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Quintessence with induced gravity

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We study the cosmic evolution in recent times within the induced gravity theory. Taking into account the experimental constraints on the parameter of the theory, we study the quintessence dynamics. We obtain the possible models and discuss the physical consequences in cosmology and particle physics.

Keywords: Cosmology; quintessence; dark energy; induced gravity.

Se estudia la evolución cosmológica en épocas actuales en la teoría de gravedad inducida. Dadas las constricciones experimentales sobre los parámetros de la teoría, se estudia la dinámica de quintaesencia del modelo. Se obtienen los posibles modelos y se discuten sus consecuencias en física de partículas y cosmología.

Descriptores: Cosmología; quintaesencia; energía oscura; gravedad inducida.

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1. Introduction

Since about nine years two supernovae groups, the Supernovae Cosmology Project and the High-Z Supernovae Search Team, has been providing evidence for an accelerated expansion of the Universe [1, 2]. This dynamics seems to has been happening since a redshift of about $z \sim 1$; for a review see Ref. 3. In recent years this discovery has gained more evidence from other independent experiments, *i.e.*, measurements from the cosmic background radiation [4] and galaxy surveys [5]. A popular way to explain this effect is either to introduce a cosmological constant or to introduce scalar fields in the dynamics of General Relativity (GR). The potential associated to this field can accomplish the desired accelerated expansion, as it does for the inflationary scenario. Since we do not know the nature of the proposed field, it has been called quintessence, presuming that it has a different origin from the other four fields that form the building blocks of the Universe: light, baryons, neutrinos and dark matter, see [6–8].

A different possibility is to couple scalar fields non-minimally to gravity, within the framework of scalar-tensor theories (STT) [9]. In the present work we use this scheme, but more precisely we employ a theory that couples to gravity (R) with a Higgs field (ϕ) through the non-minimal coupling ($\phi^2 R$). According to the Higgs mechanism, the field evolves to its energy minimum to give rise to the mass of some elementary particles. In our case, the Higgs produces a boson mass whose nature is yet unknown. That is to say, we use a theory with a Higgs mechanism of the type of the standard model of particle physics. The scalar field evolves to a constant, to generate the boson mass, and simultaneously the non-minimal coupling becomes a constant, generating General Relativity (GR). The resulting theory is called induced gravity (IG) because GR is dynamically obtained through a Higgs mechanism from a STT [10, 11].

1.1. Field equations

The STT Lagrangian is [11]:

$$\mathcal{L} = \left[\frac{\alpha}{16\pi}\phi^\dagger \phi R + \frac{1}{2}\phi^\dagger_{;\mu}\phi^{;\mu} - V(\phi^\dagger\phi) + L_M\right]\sqrt{-g}\,, \quad (1)$$

where ϕ is a Higgs field and α is an adimensional parameter to be later determined.

The Higgs potential is of Landau-Ginzburg type:

$$V(\phi) = \frac{\lambda_a}{24}\left(|\phi|^2 + 6\frac{\mu^2}{\lambda_a}\right)^2 . \quad (2)$$

The Higgs ground state, v, is given by:

$$v^2 = -\frac{6\mu^2}{\lambda_a} \quad (3)$$

with $V(v) = 0$, where λ_a is an adimensional constant, whereas $\mu^2 < 0$.

In IG, the Higgs potential $V(\phi)$ plays the role of a cosmological function during the time in which ϕ goes from an initial state to its ground state, v, whose value is determined by the Newtonian gravitational constant:

$$G = \frac{1}{\alpha v^2}\,, \quad (4)$$

through which α is also determined.

In this way the Newtonian gravitational constant is related to the boson mass (M_b), such that:

$$M_b = \sqrt{\pi} g v\,, \quad (5)$$

where g is coupling constant of the fundamental theory, i.e. standard model of particle physics.

As a consequence of these two last equations, one has that

$$\alpha = 2\pi\left(g\frac{M_{Pl}}{M_b}\right)^2 , \quad (6)$$

where $M_{Pl} \equiv 1/\sqrt{G} = 10^{19}$GeV is the Planck mass. For if we consider the standard model Higgs, one has the W-boson, such that $M_b = M_{\rm W} = 80$GeV and $g = 0.18$, therefore $\alpha \approx 10^{33}$. Such value is huge and would not pass some cosmological constraints on α that we will comment below. Therefore, here we consider a STT with an undetermined α that will correspond to another Higgs mass, different from the Higgs of the standard model of particle physics.

From Eq. (1) the gravity eqs. are:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{8\pi V(\phi^\dagger\phi)}{\alpha\phi^\dagger\phi}g_{\mu\nu} = -\frac{8\pi}{\alpha\phi^\dagger\phi}T_{\mu\nu} - \frac{8\pi}{\alpha\phi^\dagger\phi}\left[\phi^\dagger{}_{(||\mu}\phi_{||\nu)} - \frac{1}{2}\phi^\dagger_{||\lambda}\phi^{||\lambda}g_{\mu\nu}\right] - \frac{1}{\phi^\dagger\phi}\left[(\phi^\dagger\phi)_{|\mu||\nu} - (\phi^\dagger\phi)^{|\lambda}{}_{||\lambda}g_{\mu\nu}\right], \quad (7)$$

where the parenthesis stand for symmetric sum on subindices and the covariant derivative $||$ includes the gravitational covariant derivative (;) plus a component from the gauge fields [11] of the particle theory.

On the other hand, the scalar field fulfills:

$$\phi^{||\lambda}{}_{||\lambda} + \frac{\delta V}{\delta\phi^\dagger} - \frac{\alpha}{8\pi}R\phi = 2\frac{\delta L_M}{\delta\phi^\dagger}. \quad (8)$$

To analyze the cosmological solution around the background field (v), it is convenient to change variable to the excited Higgs field, χ:

$$\phi \equiv v\sqrt{1+2\chi}N, \qquad \phi^\dagger\phi = v^2(1+2\chi)N^\dagger N = v^2(1+2\chi), \quad (9)$$

where N is a constant matrix. With this definition the potential takes the simple form

$$V(\chi) = \frac{\lambda_a v^4}{6}\chi^2 = (1+\tfrac{4\pi}{3\alpha})\frac{3}{8\pi G}M_H^2\chi^2, \quad (10)$$

which vanishes in its state of lowest energy, $V(\chi=0)=0$.

Note that $V(\chi) \sim M_{Pl}^2 M_H^2\chi^2$. This mass relationship is due to the fact that Eq. (4) is necessary to get GR once the symmetry breaking takes place. Then, through Eqs. (7) and (4) one recovers GR at the ground state, which is at $\chi,\, \dot\chi \sim 0$,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi G\ \tilde{T}_{\mu\nu}, \quad (11)$$

where $\tilde{T}$ is the effective energy-momentum tensor of the IG theory, which contains a contribution from the gauge bosons; see details in [11].

Newton's gravitational constant is here a function, given by:

$$G(\chi) = \frac{1}{\alpha v^2}\frac{1}{1+2\chi} \quad (12)$$

such at $G(\chi=0)=G$, using Eq. (4).

2. FRW cosmological equations

The Universe is quite isotropic, up to one part in one hundred thousand. Therefore, to study the recent Universe evolution we use the FRW metric. Accordingly, the above gravity equations (7) go into:

$$\frac{\dot a^2 + k}{a^2} = \frac{1}{1+2\chi} \times \left(\frac{8\pi G}{3}[\rho + V(\chi)] - 2\frac{\dot a}{a}\dot\chi + \frac{4\pi}{3\alpha}\frac{\dot\chi^2}{1+2\chi}\right), \quad (13)$$

$$2\frac{\ddot a}{a} + \frac{\dot a^2 + k}{a^2} = \frac{1}{1+2\chi}\left[8\pi G(-{\rm p} + V(\chi)) - 2\ddot\chi - 4\frac{\dot a}{a}\dot\chi - \frac{4\pi}{\alpha}\frac{\dot\chi^2}{1+2\chi}\right]. \quad (14)$$

On the other hand, the Higgs equation becomes:

$$\ddot\chi + 3\frac{\dot a}{a}\dot\chi + M_H^2\chi = 0, \quad (15)$$

whereas the total energy conservation gives, for a barotropic equation of state for the matter present ${\rm p} = {\rm w}\rho$,

$$\dot\rho + 3\frac{\dot a}{a}(1+{\rm w})\rho = \frac{\dot\chi}{1+2\chi}(1-3{\rm w})\rho, \quad (16)$$

which has a source term due to the Higgs field. One can solve it in an exact form:

$$\rho a^{3(1+{\rm W})} = M_{\rm W}(1+2\chi)^{\frac{1}{2}(1-3{\rm W})}, \quad (17)$$

where $M_{\rm W}$ is an integration constant. If ${\rm w}=0$, M_0 is total mass of the Universe.

3. Initial conditions and parameter constrictions

In order to be able to integrate the above-mentioned cosmological equations we need to determine the initial conditions of integration and the cosmological parameters. From the previous equations it is inferred that it is necessary to specify: ρ, a, $\dot a$, χ, $\dot\chi$ at some initial time. Additionally, one requires the following cosmological parameters: k, w, M_H, α. These conditions and parameters will be following determined.

Since we are interested to identify our Higgs field as a quintessence field, we integrate the equations from the time of the last scattering, also called decoupling (dec), to present times. Accordingly, taking the cosmological parameters from [4], one has:

$$z_{dec} = 1089,$$
$$t_{dec} = 379{\rm kyr} = 1.19\times 10^{13}{\rm s},$$

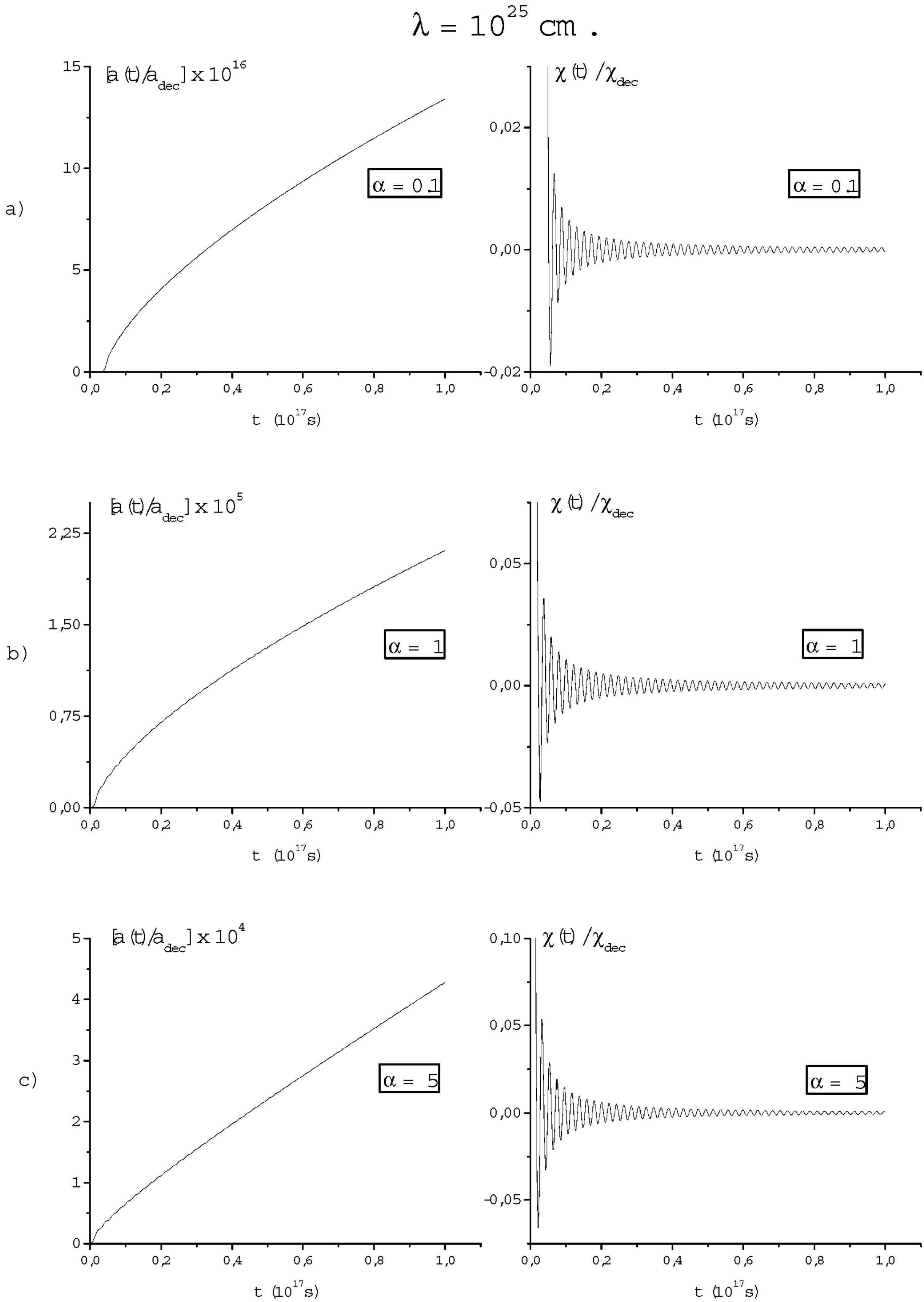

FIGURE 1A. Evolution of the scale factor, a, and the Higgs field, χ, for $\lambda = 10^{25}$cm, with $\alpha = 0.1, 1.0, 5.0$.

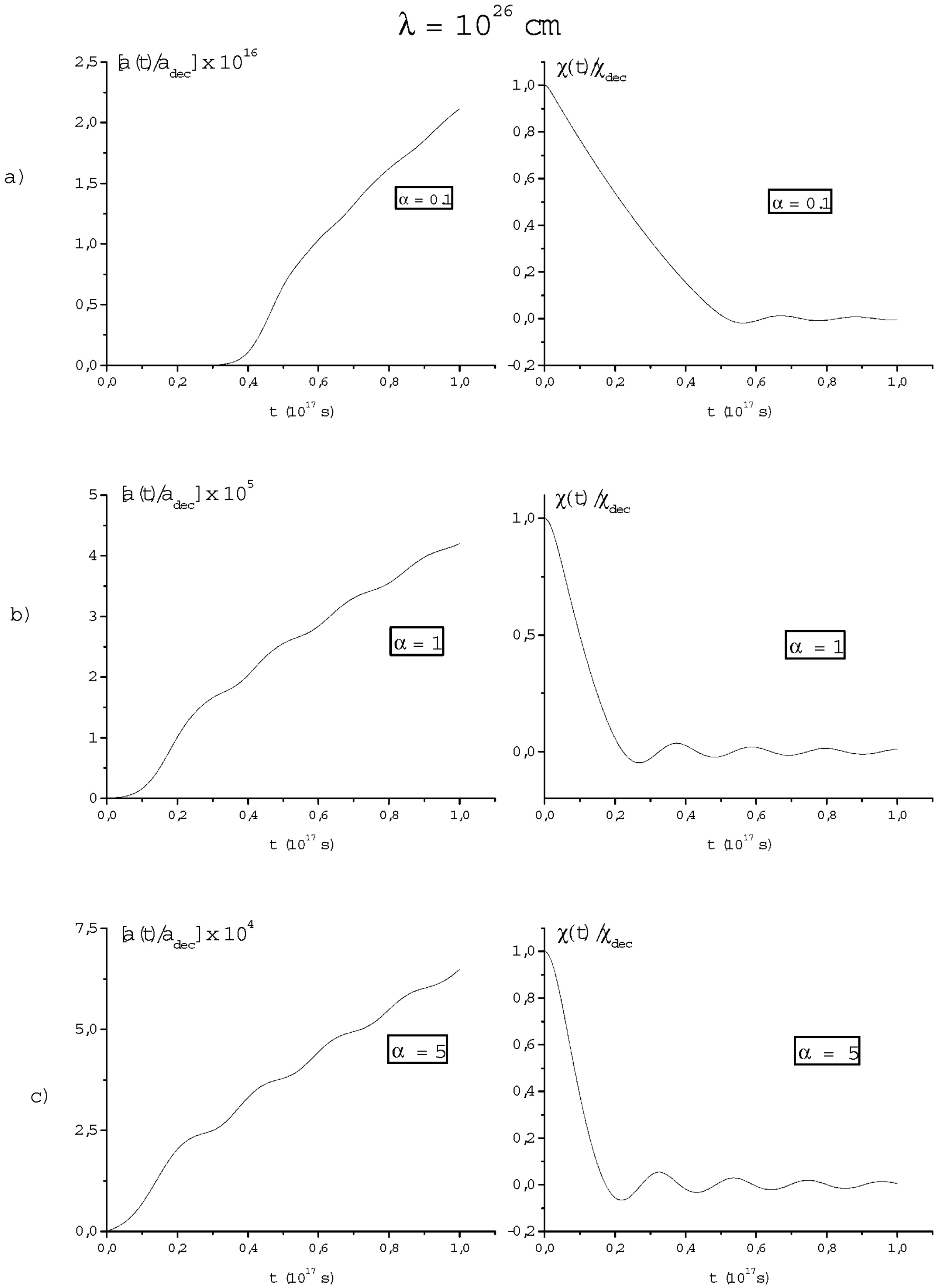

FIGURE 1B. Evolution of the scale factor, a, and the Higgs field, χ, for $\lambda = 10^{26}$cm, with $\alpha = 0.1$, 1.0, 5.0.

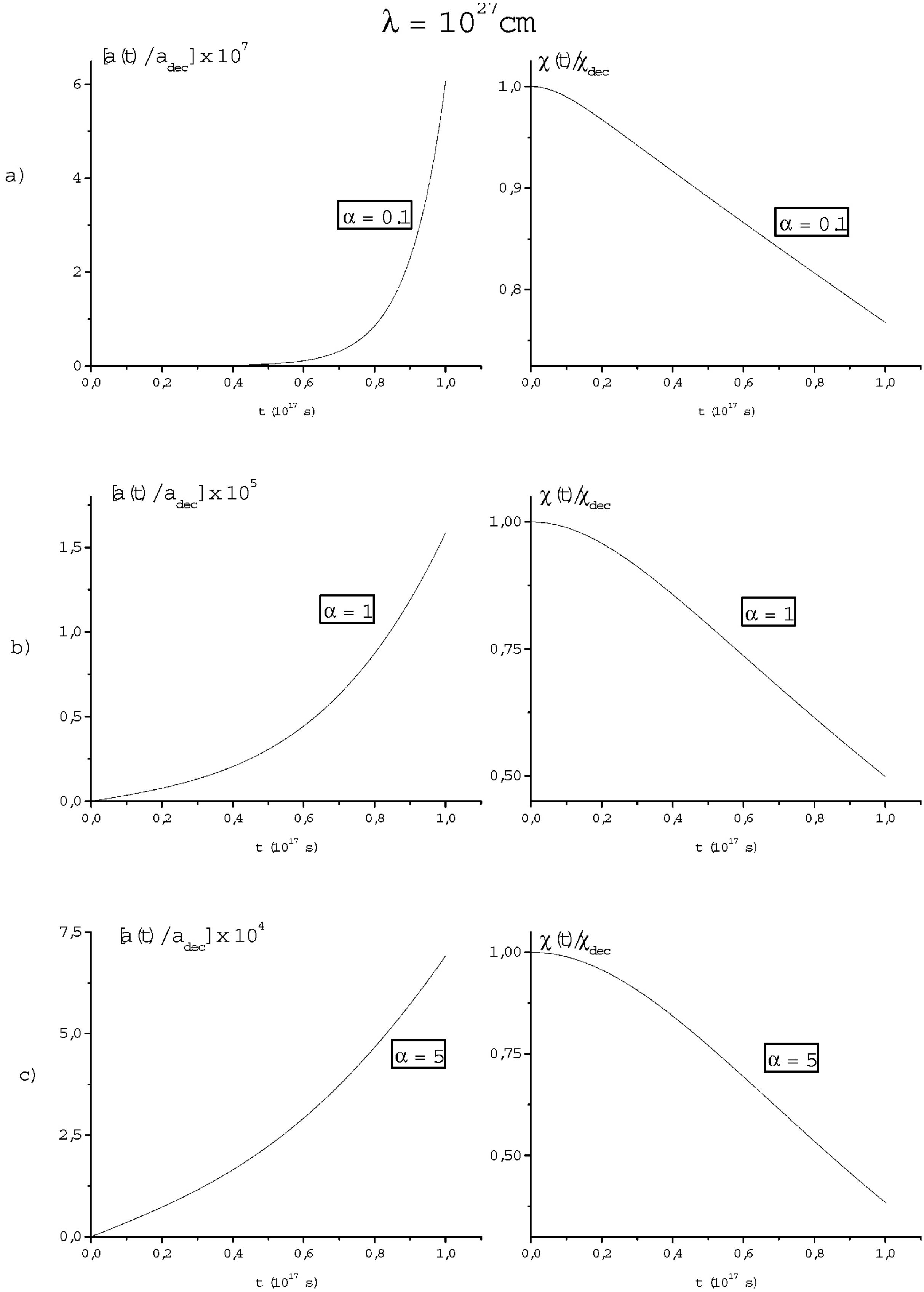

FIGURE 1C. Evolution of the scale factor, a, and the Higgs field, χ, for $\lambda = 10^{27}$cm, with $\alpha = 0.1$, 1.0, 5.0.

and given that

$$1 + z_{dec} = a_h/a_{dec},$$

thus

$$a_{dec} = a_h/1090 = 10^{25}\text{cm};$$

the subindex "h" denotes values evaluated at present. We also know that $\rho_{dec} = \rho_h \left(a_h/a_{dec}\right)^3$, and with the value for the cold dark matter density $\rho_h = 2.6 \times 10^{-28}$ g/cm^3, one has that $\rho_{dec} = 3.36 \times 10^{-19}$ g/cm^3.

Let us now to analyze the scalar field. One needs the couple ($\chi_{dec}, \dot{\chi}_{dec}$), but one has no information on these values. Therefore, we have to assume arbitrary, reasonable values. A natural scenario is to consider a value of the scalar field different from its ground state ($\chi_{\rm ground} = 0$), say $\chi_{dec} = 1$. In this way, the field will evolve to its ground state, yielding a cosmological dynamics. For its velocity, we assume $\dot{\chi}_{dec} = 0$. The results will not qualitatively depend on these values since the potential acts as an attractor to the dynamics.

Finally, the value of $\dot{a}_{dec}$ is obtained from Eq. (13). Thus, we have that $\dot{a}_{dec} \approx a_{dec}\sqrt{8\pi G\rho_{dec}/3} = 4.33 \times 10^{12}$cm/s. The quantities $\ddot{a}_{dec}$ and $\ddot{\chi}_{dec}$ will be determined by Eqs. (14) and (15), respectively.

For the cosmological parameters, we assume a flat cosmic geometry, $k = 0$, according to recent cosmological measurements [4] and for the equation of state of matter, w= 0, that is, the dominant matter from decoupling to present times is dust.

On the other hand, the Higgs mass, M_H, should be known from high energy experiments, however, it is yet unknown. Within the framework of the present theoretical scheme, it is expected that the Higgs decouples from the rest of the particles of the Universe, since Eq. (15) has no source, once it is produced [11]. Therefore, this theory predicts that no high energy experiments, such as the LHC at CERN, would detect it. Given this, we freely determine the Higgs mass through its Compton wavelength, $\lambda_C = \hbar/M_H c$. We thus associate this length-scale to cosmological distances. In this way, we assume that λ_C is as big as the apparent Universe at $z = 1$, since approximately at this time the quintessence appeared to dominate the cosmological dynamics. However, we will consider some other values for λ_C too. Thus, to have an idea of its magnitude we have that

$$\lambda_C = a_{z=1} = a_h/(1 + z_q) = a_h/2 = 5 \times 10^{27}\text{cm}.$$

Thus, the subindex "q" refers to the epoch when the accelerated expansion begins, at $z_q \approx 1$. Therefore, $M_H \approx 4 \times 10^{-31}$eV. Note that this value is much smaller than the value 10^{-3}eV that is obtained when one applies RG to a minimally coupled scalar field of mass M. That is, by considering $H^2 = (8\pi G/3)V = (8\pi/3)M^4/M_{Pl}^2$ and taking $H^{-1} = 5 \times 10^{27}$cm, one obtains $M = 10^{-3}$eV.

Finally, the parameter α can also be constrained from cosmological considerations. The main effect of α is to modify the net Newtonian constant through Eq. (12). This in turn modifies any cosmological dynamics such as Doppler peaks and structure formation. A brief discussion on these constraints can be found in Ref. 13. There it is concluded that α can be in the range $0 < \alpha \leq 1$ or around these values.

4. Recent cosmological dynamics

Next we integrate the scale factor (a) and scalar field -Higgs- (χ) evolution taking into account the above-mentioned initial values and constraints, see Sec 3. We have solved the dynamics for values of Compton wavelength $\lambda_C = 10^{25}, 10^{26}, 10^{27}$cm and an initial Universe size of $a_{dec} = 10^{25}$cm. The smallest λ_C value coincides with a_{dec}. The value $\lambda_C = 10^{27}$cm is about one tenth of the current apparent Universe. We have considered some values for the scalar field strength $\alpha = 0.1, 1.0, 5.0$, see Fig 1.

Figure 1a shows the case $\lambda_C = 10^{25}$cm for the three values of α considered. In this case the Higgs acts as a quintessence, but well before the redshift, $z_q = 1$, when is thought to begin to dominate. As a result the Universe begin its accelerated expansion- inflates- shortly after decoupling. One can see that amount of inflation (e-folds) is bigger for smaller α. The Higgs field, on the other hand, oscillates around its minimum ($\chi = 0$). The oscillations decay faster for smaller α. The asymptotic behavior of the scale factor is of the type $a \propto t^{2/3}$, which is a dust model, in accordance with the result that coherent oscillations behave as dust [12].

Figure 1b shows the case $\lambda_C = 10^{26}$cm. Here the same tendency is observed, however there is a small difference in the e-folds of inflation produced, since the bigger λ_C, the later the Universe enters to the inflationary (accelerated) dynamics. In this case, one can see the details of how the Universe enters to inflation. For instance, one observes that diminishing α provokes a later entrance to inflation. One can also see how the Universe exits the accelerated dynamics to become asymptotically dust dominated -due to Higgs oscillations again; this last assertion has been tested by integrating further in time to $t = 10^{18}s$ and comparing it with the dust solution; both solutions coincide. The scalar field in this case evolves only few times around its ground state.

Figure 1c shows the case $\lambda_C = 10^{27}$cm. Again the same tendency with the parameters is observed. In this case, inflation begins at more recent times, which matches better the Supernovae [1, 2] and WMAP data [4]. Thus, in this model inflation just began, and we are in the middle of the accelerated dynamics, as it seems to be case in our Universe.

5. Conclusions

In this work the IG theory has been considered as a model of quintessence. The physical scenario is as follows: the model is described by a scalar-tensor theory of gravitation, in which the scalar field is identified with a Higgs field. This Higgs field evolves, as a result of its symmetry breaking, from some initial value ($\chi_{dec} = 1$) to its ground state ($\chi_{dec} = 0$), around which the field oscillates and the gravitational sector becomes dynamically GR. To study the cosmological dynamics we have considered a FRW universe with flat curvature. We have integrated the evolution of the scale factor (a) and Higgs field (χ) for a range values of the parameters of the theory (λ, α). Models with $\lambda = 10^{25}$cm and 10^{26}cm turn out inadequate, since they either generate a too early accelerated era or either too many e-folds of inflation compared with the required amount. The computations made with $\lambda = 10^{27}$cm generate models that better approximate to the realistic behavior, since the epoch of quintessence dominance takes place at smaller z (later times) than compared to the models with smaller λ. On the other hand, the precise amount of e-folds can be controlled by augmenting α. However, since α is constrained by cosmological measurements [13], one has to be careful since it can not be arbitrarily augmented. In this sense, with the present chosen values for α the model does not exactly match the expected amount of e-folds of expansion of the universe since last scattering to present times. Therefore, more precise constraints on α are needed to have a definitive answer on the validity of the present model.

We conclude that the Higgs mechanism applied to the present cosmological scenario could achieve the desired cosmological dynamics, by fitting the constants of the theory to the cosmological data. However, the Higgs mass needs to be very small and therefore the coupling constant (λ_a, see Eq. 10) has to be very small too. This small numbers are unnatural from the particle physics point of view, but they are needed to accomplished the desired dynamics.

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