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Donaldson-Witten invariants for flows on four manifolds

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After a survey of the cohomological quantum field theory, we review the computation of their Donaldson-Witten invariants. These invariants are generalized for smooth flows defined on the four manifold using notion of asymptotic cycles of higher dimensions than one introduced recently by S. Schwartzman.

Keywords: Topological field theory; supersymmetric Yang-Mills; topological invariants; dynamical Systems.

Después de repasar la teoría co-homológica de campos, revisamos los cálculos de los invariantes de Donaldson-Witten. Estos invariantes son generalizados para flujos suaves definidos en cuatro variedades usando la noción de ciclos asintóticos para dimensiones mayores que uno introducido recientemente por S. Schwartzman.

Descriptores: Teoría topológica de campos; Yang-Mills supersimétrico; invariantes topológicos; sistemas dinámicos

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1. Introduction

It is known that the Jones-Witten invariants [1] can be generalized in the presence of smooth vector fields on the 3-manifold as did Verjovsky and Vila [2] using the one-dimensional asymptotic cycles defined by S. Schwartzman [3]. Donaldson develop a theory to classify 4-dimensions, manifolds using the instantonic solutions (gauge theories) a mathematical review is given in Refs. 4 and 5. E. Witten in Refs. 6 and 7, using a twisted version of a $\mathcal{N}=2$ super Yang-Mills theory in four dimensions, construct a topological quantum field theory. We will focus in this context and extend the Donaldson-Witten theory in the case where there exist flows over the four manifold.

2. Asymptotic Cycles

For the Donaldson-Witten case it is necessary to consider flows of dimension greater than one (for an introduction to the theory of dynamical systems see [8]). in this case we will be interested in cycles of dimension $p = 0, 2, 4$ and therefore we will need a foliation of the manifold and define asymptotic cycles (elements of the n-th homology group) of higher dimensions, that will help us in order to generalize the Donaldson-Witten invariants.

To give a foliation in dimension p it is necessary to give a closed subset S over a smooth manifold M, divided into subsets L_α. Then, we will endow M of a collection of closed disks $\mathbf{D}^p \times \mathbf{D}^{n-p}$, where $\mathbf{D}^p$ is the horizontal disk and $\mathbf{D}^{n-p}$ is the vertical one. These sets are called *flow boxes* (see [9]), whose interior covers all M.

The measure, the orientation and the flow lines, will define a *geometric current* (L_α, μ). Suppose that M is covered by a system of flow boxes $(\mathbf{D}^p \times \mathbf{D}^{n-p})_i$ (endowed with partitions of unity). Then, every p-form ω can be decomposed into a finite sum $\omega = \sum_i \omega_i$, where each ω_i has its own support in the i-th flow box. Now we can integrate out every ω_i over each horizontal disk $(\mathbf{D}^p \times \{y\})_i$ and obtain a continuous function f_i over $(\mathbf{D}^{n-p})_i$. Thus we can take the average of this function using the transversal measures μ

$$\langle (L_\alpha, \mu), \omega\rangle = \int\limits_{(\mathbf{D}^{n-p})_i} \mu(dy) \left(\int_{(\mathbf{D}^p\times\{y\})_i} \omega_i \right). \qquad (1)$$

This current is closed in the sense of de Rham, *i.e.*, if $\omega{=}d\phi$ where ϕ has compact support, then $\langle (L_\alpha, \mu), d\phi\rangle = 0$, since we can write $\phi = \sum_i \phi_i$. Therefore the expression

$$\int\limits_{(\mathbf{D}^p\times\{y\})_i} d\phi_i$$

vanishes. Ruelle and Sullivan [10] shown that this is precisely an element of the p-th cohomology group.

A *quantifier* is a continuous field of p-vectors over M, tangent to the orbits and invariant under the action of L. A quantifier is called positive if it is distinct from zero in every point of M and determines the orientation of the tangent space of the orbit. Some useful results that will justify the existence of cycles is given Refs. in 9 and 11: The asymptotic cycles are provided by

$$A_\mu = \int\limits_M (\omega, \nu) d\mu,$$

which is an element of $H_p(M, \mathbb{R})$. Now If ν is a positive definite quantifier and μ an invariant measure coming from a n-form ω, then $\omega \lrcorner \nu$ is closed and A_μ can be obtained by Poincaré duality, and it is an element of $H^{n-p}(M, \mathbb{R})$ determined by $\omega \lrcorner \nu$. This is an important result given by Schwartzman [9].

This is not the only way to specify a foliation. In [11], Sullivan defines structures of p-cones and operators acting over the vectors of the cones.

3. Donaldson-Witten Invariants

We will discuss briefly the Donaldson-Witten invariants. We will focus in the Witten description [6] in terms of correlation functions (expectation values of some operators).

We only consider a BRST supercharge Q and satisfy $Q^2 = 0$. Now we need a BRST invariant action, which is very important because it gives topological invariants [7]. Infinitesimal BRST transformations yield

$$\begin{aligned} [Q, A^a_\mu] &= \psi^a_\mu, \\ \{Q, \psi^a_\mu\} &= -D_\mu \phi^a, \\ [Q, \phi^a] &= 0, \end{aligned} \tag{2}$$

where A^a_μ is the gauge field, ψ^a_μ a fermionic field and ϕ a scalar field. We define a *BRST exact commutator* if it can be written as $\{Q, \Omega\}$ for some Ω. One fact is that the energy-momentum tensor $T_{\alpha\beta}$ is covariantly conserved and it can be written as one of these BRST commutators $T_{\alpha\beta} = \{Q, \lambda_{\alpha\beta}\}$, where $\lambda_{\alpha\beta}$ is some operator. Also we can observe that the lagrangian $\mathcal{L}$ is invariant if it is a BRST exact operator $\mathcal{L} = \{Q, V\}$. We define the Donaldson-Witten invariants with the path integral formalism, where the Donaldson polynomials are given by correlation functions,

$$\langle \mathcal{O} \rangle = \mathcal{Z}(\mathcal{O}) = \int \mathcal{DX} \exp\{-\mathcal{L}/\rceil^{\in}\}\mathcal{O}, \tag{3}$$

where the $\mathcal{DX}$ represent de measure of the fields of the theory, e is the coupling constant and $\mathcal{O}$ are the observables of the theory.

One of the main properties of the path integral for BRST exact operators $\{Q, \Omega\}$, is that the expectation value is zero $\big\langle \{Q, \Omega\} \big\rangle$ for all Ω, if $\mathcal{DX}$ is invariant under supersymmetry. An important fact, is that in order to $\mathcal{Z}$ be a topological invariant (metric independent) is necessary that satisfies the property $\big\langle \{Q, \mathcal{O}\} \big\rangle = 0$, for all $\mathcal{O}$ (Donaldson-Witten polynomials). With this property we can prove that the correlation functions are independent of the metric $g_{\mu\nu}$ on M and the coupling constant e, i.e. $\delta_g \mathcal{Z} = 0$ and $\delta_e \mathcal{Z} = 0$. Then $\mathcal{Z}$ can be evaluated when e is small and give us topological invariants. The observables of the theory can be constructed as,

$$dW_{\gamma_{k-1}} = \{Q, W_{\gamma_k}\}, \tag{4}$$

with $W_{\gamma_0}(x) = (1/8\pi^2)\mathrm{Tr}\phi^2(\mathrm{x})$. This observables are not BRST invariant, but if we consider the product $(\cdot,\cdot) : H_k(M) \times H^k(M) \to \mathbb{R}$

$$\mathcal{O}^{\gamma_k} \equiv (\gamma_k, W_{\gamma_k}) = \int_{\gamma_k} W_{\gamma_k}, \tag{5}$$

for $k = 0$ $W_{\gamma_0} = \mathcal{O}^{\gamma_0}$. This define BRST invariants, *i.e.*

$$\{Q, \mathcal{O}^{\gamma_k}\} = \int_{\gamma_k} \{Q, W_{\gamma_k}\} = \int_{\gamma_k} dW_{\gamma_{k-1}} = 0. \tag{6}$$

For that reason the BRST commutator of $\mathcal{O}_{\gamma_k}$ only depends of the homology class of γ, then the correlation functions are written as

$$\langle \mathcal{O}^{\gamma_{k_1}} \ldots \mathcal{O}^{\gamma_{k_r}} \rangle = \int (\mathcal{DX}) \exp(-\mathcal{L}/e^2) \prod_{i=1}^{r} \int_{\gamma_{k_i}} W_{\gamma_{k_i}}. \tag{7}$$

At this point we have defined the Donaldson invariants on a 4-manifold. In a natural way we will extend this invariants to the moduli space.

We choose a moduli space $\mathcal{M}$ such that $d(\mathcal{M}) = n$ where n is a positive integer, that supposition assume that the fields (ϕ, λ) don't have zero modes, the only zero modes are those of the gauge field A_α tangent to $\mathcal{M}$ therefore the zero modes are the associated to ψ_α. After integrating in the limit of weak coupling ($e \to 0$) the non-zero modes, we obtain the measure $da_1 \ldots da_n d\psi_1 \ldots d\psi_n$, where a_i, ψ_j are the bosonic and fermionic zero modes. The effective functional $\mathcal{O}'$ that only depends of the zero modes is $\mathcal{O}' = \Phi_{i_1,\ldots i_n}(a^k)\psi^{i_1} \ldots \psi^{i_n}$, where Φ is an n-form in $\mathcal{M}$. Substituting the last expression and the measure in (7) we obtain $\mathcal{Z}(\mathcal{O}) = \int_{\mathcal{M}} \Phi$ explicitly

$$\mathcal{Z}(\mathcal{O}) = \int_{\mathcal{M}} da_1 \ldots da_n d\psi^1 \ldots d\psi^n \Phi_{i_1,\ldots,i_n} \psi^{i_1} \ldots \psi^{i_n}. \tag{8}$$

Suppose that $\mathcal{O} = \mathcal{O}_1 \ldots \mathcal{O}_\ell$ with $\sum_{r=1}^{\ell} n_r = n$ where n_r is the number of zero modes of $\mathcal{O}^{r_k}$. Integrating out the non-zero modes of every one, we obtain $\mathcal{O}'_r = \Phi^{(r)}_{i_1,\ldots i_{n_r}}(a^k)\psi^{i_1} \ldots \psi^{i_{n_r}}$, with $(1 \leq r \leq \ell)$ where $\Phi^{(r)}_{i_1,\ldots i_{n_r}}$ is a n_r-form in $\mathcal{M}$, and we can write $\Phi = \Phi^{(1)} \wedge \ldots \wedge \Phi^{(\ell)}$. Integrating out over the non-zero modes and substituting in (7) we obtain

$$\mathcal{Z}(\mathcal{O}_{\alpha_1} \ldots \mathcal{O}_{\alpha_\ell}) = \int_{\mathcal{M}} \Phi^{(\alpha_1)} \wedge \ldots \wedge \Phi^{(\alpha_\ell)}. \tag{9}$$

So the differential forms in $\mathcal{M}$ are

$$\Phi^{(\gamma_0)} = \frac{1}{8\pi^2} \mathrm{Tr} \langle \phi \rangle^2, \tag{10}$$

$$\Phi^{(\gamma_1)} = \int_{\gamma_1} \mathrm{Tr} \left(\frac{1}{4\pi^2} \langle \phi \wedge \psi \rangle \right), \tag{11}$$

$$\Phi^{(\gamma_2)} = \int_{\gamma_2} \frac{1}{4\pi^2} \mathrm{Tr}\Big(-\mathrm{i}\psi \wedge \psi + \langle \phi \rangle \mathrm{F} \Big), \tag{12}$$

$$\Phi^{(\gamma_3)} = \int_{\gamma_3} \frac{1}{4\pi^2} \mathrm{Tr}\Big(\psi \wedge \mathrm{F} \Big), \tag{13}$$

$$\Phi^{(\gamma_4)} = \frac{1}{8\pi^2} \int_M \mathrm{Tr}\Big(\mathrm{F} \wedge \mathrm{F} \Big). \tag{14}$$

Here the $W'_{\gamma_k}s$, has ghost number $U = 4 - k_\gamma$, therefore for each $\mathcal{O}^{\gamma_k}$ we can associate a $(4 - k_\gamma)$-form $\Phi^{(\gamma_k)}$ over $\mathcal{M}$. This correspond to the *Donaldson map*

$H_k(M) \to H^{4-k}(\mathcal{M})$. Witten [6] proved that this elements are in the cohomology class of $\mathcal{M}$. Finally we will take the following convention for 2-cycles γ_2 we will denote it by Σ, whose codimension is 2 and the Donaldson-Witten invariants (the intersection number)

$$\langle \mathcal{O}^{\gamma_{k_1}} \ldots \mathcal{O}^{\gamma_{k_r}} \rangle = \# \left(H_{\gamma_{k_1}} \cap \ldots \cap H_{\gamma_{k_r}} \right). \qquad (15)$$

4. Donaldson-Witten Invariants for Flows

Now we generalize the Donaldson-Witten invariants, when there exist flows over a manifold M.

Consider a positive quatifier ν_p over M and a system of flow boxes $\mathbf{D}_1^p \times \mathbf{D}_1^{n-p}, \ldots, \mathbf{D}_j^p \times \mathbf{D}_j^{n-p}$ such that this cover all the manifold. Here p is the dimension of each disk. Exist a set of transversal measures $\tau_1, \ldots, \tau_j$ with support in the Borel sets of each transversal disk $\mathbf{D}_1^{n-p}, \ldots, \mathbf{D}_j^{n-p}$, then the geometric current is given by

$$\langle (L_\alpha, \tau), W_{\gamma_p} \rangle = \sum_i \int_{(\mathbf{D}^{n-p})_i} \tau_i(dy) \times \left(\int_{(\mathbf{D}^p \times \{y\})_i} W_{\gamma_p, i} \right), \qquad (16)$$

where $W_{\gamma_p,i}$ are the differential forms in (4) associated to the observables with support in the i-th flow box *i.e.*, $W_{\gamma_p} = \sum_{i=1}^{j} W_{\gamma_p,i}$.

These currents are closed and they are the analogous to the "winding number" [2] of one-dimensional cycles and by duality defines an asymptotic cycle in $H_p(M, \mathbb{R})$ which is a functional $H^p(M; \mathbb{R}) \to \mathbb{R}$ and we define the "asymptotic" operator inspired in terms of the winding cycles [2] as follows, when we do not have a quantifier, we will return to the case without flows. Let a positive quantifier $\nu_1, \ldots, \nu_4$ of dimension $1, \ldots, 4$ respectively and together with the currents and flow boxes define the *"asymptotic" observable* as

$$\mathcal{O}^{\gamma_k}(\mu, \nu_k) = \int_M \widetilde{W}_{\gamma_k} d\mu = \int_M W_{\gamma_k} \lrcorner \nu_1 d\mu$$

(here $\widetilde{W}_{\gamma_i}$ denotes the contraction $W_{\gamma_i} \lrcorner \nu_i$ and $d\mu$ is the volume form given by a 4-form). We can interpret the integral as averaged cycles in the flow boxes.

Also we are working in a smooth manifold, then every volume element is given by a 4-form $d\mu = \omega$. Making use of the Schwartzman theorem, $\omega \lrcorner \nu_p$ is a closed $(4-p)$-form, from which we will obtain a asymptotic p-cycle by the Poincaré duality [an element of the $H_p(M, \mathbb{R})$].

Remark: In this case we only consider the case where W_{γ_p} and ν_p are a p-form and a p-vector respectively, otherwise the observables $\mathcal{O}^{\gamma_k}(\mu, \nu_k)$, will be indefinite

$$\mathcal{O}^{\gamma_1}(\mu, \nu_1) = \int_M \mathrm{Tr} \frac{1}{4\pi^2} (\phi\psi) \lrcorner \nu_1 \mathrm{d}\mu, \qquad (17)$$

$$\mathcal{O}^{\gamma_2}(\mu, \nu_2) = \int_M \mathrm{Tr} \frac{1}{4\pi^2} (-\mathrm{i}\psi \wedge \psi + \phi \mathrm{F}) \lrcorner \nu_2 \mathrm{d}\mu, \qquad (18)$$

$$\mathcal{O}^{\gamma_3}(\mu, \nu_3) = \int_M \mathrm{Tr} \frac{1}{4\pi^2} (\psi \wedge \mathrm{F}) \lrcorner \nu_3 \mathrm{d}\mu, \qquad (19)$$

$$\mathcal{O}^{\gamma_4}(\mu, \nu_4) = \int_M \mathrm{Tr} \frac{1}{8\pi^2} (\mathrm{F} \wedge \mathrm{F}) \lrcorner \nu_4 \mathrm{d}\mu. \qquad (20)$$

These asymptotic observables satisfy the following properties:

$$\{Q, \int_{\gamma_{k+1}} W_{\gamma_{k+1}}\} = \{Q, \int_M W_{\gamma_{k+1}} \lrcorner \nu_{k+1} d\mu\}$$
$$= \int_M \{Q, W_{\gamma_{k+1}}\} \lrcorner \nu_{k+1} d\mu = \int_M dW_{\gamma_k} \lrcorner \nu_{k+1} d\mu = 0. \qquad (21)$$

Here we have used the fact that the measure is invariant with respect to the flow. These asymptotic observables are BRST invariant, this is an important property because the expectation values of the observables $\mathcal{O}^{k_j}$ will be topological invariants (of the dynamical system) and they are independent of the cohomology class.

We will use the following notation for cycles of different dimension. The observables will be denoted by $\mathcal{O}^{k_j}(\mu, \nu_{k_j})$, where k_j take values $0, 1, \ldots, 4$, and they satisfy $\sum_{j=1}^{r} k_j = d(\mathcal{M})$.

For a simply connected 4-dimensional closed and oriented manifold M with quantifiers ν_i con $i = 0, 1, \ldots, 4$, and a probability invariant measure μ, we define the correlation functions as

$$\mathcal{Z}_{\nu\mu}(\mathcal{O}^{k_1}(\mu, \nu_{k_1}), \ldots, \mathcal{O}^{k_r}(\mu, \nu_{k_r})) = \int (\mathcal{DX}) \exp(-\mathcal{L}/e^2) \prod_{j=1}^{r} \int_M \widetilde{W}_{k_j} d\mu. \qquad (22)$$

This expression is reduced to the case of Donaldson-Witten (7), when the measure is localized on the cycles. This means if

$$\mu = \sum_{i=1}^{r} \mu_i$$

where each μ_i is distributed uniformly over the cycles. If t_a and t_b are generators of the Lie algebra $su(N)$ they satisfy $\mathrm{Tr}(\mathrm{t_a t_b}) = \mathrm{N}\delta_{\mathrm{ab}}$. For our case we have $N = 2$ for example for 2-cycles

$$\mathcal{O}^{\gamma_2} = \int_{\gamma_2} \frac{1}{2\pi^2} (-i\psi^a \wedge \psi^a + \phi^a F^a). \qquad (23)$$

Our case is non-abelian ($su(2)$), but it doesn't present any problem, because we are working at the level of the Lie algebra and not directly with the group, therefore we don't have the problem that appears in the Jones-Witten case [2].

Now proceed to perform the integral over the non-zero modes, as in the case without flows. Denote this observable by $\mathcal{O'}^{k_j}(\mu,\nu_{k_j}) = \tilde{\Phi}_{i_1,\ldots,i_n}(a_i,\nu_{k_j})\psi^{i_1}\ldots\psi^{i_n}$, where a_i denotes the zero modes of the gauge field and ψ are the zero modes of the fermionic field, $\tilde{\Phi}(a,\nu_{k_j})$ is a function that only depends of the zero modes of the gauge field and contains the information of the flow. Once we integrate out over the fermionic degrees of freedom we obtain

$$\mathcal{Z}(\mathcal{O}'^{k_j}(\mu,\nu_{k_j})) = \int\limits_{\mathcal{M}} \tilde{\Phi}_{1,\ldots,n} da_1 \ldots da_n. \qquad (24)$$

where $\tilde{\Phi}_\mu^{k_j}$ is a n-form in the moduli space. The zero modes of the gauge field can be regarded as a basis of a n-form in the moduli space *i.e.*, $da_1 \ldots da_n$ can be used as a volume element.

On the other hand, we know that the moduli space $\mathcal{M}$ has M as boundary, we suppose $a_1,\ldots,a_4$ are the coordinates over M, therefore is possible to see the moduli space locally as a foliation (collar) $M \times F$, where F is a manifold of dimension $n-4$. The total dimension is determined by $d(\mathcal{M}) = 8p_1(E) - \frac{3}{2}(\chi(M)+\sigma(M)))$.

We would be able to induce "flow boxes" over the moduli space. Given the volume element in (24) $da_1,\ldots,da_n$, which is a n-form, where the first four coordinates correspond to the space-time, we say $da_1\wedge\ldots\wedge da_4 = \alpha$ define a volume element in M. Given the quantifier ν_{k_j}, together with α will define an asymptotic cycle of the Schwartzman theorem, transversal to F, therefore $H^n(\mathcal{M})$ can be separated in a part associated to the flow $H^{k_j}(\mathcal{M})$ (the Ruelle-Sullivan class) and a $(n-k_j)$-form where it doesn't exist flow, i.e we get

$$\int\limits_{\mathcal{M}} \tilde{\Phi}_{1,\ldots,n} da_1 \ldots da_n$$
$$= \sum_i \int\limits_{F_i} \left[\int\limits_{\mathbf{B}_i\times\{y\}} \tilde{\Phi}_{1,\ldots,n} da_1 da_2 da_3 da_4 \right] da_5 \ldots da_n. \quad (25)$$

This expression is analogous to the currents defined by (16), where $\mathbf{B}$ is a flow box contained in M and y belongs to F. Consider the expression between the bracket, as a winding cycle associated to the flow of the boundary of $\mathcal{M}$, as follows:

$$A_{\mu_4} = \int\limits_B \tilde{\Phi}_{1,\ldots,n} \lrcorner \nu_k d\mu_4, \qquad (26)$$

this expression define the asymptotic cycle in the moduli space, in the sense that $A_{\mu_4} \times F$, where $d\mu_4 = da_1 da_2 da_3 da_4$.

Now we proceed to integrate out the zero modes, completely analogous to the case without flows. We going back again to the Donaldson map, since the path integral is integrated over the fields, not over the space-time coordinates. Finally we obtain the map $H_k(M) \to H^{4-k}(\mathcal{M})$ given by:

$$\Phi_\mu^{\gamma_1} = \int\limits_M \mathrm{Tr}\frac{1}{4\pi^2}(\langle\phi\rangle\psi)\lrcorner\nu_1 \mathrm{d}\mu, \qquad (27)$$

$$\Phi_\mu^{\gamma_2} = \int\limits_M \mathrm{Tr}\frac{1}{4\pi^2}(-\mathrm{i}\psi\wedge\psi + \langle\phi\rangle\mathrm{F})\lrcorner\nu_2 \mathrm{d}\mu, \qquad (28)$$

$$\Phi_\mu^{\gamma_3} = \int_M \mathrm{Tr}\frac{1}{4\pi^2}(\psi\wedge\mathrm{F})\lrcorner\nu_3 \mathrm{d}\mu, \qquad (29)$$

$$\Phi_\mu^{\gamma_4} = \int\limits_M \mathrm{Tr}\frac{1}{8\pi^2}(\mathrm{F}\wedge\mathrm{F})\lrcorner\nu_4 \mathrm{d}\mu. \qquad (30)$$

As it was seen previously these forms contain information about the asymptotic operators of M that are induced locally over the boundary of the moduli space. Now we will define the intersection number analogous in the case without flows

$$\langle \mathcal{O}^{\gamma_{k_1}}(\mu,\nu_{k_1})\ldots\mathcal{O}^{\gamma_{k_r}}(\mu,\nu_{k_1})\rangle$$
$$= \#\left(H_{\tilde{\gamma}_{k_1}} \cap \ldots \cap H_{\tilde{\gamma}_{k_r}}\right). \qquad (31)$$

Here H_{γ_i} are the asymptotic cycles in $\mathcal{M}$ dual to $\tilde{\Phi}_\mu^{\gamma_i}$.

5. Conclusions

We have extended the Donaldson-Witten invariants in the case that exist flows over the four manifold. We recover the case without flows when the measure are localized on the cycles. The measure defined over all the manifold give an average of the flow. In a future work we will generalize the Donaldson-Witten invariants on Kähler manifolds, and the Seiberg-Witten invariants.

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1. E. Witten, *Commun. Math. Phys.* **121** (1989) 351.
2. A. Verjovsky and R. Vila Freyer, *Commun. Math. Phys.* **163** (1994) 73.
3. S. Schwartzman, *Ann. Math.* **66** (1957) 270.
4. M. Freedman and F. Quinn, *Topology of Four Manifolds*, (Princeton University Press, Princeton, 1990) Math. Series No. 39.
5. S.K. Donaldson and P. Kronheimer, *The Geometry of Four Manifolds*, (Oxford University Press, Oxford, 1990).
6. E. Witten, *Commun. Math. Phys.* **117** (1988) 353.
7. E. Witten, *Int. J. Mod. Phys. A* **6** (1991) 2775.
8. D.K. Arrowsmith and C.M. Place, *An Introduction to Dynamical Systems*, (Cambridge University Press, Cambridge 1990).
9. S. Schwartzman, *Canad. J. Math.* **55** (2003) 636.
10. D. Ruelle and D. Sullivan, *Topology* **14** (1975) 319.
11. D. Sullivan, *Invent. Math.* **36** (1976) 225.