaks on the σ_E curve as a reflection of the existence of multiple minima in the potential energy surface of the fissioning system near the scission configuration. Their conclusion is that for a given mass and charge split; the multiple minima of the potential energy map do not seem to be sufficient to account for the broadening of the experimental kinetic energy distribution. In order to interpret the supposed peak on $\sigma_E(A)$ around $A = 122$, they take into account the superposition of two or three different charge splits for a given mass division.

However, more recent theoretical calculations made by Faust *et al.* [5] do not suggest any peak in SD for the distribution of primary fragment kinetic energy σ_E around $A = 122$.

In order to clarify the apparent controversy between results obtained by Belhafaf *et al.* and Faust *et al.*, respectively, it is crucial to find the relation between the primary and the final kinetic energy distributions; the relation between the primary ($Y(A)$) and the final mass yield ($Y(m)$); as well as the relation between the average value of the number of emitted neutron ($\bar{\nu}$) as a function of the primary fragment mass and the values corresponding to the experimental results. To address this question, we present a Monte-Carlo simulation for an experiment measuring kinetic energy and mass distribution of final fragments from thermal neutron induced fission of ^{233}U *i.e.* low energy fission of ^{234}U.

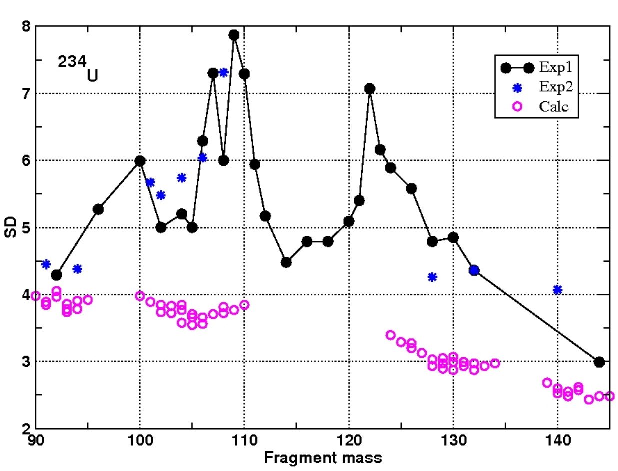

FIGURE 1. SD of the final fragment kinetic energy distribution as a function of the final mass m ($\bullet$ and $*$) as measured by Belhafaf *et al.* [4], and Faust *et al.* [5], respectively; and SD as a function of primary mass ($\circ$) as calculated by Faust *et al.* [5]

2. Monte Carlo simulation model

In our Monte Carlo simulation, the input quantities are the primary fragment yield (Y), the average kinetic energy ($\bar{E}$), the SD of the kinetic energy distribution (σ_E) and the average number of emitted neutrons ($\bar{\nu}$) as a function of primary fragment mass (A). The output of the simulation for the final fragments are the yield (Y), the SD of the kinetic energy distribution (σ_e) and the average number of emitted neutrons ($\bar{\nu}$) as a function of the final fragment mass m. The idea is to see if a $[\sigma_E(A)]$ curve without structure as input can produce a $[\sigma_e(m)]$ curve with structures.

2.1. Fragment kinetic energy and neutron multiplicity

In order to simplify calculation, we also make some assumptions in relation to neutron emission. We assume that

i) the E values have a Gaussian distribution,

ii) the average number of emitted neutrons ($\bar{\nu}$) corresponds to the fragments with the average value of kinetic energy $\bar{E}$, and

iii) there is a negative linear relation between the number of neutrons emitted and the fragment kinetic energy

(this assumption is based on the fact that the number emitted increases with the excitation energy, which decreases when the kinetic energy increases); then

$$\nu = \bar{\nu}\left[1 - \beta\left(\frac{E - \bar{E}}{\sigma_E}\right)\right], \tag{1}$$

where β is a free parameter.

2.2. Simulation process

We make several iterative simulations. After each simulation result (final distribution), we change input values (primary distribution) for a new simulation in order to get output values (new final distribution) closer to experimental results. For the first simulation, we take Y and $\bar{E}$ from Ref. 4. The first SD σ_E curve is an extrapolation of calculation results obtained by Faust *et al.* [5]. Then, we adjust $Y(A)$, $\nu(A)$, $\bar{E}(A)$ and $\sigma_E(A)$ in order to get $Y(m)$, $\bar{\nu}$, $\bar{e}(m)$, $\sigma_e(m)$ in agreement with experimental data.

In the simulation, for each primary mass A, the kinetic energy of the fission fragments is chosen randomly from a Gaussian distribution

$$P(E) = \frac{1}{\sqrt{2\pi}\sigma_E}\exp\left[-\frac{(E - \overline{E})^2}{2\sigma_E^2}\right], \tag{2}$$

where $P(E)$ is the probability density of energy with average value $\overline{E}$ and SD σ_E.

For each E value, the simulated number of neutrons N is calculated taking into account the relation (1). The final mass of the fragment will be

$$m = A - N. \tag{3}$$

Furthermore, assuming that the fragments lose energy only by neutron evaporation and not by gamma emission or any other process, and neglecting the recoil effect due to neutron emission, then the kinetic energy $e(m)$ of the final fragment will be given by

$$e(m) = (1 - \frac{N}{A})E. \tag{4}$$

With the ensemble of values corresponding to m, e and N, we calculate $Y(m)$, $\bar{e}(m)$, $\sigma_e(m)$ and $\nu(m)$. To obtain acceptable statistics during the simulation, we have considered a total number of fission events of ^{234}U of the order of 10^8, and we have computed the SD of all the relevant quantities by means of the following expression:

$$\sigma^2(m) = \frac{\sum_{j=1}^{N_j(m)} e_j^2(m)}{N_j(m)} - \bar{e}^2(m), \tag{5}$$

where $\bar{e}(m)$ is the average value of the kinetic energy of final fragments with a given mass m, and $N_j(m)$ is the number of fission events corresponding to that mass.

3. Results and interpretation

The simulated final mass yield curve $Y(m)$ and the primary mass yield $Y(A)$ are illustrated in Fig. 2. As expected, due to neutron emission, the $Y(m)$ curve is shifted from $Y(A)$ towards smaller fragment masses.

The simulated curve of the average number of emitted neutrons $\bar{\nu}(m)$ is shifted from $\bar{\nu}(A)$ in a similar way to $Y(m)$ with respect to $Y(A)$ (see Fig. 3).

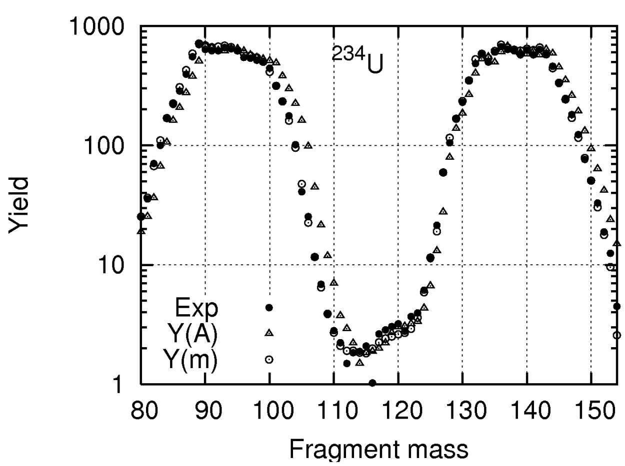

FIGURE 2. Simulation results for the primary (△) and final (⊙) mass yields are presented together with experimental data (•), taken from Ref. 4.

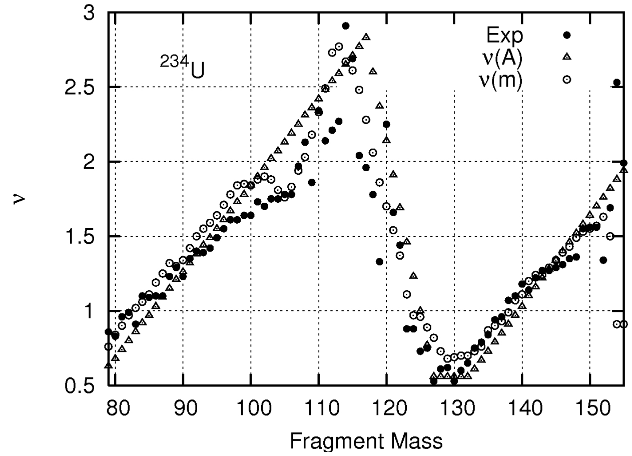

FIGURE 3.The average number of emitted neutrons from fission of ^{234}U: as a function of the primary (△) and final fragment mass (⊙), both as a result of simulation and experimental data (•), taken from Ref. 6.

As stated in Sec. 2, the primary kinetic energy ($E(A)$) is generated from a Gaussian distribution, while the final kinetic energy ($e(m)$) is calculated through Eq. (4).

The plots of the simulated average kinetic energy for the primary and final fragments as a function of their corresponding masses are shown in Fig. 4. In general, the simulated average final kinetic energy curve as a function of the final mass ($\bar{e}(m)$) displays a shift roughly similar to that of the $Y(m)$ curve. It is clearly noticed that shifts on $Y(m)$ and $\bar{e}(m)$ relative to $Y(A)$ and $\bar{E}(A)$, respectively, are greater for higher neutron multiplicity.

Furthermore, Fig. 5 displays the standard deviation of the kinetic energy distribution of the primary fragments and the SD of the kinetic energy of the final fragments ($\sigma_e(m)$). The simulated ($\sigma_E(A)$) curve does not present any peak.

The plots of $\sigma_e(m)$ reveal the presence of a pronounced peak around $m = 109$, in agreement with the experimental results obtained by Belhafaf *et al.* [4] and Faust *et al.* [5], respectively.

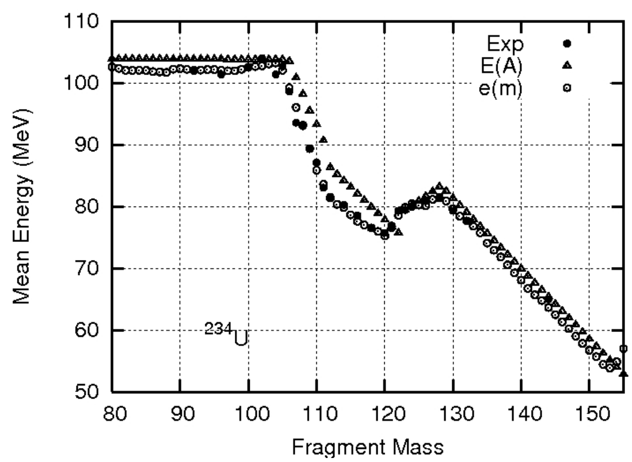

FIGURE 4. Average kinetic energy of the primary (△) and the final fragment (⊙), as a result of simulation in this work, to be compared to experimental data (•) from [4].

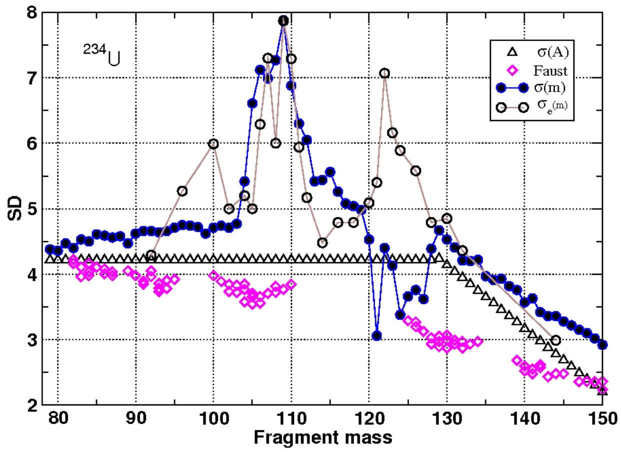

FIGURE 5. SD of primary fragments kinetic energy distribution (△), as simulated in this work, compared to results of calculations (◇) made by Faust *et al.* [5]; and SD of final fragment kinetic energy distribution (○) compared to experimental data (●) as measured by Belhafaf *et al.* [4].

The peak on the SD around $m = 122$ resulting from our simulation is not as great as that obtained by Belhafaf *et al.* Moreover, a depletion on the SD in the mass region from $m = 121$ to $m = 129$ is obtained as a result of simulation.

These results were obtained with a simulated primary fragment ($\sigma_E(A)$) without peaks in the range of fragment masses A from 90 to 145 [see Fig. 5 (△)]. If one simulates an additional source of energy dispersion in σ_E, without any peak, no peak will be observed on σ_e.

Both the shape and height of the peaks of $\sigma_e(m)$ are sensitive to the value of parameter β appearing in Eq. (1). The effect of β on peak depends to a great extent on mass region. For the region $m = 109$, a higher value of β will produce a larger peak of σ_e. The simulated results for $\sigma_e(m)$ presented in Fig. 5 were obtained with β=0.35. The presence of a peak at $m = 109$ could be associated with neutron emission characteristics (approximately $\bar{\nu} = 2$) and a very sharp fall in kinetic energy from E =96 MeV to E =90 MeV, corresponding to A=109 and A=111, respectively. A similar result was obtained for low energy fission of ^{236}U [7].

4. Schematic analytical interpretation

We are going to use a simple analytical way to interpret the effects of neutron emission, examining separately the influence of $\bar{E}(A)$ and $Y(A)$ variations, respectively, on $\sigma_e(m)$.

4.1. Influence of average kinetic energy variation on standard deviation of final kinetic energy distribution

In order to analytically evaluate the influence of the variation of $\bar{E}$ on $\sigma_e(m)$, we assume that i) Y(A) and $\sigma_E(A)$ are constant and ii) fragments with $E > \bar{E}$ do not emit neutrons and fragments with $E < \bar{E}$ emit one neutron. Then, for each final mass m there is a contribution from fragments with primary

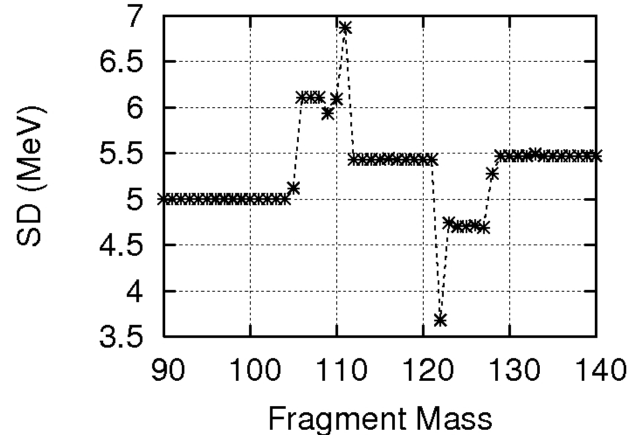

FIGURE 6. SD of final fragments kinetic energy distribution calculated under the assumption that i) Y(A) is constant, ii) $\sigma_E(A) = 5$ MeV. $E(A)$ values are taken from Fig. 4.

mass m that do not emit any neutron and from fragments with primary mass $m + 1$ that emit one neutron. With these conditions we can show that,

$$\sigma_e(m) = \left[\sigma_E^2 - \sqrt{\frac{2}{\pi}}\sigma_E \Delta\bar{E} + \left(\frac{\Delta\bar{E}}{2}\right)^2\right]^{\frac{1}{2}}, \qquad (6)$$

where $\Delta\bar{E} = \bar{E}(m+1) - \bar{E}(m)$.

As we can see on Fig. 6, the $\sigma_e(m)$ curve, calculated with relation (6), presents a peak around $m = 109$ in agreement with the experimental data. In that region $\Delta\bar{E} < 0$, so from relation (6) it follows that $\sigma_e(m) > \sigma_E(A)$.

The depletion on the simulated $\sigma_e(m)$ on the mass region between $m = 121$ and $m = 129$ is explained by the fact that in that mass region, $\Delta\bar{E} > 0$. Using relation (6), we obtain that $\sigma_e(m) < \sigma_E(A)$.

If we assume that fragments with $E > \bar{E}$ emit one neutron and fragments with $E < \bar{E}$ do not emit neutrons, then

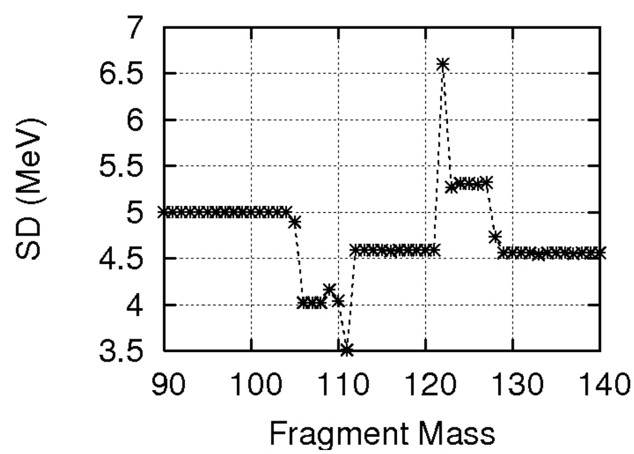

FIGURE 7. SD of final fragments kinetic energy distribution, calculated under the assumption thar ii) Y(A) is constant, ii) $\sigma_E(A)$ = 5 MeV. iii) fragments with $E > \bar{E}$ emit one neutron and fragments with $E < \bar{E}$ do not emit neutrons. $E(A)$ values are taken from Fig. 4.

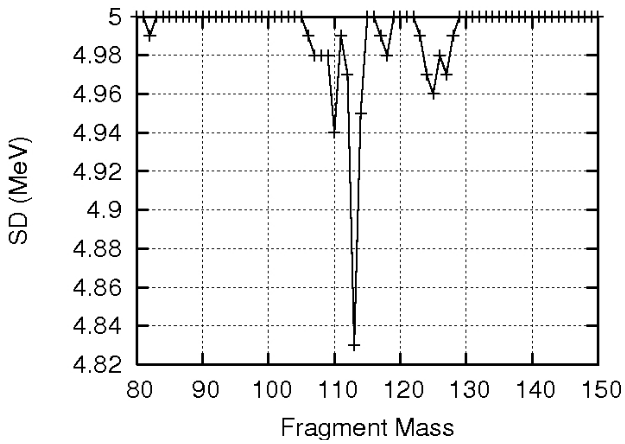

FIGURE 8. SD of final fragments kinetic energy distribution, calculated under the assumption that i) $\sigma_E(A)$=5 MeV (ii) $\bar{E}(m{+}1)$=$\bar{E}(m)$ iii) fragments with $E > \bar{E}$ do not emit neutrons and fragments with $E < \bar{E}$ emit one neutron. $Y(A)$ values are taken from Fig. 2.

for each final mass m there is a contribution from fragments with primary mass m that do not emit any neutron and from fragments with primary mass $m+1$ that emit one neutron. With these conditions we get,

$$\sigma_e(m) = \left[\sigma_E^2 + \sqrt{\frac{2}{\pi}}\sigma_E \Delta\bar{E} + \left(\frac{\Delta\bar{E}}{2}\right)^2\right]^{\frac{1}{2}}, \qquad (7)$$

where $\Delta\bar{E} = \bar{E}(m+1) - \bar{E}(m)$

This results shows how sensitive $\sigma_e(m)$ is to variation of $\bar{E}$ as a function of A.

As we can see in Fig. 7, the $\sigma_e(m)$ curve, calculated with relation (7), presents a peak around $m = 122$ in agreement with the experimental data. In that region $\Delta\bar{E} > 0$, so that from relation (7) it follows that $\sigma_e(m) > \sigma_E(A)$. The depletion in the simulated $\sigma_e(m)$on the mass region around $m = 109$ is explained by the fact that in this mass region, $\Delta\bar{E} < 0$. Using relation (7), we obtain that $\sigma_e(m) < \sigma_E(A)$.

4.2. Influence of mass yield variation on standard deviation of final kinetic energy distribution

In order to analytically evaluate the influence of the variation of $Y(A)$ on $\sigma_e(m)$, we assume that

i) $Y(A+1) = r\,Y(A)$,

ii) $\sigma_E(A)$ are constant,

iii) $\bar{E}(m+1) = \bar{E}(m)$ and

iv) neutron emission have no recoil effect on fragment kinetic energy.

Then we can show that,

$$\sigma_e(m) = \sigma_E\left[1 - \frac{2}{\pi}\left(\frac{1-r}{1+r}\right)^2\right]^{\frac{1}{2}}, \qquad (8)$$

The SD curve calculated with relation (8) is presented in Fig. 8. We can get a peak at $m = 122$ assuming that around this mass Y increases very rapidly with A except at $Y(123) = Y(122)$. However, we can not reproduce the pronounced peak obtained by Belhafaf *et al.* [4]

5. Conclusions

From results of our Monte Carlo simulation (validated by an simple analytical model) of an experiment measuring final mass and kinetic energy distribution of fragment from low energy fission of ^{234}U, we may conclude that:

i) there is no structures on the standard deviation of primary kinetic energy energy as a function of primary mass;

ii) the peak around $m = 109$ on standard deviation of final fragment kinetic energy as a function of final mass $\sigma_e(m)$, observed by Belhafaf *et al.* [4] and Faust *et al.* [5], are a result of high multiplicity of neutron emission and the variation of average kinetic energy in the neighboring of that mass value;

iii) our assumption of non-existence of a peak on standard deviation of primary kinetic energy distribution as a function of primary mass agrees with results of theoretical calculations obtained by Faust *et al.* [5], as opposed to the assumption of the existence of a peak around around $A = 122$ proposed by Belhafaf *et al.* [4].

This result suggests that shell effects on potential energy of the fissioning system ^{234}U are not necessarily reflected in the standard deviation of primary fragments kinetic energy as a function of mass.

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