The universe as a brane

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If string theory is on the right track as a description of nature, then our observable universe may be a 3-dimensional membrane surface in a higher-dimensional spacetime. The brane-world picture of the universe, as inspired by string theory, opens up new vistas in cosmology. The extra polarizations of the higher-dimensional graviton, and the additional fields arising from the extra dimensions, bring new features that may leave detectable signatures. Moreover, the hope is that these new ingredients will shed light on the major puzzles associated with the standard cosmological model – in particular, the problem of explaining inflation and the dark energy problem.

Keywords: Cosmological models; dark energy; brane-world models.

Si la teoría de cuerdas es una descripción correcta de la naturaleza, entonces nuestro universo observable puede ser una membrana tridimensional en un espacio-tiempo de mayor dimensionalidad. La visión del universo basada en mundos brana, inspirada en la teoría de cuerdas, abre nuevas posibilidades en la cosmología. Las polarizaciones extra del gravitón en mas dimensiones, y los campos adicionales que aparecen debido a las dimensiones extra, tienen como consecuencia nuevas propiedades que producir señales detectables. Además, existe la esperanza de que estos nuevos ingredientes iluminen algunos de los mayores problemas asociados con el modelo cosmológico estándar - en particular, el problema del origen de la inflación y la energía obscura.

Descriptores: Modelos cosmológicos; energía obscura; modelos de mundos brana.

PACS: 95.36.+x; 98.80.-k; 98.80.Cq; 98.80.Qc

1. Introduction

The current "standard model" of cosmology – the inflationary cold dark matter model with cosmological constant (LCDM), based on general relativity and particle physics (the minimal supersymmetric extension of the Standard Model) – provides an excellent fit to the wealth of high-precision observational data [1]. In particular, independent data sets from CMB anisotropies, galaxy surveys and SNe redshifts, provide a consistent set of model parameters. For the fundamental energy density parameters, this is shown in Fig. 1. The data indicates that the cosmic energy budget is given by

$$\Omega_\Lambda \approx 0.7 \,, \ \ \Omega_M \approx 0.3 \,, \tag{1}$$

so that the universe is undergoing a late-time acceleration. The data further indicates that the universe is (nearly) spatially flat, and that the primordial perturbations are (nearly) scale-invariant, adiabatic and Gaussian.

This standard model is remarkably successful, but we know that its theoretical foundation, general relativity, breaks down at high enough energies, usually taken to be at the Planck scale,

$$E \gtrsim M_p \sim 10^{16} \,\text{TeV} \,. \tag{2}$$

The classical singularities predicted by general relativity in gravitational collapse and in the hot big bang will be removed by quantum gravity. But even below the fundamental energy scale that marks the transition to quantum gravity, significant corrections to general relativity will arise. These corrections could have a major impact on the behaviour of gravitational collapse, black holes and the early universe, and they could leave a trace – a "smoking gun" – in various observations

and experiments. Thus it is important to estimate these corrections and develop tests for detecting them or ruling them out. In this way, quantum gravity can begin to be subject to testing by astrophysical and cosmological observations.

The LCDM model can only provide limited insight into the very early universe. Indeed, the crucial role played by inflation belies the fact that inflation remains an effective
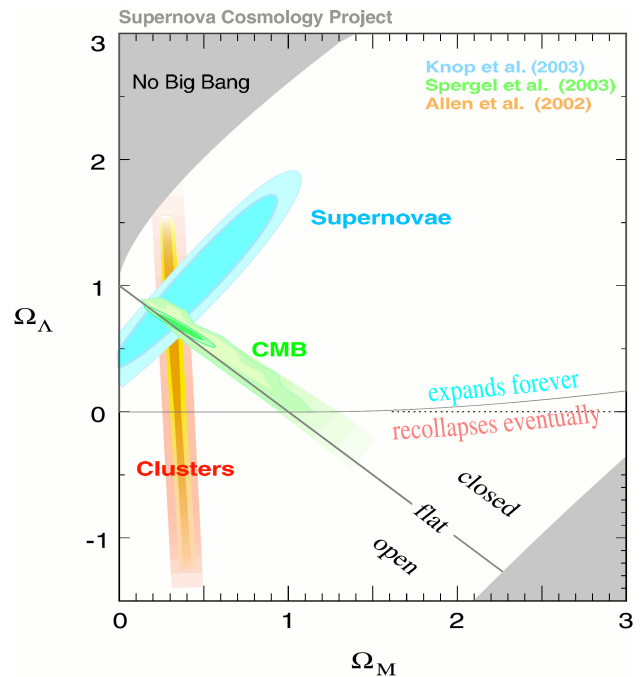

FIGURE 1. Observational constraints in the $(\Omega_\Lambda, \Omega_M)$ plane (from Ref. 2).

theory without yet a basis in fundamental theory. A quantum gravity theory will be able to probe higher energies and earlier times, and should provide a consistent basis for inflation, or an alternative that replaces inflation within the standard cosmological model.

An even bigger theoretical problem than inflation is that of the recent accelerated expansion of the universe. Within the framework of general relativity, the acceleration must originate from a dark energy field with effectively negative pressure ($w \equiv p/\rho < -1/3$), such as vacuum energy or a slow-rolling scalar field ("quintessence"). So far, none of the available models has a natural explanation. For the simplest option of vacuum energy, in the LCDM model, the incredibly small,

$$\rho_{\Lambda,\text{obs}} = \frac{\Lambda}{8\pi G} \sim H_0^2 M_P^2 \ll \rho_{\Lambda,\text{theory}}, \qquad (3)$$

and incredibly fine-tuned,

$$\Omega_\Lambda \sim \Omega_M, \qquad (4)$$

value of the cosmological constant cannot be explained by current particle physics. Quantum gravity will hopefully provide a solution to the problems of vacuum energy and fine-tuning.

Alternatively, it is possible that there is no dark energy, but instead a low-energy/ large-scale modification to general relativity that accounts for late-time acceleration. An infra-red modification to general relativity could emerge within the framework of quantum gravity, in addition to the ultraviolet modification that must arise at high energies in the very early universe.

The LCDM model is a framework seeking a fundamental theory. Cosmological observations are pointing towards the gaps that need to be filled by quantum gravity – and it is probable that further puzzles will arise from future observational advances. In this sense, cosmology has become a driving force in quantum gravity theory. Observations are ahead of theory, since no candidate theory is available that is yet able to make cosmological predictions. This includes the leading candidate theory, string theory.

String theory removes the infinities of quantum field theory and unifies the fundamental interactions, including gravity. But there is a price – the theory is only consistent in 9 space dimensions. There are five distinct 1+9-dimensional string theories, all giving quantum theories of gravity. Duality transformations relate these theories and the 1+10-dimensional supergravity theory, leading to the conjecture that all of these theories arise as different limits of a single theory, known as M theory. The 11th dimension in M theory is related to the string coupling strength; at low energies, M theory can be approximated by 1+10-dimensional supergravity.

It was also discovered that p-branes, which are extended objects of higher dimension than strings (1-branes), play a fundamental role in the theory. In the weak coupling limit, p-branes ($p > 1$) become infinitely heavy, so that they do not appear in the perturbative theory. Of particular importance among p-branes are the D-branes, on which open strings can end. Roughly speaking, open strings, which describe the non-gravitational sector, are attached at their endpoints to branes, while the closed strings of the gravitational sector can move freely in the higher-dimensional "bulk" spacetime. Classically, this is realised via the localization of matter and radiation fields on the brane, with gravity propagating in the bulk.

The implementation of string theory in cosmology is extremely difficult, given the complexity of the theory. There has been some recent progress in constructing inflationary models [4], but there is still a long way to go. This situation motivates the development of phenomenology, as an intermediary between observations and fundamental theory. (Indeed, the development of inflationary cosmology has been a very valuable exercise in phenomenology.) Brane-world cosmological models inherit key aspects of string theory, but do not attempt to impose the full machinery of the theory. Instead, drastic simplifications are introduced in order to be able to construct cosmological models that can be used to compute observational predictions (see Ref. 5 for reviews in this spirit). Cosmological data can then be used to constrain the brane-world models, and hopefully thus provide constraints on string theory, as well as pointers for the further development of string theory.

It turns out that even the simplest brane-world models are remarkably rich – and the computation of their cosmological perturbations is remarkably complicated, and still incomplete. Here I will describe two brane-world cosmologies – those of Randall-Sundrum (RS) type [6] and those of Dvali-Gabadadze-Porrati (DGP) type [7]. Both are 5-dimensional models, with an infinite extra dimension. (We effectively assume that 5 of the extra dimensions in the "parent" string theory may be ignored at low enough energies.)

2. KK modes of the graviton

The brane-world mechanism, whereby matter is confined to the brane while gravity accesses the bulk, means that extra dimensions can be much larger than in the conventional Kaluza-Klein (KK) mechanism, where matter and gravity both access all dimensions. The dilution of gravity via the bulk effectively weakens gravity on the brane, so that the true, higher-dimensional Planck scale, M_{4+d}, can be significantly lower than the effective 4D Planck scale M_p. Since higher-dimensional gravity effects would have shown up in particle colliders, the true Planck scale should be $\gtrsim 1$ TeV.

The higher-dimensional graviton has massive 4D modes felt on the brane, known as KK modes, in addition to the massless mode of 4D gravity. From a geometric viewpoint, the KK modes can also be understood via the fact that the projection of the null graviton 5-momentum $p_a^{(5)}$ onto the brane is timelike. If the unit normal to the brane is n_a, then the induced metric on the brane is

$$g_{ab} = g_{ab}^{(5)} - n_a n_b, \; g_{ab}^{(5)} n^a n^b = 1, \; g_{ab} n^b = 0, \quad (5)$$

and the 5-momentum may be decomposed as

$$p_a^{(5)} = mn_a + p_a \,, \; p_a n^a = 0 \,, \; m = p_a^{(5)} n^a \,, \qquad (6)$$

where $p^a = g^{ab} p_b^{(5)}$ is the projection along the brane, depending on the orientation of the 5-momentum relative to the brane. The effective 4-momentum of the 5D graviton is thus p_a. Expanding $g_{ab}^{(5)} p_{(5)}^a p_{(5)}^b = 0$, we find that

$$g_{ab} p^a p^b = -m^2 \,. \qquad (7)$$

It follows that the 5D graviton has an effective mass m on the brane. The usual 4D graviton corresponds to the zero mode, $m = 0$, when $p_a^{(5)}$ is tangent to the brane.

The extra dimensions lead to new scalar and vector degrees of freedom on the brane. The spin-2 5D graviton is represented by a metric perturbation $h_{ab}^{(5)}$ that is transverse traceless:

$$g_{ab}^{(5)} \to g_{ab}^{(5)} + h_{ab}^{(5)} \,, \; h^{(5)a}{}_a = 0 = \partial_b h^{(5)b}{}_a \,. \qquad (8)$$

In a suitable gauge, $h_{ab}^{(5)}$ contains a 3D transverse traceless perturbation h_{ij}, a 3D transverse vector perturbation Σ_i and a scalar perturbation β, which each satisfy the 5D wave equation:

$$h^i{}_i = 0 = \partial_j h^{ij} \,, \; \partial_i \Sigma^i = 0 \,, \qquad (9)$$

$$(\nabla_\mu \nabla^\mu + \partial_y^2) \begin{pmatrix} \beta \\ \Sigma_i \\ h_{ij} \end{pmatrix} = 0 \,. \qquad (10)$$

The 5 degrees of freedom (polarizations) in the 5D graviton are thus split into 2 (h_{ij}) + 2 (Σ_i) +1 (β) degrees of freedom in 4D. On the brane, the 5D graviton field is felt as

- a 4D spin-2 graviton h_{ij} (2 polarizations)

- a 4D spin-1 gravi-vector (gravi-photon) Σ_i (2 polarizations)

- a 4D spin-0 gravi-scalar β.

The massive modes of the 5D graviton are represented via massive modes in all 3 of these fields on the brane. The standard 4D graviton corresponds to the massless zero-mode of h_{ij}.

In the general case of d extra dimensions, the number of degrees of freedom in the graviton follows from the irreducible tensor representations of the rotation group as $\frac{1}{2}(d+1)(d+4)$.

3. RS type brane-worlds: ultraviolet modifications to GR

RS brane-worlds do not rely on compactification to localize gravity at the brane, but on the curvature of the bulk. What prevents gravity from "leaking" into the extra dimension at low energies is a negative bulk cosmological constant,

$$\Lambda_5 = -\frac{6}{\ell^2} \,, \qquad (11)$$

where ℓ is the curvature radius of 5D anti de Sitter spacetime, AdS$_5$. For a vacuum brane, the metric takes the form

$$ds_{(5)}^2 = dy^2 + e^{-2|y|/\ell} \left[-dt^2 + d\vec{x}^2 \right] \,, \qquad (12)$$

which is a solution of the 5D Einstein equations,

$$G_{ab}^{(5)} = -\Lambda_5 g_{ab}^{(5)} \,. \qquad (13)$$

The brane gravitates with self-gravity in the form of brane tension σ, where

$$\sigma = \frac{3M_p^2}{4\pi\ell^2} \,, \; M_p^2 = M_5^3 \ell \,. \qquad (14)$$

On the brane, the negative Λ_5 is offset by the positive brane tension σ.

To see how gravity is localized at low energies, we consider the 5D graviton perturbations of the metric in a convenient gauge:

$$h_{ay}^{(5)} = 0 = h^{(5)\mu}{}_\mu = \partial^\nu h_{\mu\nu}^{(5)} \,. \qquad (15)$$

We split the amplitude h into 3D Fourier modes, and the linearized 5D Einstein equations lead to the wave equation $(y > 0)$

$$e^{2y/\ell} \left[\ddot{h} + k^2 h \right] = h'' - \frac{4}{\ell} h' \,. \qquad (16)$$

Separability means we can write

$$h(t,y) = \int dm \, \varphi_m(t) \, h_m(y) \,, \qquad (17)$$

and the wave equation reduces to

$$\ddot{\varphi}_m + (m^2 + k^2)\varphi_m = 0 \,, \qquad (18)$$

$$h_m'' - \frac{4}{\ell} h_m' + e^{2y/\ell} h_m = 0 \,. \qquad (19)$$

The zero mode solution is

$$\varphi_0(t) = A_{0+} e^{+ikt} + A_{0-} e^{-ikt} \,, \qquad (20)$$

$$h_0(y) = B_0 + C_0 e^{4y/\ell} \,, \qquad (21)$$

and the continuum of massive KK modes ($m > 0$) is

$$\varphi_m(t) = A_{m+} \exp\left(+i\sqrt{m^2 + k^2} t \right)$$
$$+ A_{m-} \exp\left(-i\sqrt{m^2 + k^2} t \right) \,, \qquad (22)$$

$$h_m(y) = e^{2y/\ell} \left[B_m J_2\left(m\ell e^{y/\ell} \right) + C_m Y_2\left(m\ell e^{y/\ell} \right) \right] \,, \qquad (23)$$

where J_2, Y_2 are Bessel functions.

The boundary condition for the perturbations is $h'(t, 0) = 0$, which implies

$$C_0 = 0, \quad C_m = -\frac{J_1(m\ell)}{Y_1(m\ell)} B_m. \qquad (24)$$

The zero mode is normalizable, since

$$\left| \int_0^\infty B_0 e^{-2y/\ell} dy \right| < \infty. \qquad (25)$$

Its contribution to the gravitational potential $V = 1/2h_{00}^{(5)}$ gives the 4D result, $V \propto r^{-1}$. The contribution of the massive KK modes sums to a correction of the 4D potential. For $r \ll \ell$, one obtains

$$V(r) \approx \frac{GM\ell}{r^2}, \qquad (26)$$

which simply reflects the fact that the potential becomes truly 5D on small scales. For $r \gg \ell$,

$$V(r) \approx \frac{GM}{r} \left(1 + \frac{2\ell^2}{3r^2} \right), \qquad (27)$$

which gives the small correction to 4D gravity at low energies from extra-dimensional effects.

The RS cosmological model has a Friedman-Robertson-Walker brane in AdS_5. On the brane, the standard conservation equation holds,

$$\dot{\rho} + 3H(\rho + p) = 0, \qquad (28)$$

but the Friedmann equation is modified by an ultraviolet correction:

$$H^2 = \frac{8\pi}{3M_p^2} \rho \left(1 + \frac{\rho}{2\sigma} \right) + \frac{\Lambda}{3} - \frac{K}{a^2}. \qquad (29)$$

The ρ^2/σ term is the ultraviolet term. At high energies, $\rho \gg \sigma$, gravity "leaks" off the brane and behaves increasingly as 5D gravity – the massive KK modes dominate over the zero-mode. At low energies, the zero-mode dominates over the massive modes, and the standard Friedman equation is recovered. Since $\sigma \gg 1\,\mathrm{MeV}$, standard cosmology applies well before nucleosynthesis.

When $\rho \gg \sigma$, or equivalently $H\ell \gg 1$, in the early universe, then $H^2 \propto \rho^2$. This means a given energy density produces a greater rate of expansion than it would in general relativity. As a consequence, inflation in the early universe is modified in interesting ways.

In the the slow-roll approximation, for a 4D inflaton ϕ with potential $V(\phi)$,

$$H^2 \approx \frac{8\pi}{3M_p^2} V \left[1 + \frac{V}{2\sigma} \right], \qquad (30)$$

$$\dot{\phi} \approx -\frac{V'}{3H}. \qquad (31)$$

The brane-world correction term V/σ in Eq. (30) serves to enhance the Hubble rate for a given potential energy, relative to general relativity. Thus there is enhanced Hubble "friction" in Eq. (31), and brane-world effects will reinforce slow-roll at the same potential energy. The slow-roll parameters at high energies are modified relative to the general relativity parameters as

$$\epsilon \approx \epsilon_{\mathrm{gr}} \left[\frac{4\sigma}{V} \right], \quad \eta \approx \eta_{\mathrm{gr}} \left[\frac{2\sigma}{V} \right]. \qquad (32)$$

In particular, this means that steep potentials which do not give inflation in general relativity, can inflate the brane-world at high energy and then naturally stop inflating when V drops below σ. These models can be constrained because they typically end inflation in a kinetic-dominated regime and thus generate a blue spectrum of gravitational waves, which can disturb nucleosynthesis. They also allow for the novel possibility that the inflaton could act as dark matter or quintessence at low energies.

RS modifications at high energy mean that the same number of e-folds can be obtained as in general relativity for a much lower value of the initial inflaton and inflationary potential. This alters the relation between the inflationary parameters and the observations, so that constraints on various inflationary potentials are modified. As an illustration, the constraints on ϕ^2 and ϕ^4 potentials are shown in Fig. 2.

In addition, the amplitude of scalar perturbations is enhanced relative to the standard result at a fixed value of ϕ for a given potential [9]:

$$A_s^2 \approx \left(\frac{512\pi}{75M_p^6} \frac{V^3}{V'^2} \right) \left[\frac{V}{2\sigma} \right]^3 \bigg|_{k=aH}. \qquad (33)$$
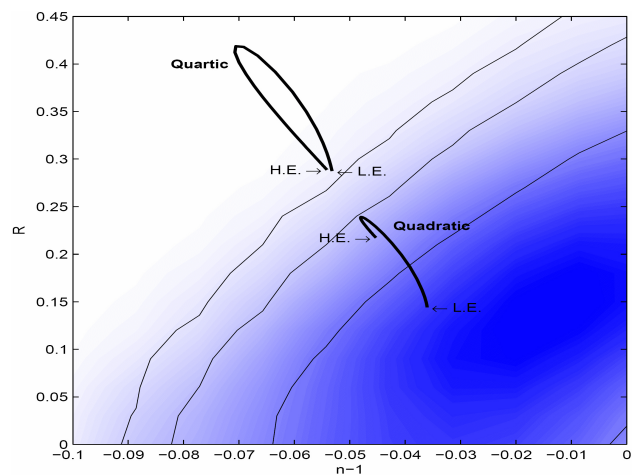

FIGURE 2. Constraints from CMB and large-scale structure data on inflation models with quadratic and quartic potentials, where R is the ratio of tensor to scalar amplitudes and n is the scalar spectral index. The high energy (H.E.) and low energy (L.E.) limits are shown, with intermediate energies in between, and the 1-, 2- and 3-σ contours are also shown. (From Ref. 8.)

High-energy inflation on the brane also generates a zero-mode (4D graviton mode) of tensor perturbations, and stretches it to super-Hubble scales. This zero-mode has the same qualitative features as in general relativity, remaining frozen at constant amplitude while beyond the Hubble horizon. Its amplitude is enhanced at high energies, although the enhancement is much less than for scalar perturbations [10]:

$$A_t^2 \approx \left(\frac{32V}{75M_p^2} \right) \left[\frac{3V^2}{4\sigma^2} \right], \qquad (34)$$

$$\frac{A_t^2}{A_s^2} \approx \left(\frac{M_p^2}{16\pi} \frac{V'^2}{V^2} \right) \left[\frac{6\sigma}{V} \right]. \qquad (35)$$

Equation (35) means that RS brane-world effects suppress the large-scale tensor contribution to CMB anisotropies.

To lowest order in the slow-roll approximation, we can neglect the KK modes of the inflationary scalar and tensor perturbations. Modifications to the CMB spectrum will arise from KK corrections to the primordial initial conditions for the perturbations, as well as from KK modes that are generated after inflation. The computation of these effects is still an open problem, given the complexity of the 5D perturbation problem [11]. In this problem, the anisotropic 5D gravitational field imprints an effective anisotropic stress on the brane, which plays a crucial role.

4. DGP type brane-worlds: infrared modifications to GR

Could the late-time acceleration of the universe be a gravitational effect? (Note that this would not remove the problem of explaining why the vacuum energy does not gravitate.) An historical precedent is provided by attempts to explain the anomalous precession of Mercury's perihelion by a "dark planet". In the end, it was discovered that a modification to Newtonian gravity was needed.

An alternative to dark energy plus general relativity is provided by models where the acceleration is due to modifications of gravity on very large scales, $r \gtrsim H_0^{-1}$. It is very difficult to produce infrared corrections to general relativity by modifying the 4D Einstein-Hilbert action; typically, instabilities arise or the action has no natural motivation. The DGP brane-world offers an alternative higher-dimensional (and fully covariant) approach to the problem.

In the DGP case the brane has no tension and the bulk is 5D Minkowski spacetime. Unlike the AdS bulk of the RS model, the Minkowski bulk has infinite volume. Consequently, there is no normalizable zero-mode of the graviton in the DGP brane-world. Gravity leaks off the 4D Minkowski brane into the bulk at large scales. At small scales, gravity is effectively bound to the brane and 4D Newtonian dynamics is recovered to a good approximation. The transition from 4- to 5D behaviour is governed by a crossover scale r_c; the weak-field gravitational potential behaves as

$$\Psi \sim \begin{cases} r^{-1} & \text{for} \quad r < r_c \\ r^{-2} & \text{for} \quad r > r_c \end{cases} \qquad (36)$$

For a Friedman-Robertson-Walker brane in a Minkowski bulk, gravity leakage at late times initiates acceleration – not due to any negative pressure field, but due to the weakening of gravity on the brane. 4D gravity is recovered at high energy via the lightest KK modes of the graviton, effectively via an ultralight metastable graviton.

The energy conservation equation remains the same as in general relativity, but the Friedman equation is modified:

$$\dot{\rho} + 3H(\rho + p) = 0, \qquad (37)$$

$$H^2 - \frac{H}{r_c} = \frac{8\pi G}{3} \rho. \qquad (38)$$

This shows that at late times in a CDM universe, with $\rho \propto a^{-3} \to 0$, we have

$$H \to H_\infty = \frac{1}{r_c}. \qquad (39)$$

Since $H_0 > H_\infty$, in order to achieve acceleration at late times, we require $r_c \gtrsim H_0^{-1}$, and this is confirmed by fitting SNe observations, as shown in Fig. 3.

LCDM and DGP can both account for the SNe observations, with the fine-tuned values $\Lambda \sim H_0^2$ and $r_c \sim H_0^{-1}$ respectively. This degeneracy may be broken by observations based on structure formation, since the two models suppress the growth of density perturbations in different ways. The
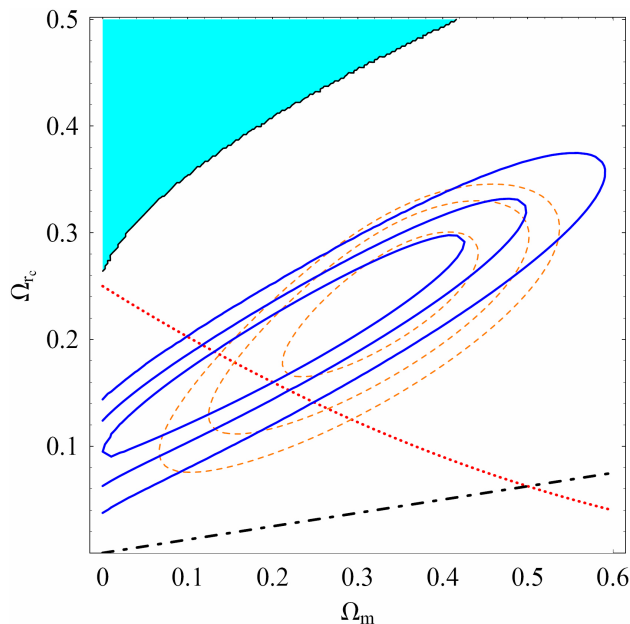
FIGURE 3. Supernovae constraints on DGP models, where $\Omega_{r_c} = 1/4r_c^2 H_0^2$. The dotted (red) curve is the flat case, with closed models above and open models below; the dot-dashed (black) curve demarcates models that are currently accelerating (above) from those decelerating (below); the shaded (blue) region contains expanding models that bounce in the past, i.e. do not have a big bang. The likelihood contours are fits to the SNe Gold data [dashed (brown)] and Legacy data [solid (blue)]. (From Ref. 12.)

distance-based SNe observations draw only upon the background 4D Friedman equation (38) in DGP models, and therefore there are quintessence models in general relativity that can produce precisely the same SNe redshifts as DGP. By contrast, structure formation observations require the 5D perturbations in DGP, and one cannot find equivalent general relativity models.

For LCDM, the analysis of density perturbations is well understood. For DGP it is much more subtle and complicated. Although matter is confined to the 4D brane, gravity is fundamentally 5D, and the bulk gravitational field responds to and backreacts on density perturbations. The evolution of density perturbations requires an analysis based on the 5D nature of gravity. In particular, the 5D gravitational field produces an anisotropic stress on the 4D universe. Some previous results are based on inappropriately neglecting this stress and all 5D effects – as a consequence, the 4D Bianchi identity on the brane is violated, $i.e.$, $\nabla^\nu G_{\mu\nu} \neq 0$. When the 5D effects are incorporated [13], the 4D Bianchi identity is satisfied (The results of Ref. 13 confirm and generalize those of Ref. 14.). The modified evolution equation for density perturbations is

$$\ddot{\Delta} + 2H\dot{\Delta} = 4\pi G \left\{ 1 - \frac{(2Hr_c - 1)}{3[2(Hr_c)^2 - 2Hr_c + 1]} \right\} \rho\Delta \,.$$
(40)

The linear growth factor, $g(a) = \Delta(a)/a$ ($i.e.$, normalized to the flat CDM case, $\Delta \propto a$), is shown in Fig. 4.

It must be emphasized that these results apply on subhorizon scales. On superhorizon scales, where the 5D effects are strongest, the problem has yet to be solved. This solution is necessary before one can compute the large-angle CMB anisotropies. It should also be remarked that the late-time asymptotic de Sitter solution in DGP cosmological models has a ghost [15], which may have implications for the analysis of density perturbations.
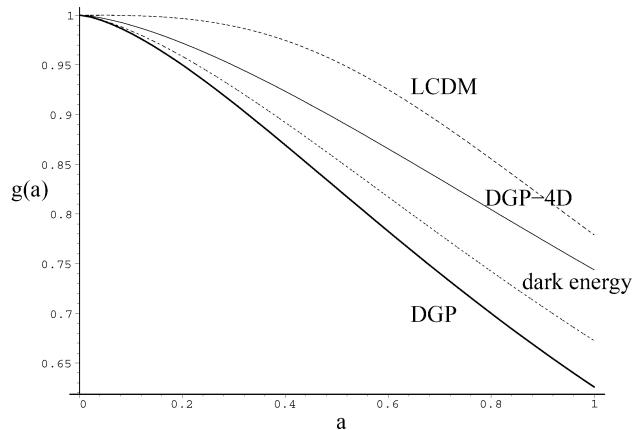

FIGURE 4. The growth factor $g(a) = \Delta(a)/a$ for LCDM (long dashed) and DGP (solid, thick), as well as for a dark energy model with the same expansion history as DGP (short dashed). DGP-4D (solid, thin) shows the incorrect result in which the 5D effects are set to zero. (From Ref. 13.)

5. Conclusion

In conclusion, brane-world models that are inspired by ideas from string theory provide a rich and interesting phenomenology, where higher-dimensional gravity effects in the early and late universe can be explored, and predictions can be computed for comparison with high-precision cosmological data. Even for the simplest models, of RS and DGP type, brane-world cosmology can bring new effects in inflation and structure formation, and new ideas for dark energy. In the RS and DGP type models, the 5D graviton, i.e., its KK modes, plays a crucial role, which has been emphasized in this review.

Acknowledgements

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