String theory: a decade of progress

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String theory aims at providing a complete description of the fundamental structure of our universe. Independently of whether or not it is eventually able to achieve this ambitious goal, over the years it has already proven to be an enormously rich theoretical structure, with several points of contact with other problems of interest in modern theoretical physics. In this article we review the basic ideas of string theory, and provide a brief overview of the achievements of the past ten years, which have radically improved our understanding of the theory.

Keywords: String theory; dualities; black holes.

La teoría de cuerdas busca dar una descripción completa de la estructura fundamental de nuestro universo. Independientemente de si logra o no eventualmente alcanzar esta ambiciosa meta, en el transcurso de los años ha demostrado ya ser una estructura teórica enormemente rica, con varios puntos de contacto con otros problemas de interés en la física teórica moderna. Este artículo describe las ideas básicas de la teoría de cuerdas, y ofrece un breve panorama de los logros de los últimos diez años, los cuales han mejorado radicalmente nuestra comprensión de la teoría.

Descriptores: Teoría de cuerdas; dualidades; dualidad norma-gravedad; agujeros negros.

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1. Basic String Theory

The basic proposal of string theory [1, 2] is to interpret the various elementary particles known to us not as distinct point-like objects, but as different manifestations of a single one-dimensional object, a string. Upon quantization, the different modes of oscillation of the string give rise to an infinite tower of states with progressively higher masses, each state having the properties of a specific type of particle and thus being interpretable as a fluctuation in an associated field. For a *closed* string, one obtains a free mass spectrum $m = 0, \sqrt{4}/l_s, \sqrt{8}/l_s, \ldots$, with l_s the only dimensionful parameter of the theory, known as the string length.

In conventional models l_s is close to the Planck length, 10^{-33} cm (corresponding to an energy scale $\sim 10^{19}$ GeV); but in recent years alternative scenarios have been considered in which l_s could perhaps be as large as 10^{-17} cm (i.e., the string scale be as low as $\sim 10^3$ GeV), on the verge of experimental detection. Either way, up to now we would have observed only the states that are (in a first approximation) massless. Closed strings yield massless states that correspond to a metric $g_{\mu\nu}$, a scalar field φ known as the dilaton, and antisymmetric tensor fields (generalized gauge fields) $B_{\mu\nu}$ and $C_{\mu_1\ldots\mu_{p+1}}$ for various values of p, plus fermionic partners for all of these fields. This is precisely the content of the field-theoretic generalization of General Relativity known as supergravity, which is then the lowest-energy approximation to string theory. There is a generalization of the Feynman diagram expansion which allows the computation of string scattering amplitudes, from which the interactions between the massless (as well as massive) modes can be deduced. The basic string interaction is a sort of cubic vertex, which allows a string to split into two (or the reverse).

The discovery that the graviton, in particular, emerges naturally as a mode of oscillation of the string provided for the first time a *perturbative* description of quantum gravity that is free of ultraviolet divergences. The fact that, at this level, the description is perturbative implies in particular that prior to any computation a background must be chosen, in order to consider the propagation of a small number of weakly-interacting strings on it. It is important, however, to keep in mind that, just like point particles represent small fluctuations about a chosen background value for the corresponding field, strings can explicitly be seen to correspond to small excitations of the selected background. What we are studying, then, is not 'just' a string moving on a fixed background, but the background itself (a rather drastic generalization of spacetime), undergoing small fluctuations.

Starting from closed strings, (and, as we will see below, much more naturally from *open* strings,) it is also possible to obtain non-Abelian gauge fields. A single basic object, the string, is thus capable of generating the basic ingredients of the Standard Model plus gravity. And, remarkably, the unique interaction of the theory, which allows a string to split into two (or the reverse), is able to reproduce the gravitational, Yang-Mills and Yukawa interactions needed to describe the world around us. Moreover, there are *no* free parameters in the theory. In particular, the dimensionless string coupling constant g_s, which controls the strength of the string-splitting interaction (and consequently the gravitational and Yang-Mills couplings, $\sqrt{G_N} \propto g_s$ and $g_{YM} \propto \sqrt{g_s}$), is determined by the expectation value of the dilaton field: $g_s = \exp\varphi$.

For a string theory to be consistent (*e.g.*, for it to predict only *non-negative* probabilities), the strings must live in more than the $3+1$ dimensions x^μ ($\mu = 0, 1, 2, 3$) that are evident

TABLE I. Properties of different string theories.

Superstring Theory:	**I**	**IIA**	**IIB**	**HO**	**HE**
Can strings break open?	Yes	No	No	No	No
Are strings oriented?	No	Yes	Yes	Yes	Yes
How many (10-dim) supersymmetries?	$\mathcal{N}=1$	$\mathcal{N}=2$	$\mathcal{N}=2$	$\mathcal{N}=1$	$\mathcal{N}=1$
Non-Abelian gauge group?	$SO(32)$	None	None	$SO(32)$	$E_8\times E_8$
Is the theory parity invariant?	No	Yes	No	No	No

to us. There are in fact additional dimensions of two different types.

First, there are six additional *bosonic* (i.e., ordinary) dimensions, $\vec{x}\equiv\{x^a\}$, with $a=4,\ldots,9$. These must be hidden from us in some way, and the conventional proposal is that they are compact and very small. Our universe would thus be somewhat analogous to the surface of a garden hose, with three spatial dimensions that are of astronomical size (say, $\sim 10^{10}$ light years), and six dimensions whose size is of order the Planck length ($\sim 10^{-33}$ cm).

We have known since the days of Kaluza and Klein that under such circumstances each ten-dimensional field would be understood from the four-dimensional perspective as an infinite tower of fields with progressively higher masses and charges. To illustrate the idea, consider the simplest case, where the additional dimensions form a straight six-dimensional torus with radii R_a. For a field ϕ to be periodic along these directions, the corresponding momenta must then be discrete, $p_a=n_a/R_a$, and we can Fourier decompose

$$\phi(x^\mu,\vec{x})=\sum_{\vec{n}}\exp(i\vec{p}\cdot\vec{x})\phi_{\vec{n}}(x^\mu).$$

Each component field $\phi_{\vec{n}}$, with definite momenta along the hidden dimensions, is interpreted from the four-dimensional viewpoint as a field with a definite mass $m_{\vec{n}}$. *E.g.*, if ϕ is massless in the ten-dimensional sense ($p^\mu p_\mu+\vec{p}^2$=0), then the mass of $\phi_{\vec{n}}$ i $m_{\vec{n}}\equiv\sqrt{-p^\mu p_\mu}=|\vec{p}|$. This implies that, for small enough R_a, only the $\vec{n}=0$ modes will be experimentally accessible. Choosing a more complicated topology for the six hidden dimensions, it is possible to obtain not one but, *e.g.*, *three* light families of particles— the number of fermion generations can be determined topologically!

In addition to this decomposition according to momenta in the compact dimensions, a tensor field gets broken up into various lower-dimensional fields, depending on whether its indices take values along the manifest or the hidden dimensions. The ten-dimensional metric, in particular, gives rise not only to a four-dimensional metric $g_{\mu\nu}$, but also to six four-dimensional vectors $A^{(a)}_\mu\equiv g_{\mu a}$, and 21 scalar fields $\sigma^{(ab)}\equiv g_{ab}$. Since the momenta p_a couple to the metric components $g_{\mu a}$, the mode $\phi_{\vec{n}}$ corresponds to a particle in four dimensions that carries charge n_a under the gauge field $A^{(a)}_\mu$.

What we have said up to now applies equally well to fields describing point particles and to those associated with the states of a string; but for (closed) string states there is an additional source of multiplicity, as each string can *wind* around compact direction x^a an arbitrary integer number of times. From the lower-dimensional perspective, these winding numbers $w_a\in\mathbf{Z}$ are simply perceived as charges under the additional gauge fields $A'^{(a)}_\mu\equiv B_{\mu a}$.

Second, there exist $\mathcal{N}=1$ or 2 additional dimensions that are *fermionic*, meaning in particular that the corresponding coordinates θ^A ($A=1$ or $A=1,2$) *anticommute*: $\theta^A\theta^B=-\theta^B\theta^A$. This implies that each of the coordinates squares to zero, and therefore the Taylor expansion of a field that is a function of all the bosonic *and* fermionic coordinates terminates, $\Phi(x,\theta)\sim\phi_{(1)}(x)+\theta^A\psi_{(A)}(x)+\theta^1\theta^2\phi_{(2)}(x)$. If the 'superfield' Φ is bosonic, it follows that the two fields $\phi_{(A)}$ are bosonic and the $\psi_{(A)}$ fermionic. This one-to-one pairing of bosonic and fermionic states, together with the invariance of the physics under transformations that 'rotate' from one to the other, is what is known as *supersymmetry*[i], a property that (in moderate amounts) is appealing from the theoretical perspective and for which we might even have some indirect phenomenological evidence. The search for this (necessarily broken) symmetry will be one of the main tasks of the Large Hadron Collider (LHC), currently under construction at CERN.

Up to 1994, there were five known consistent string theories, all ten-dimensional and supersymmetric[ii]. Their definitions differ at a technical level; but they can be easily distinguished from one another by listing a few of their physical properties, as we do in Table I.

2. Branes

Our understanding has improved dramatically over the past ten years. The point of departure was the discovery that in string theories there are other objects besides strings. In particular, the string equations of motion (which for slowly-varying fields are simply those of supergravity) allow *solitonic* solutions that are extended in some number of spatial dimensions. Solitons are known to exist already in many non-interacting field theories: due to the nonlinear nature of the equations of motion, there exist localized configurations in which a lump of field holds itself together. In the string theory context, one finds (in particular, static) configurations in which the energy density is uniformly spread out along p spatial dimensions but localized along the remaining $9 - p$ dimensions, in such a way that the *tension* $T_p \equiv$ energy/p-volume is finite. Such a configuration is then interpreted as a p-dimensional object, a p-brane.

One example is the so-called NS5-brane (present in all theories but I) [8]. This is a five-dimensional object, so in the corresponding static solution all fields are by definition independent of the $5+1$ coordinates that parametrize the spacetime trajectory swept out by of the brane (known as the brane's 'worldvolume'), x^α with $\alpha = 0, 1, \ldots, 5$. The NS5 solution involves non-trivial metric, dilaton, and anti-symmetric tensor fields that depend only on the radial direction r away from the brane, $g_{\mu\nu}(r)$, $\varphi(r)$, and $B_{\mu\nu}(r)$. As we stated before, $B_{\mu\nu}$ is a generalized gauge field (a tensor, instead of vector, potential), and the string carries an *electric* charge under it (the winding number). The NS5-brane, on the other hand, carries a *magnetic* charge under $B_{\mu\nu}$[iii]. There is a natural generalization of the Dirac quantization condition [9], and so, in appropriate units, the NS5 charge is an integer, $Q^{(B)} = N$. The geometry of the solution is asymptotically flat at $r \to \infty$ (where $e^\varphi \to$ const. $\equiv g_s$), but starting at $r \sim \sqrt{N} l_s$ there is a 'throat' that extends down to an event horizon at $r = 0$ (where $e^\varphi \to \infty$). The solution in questions is thus a *black* fivebrane, i.e., an extended generalization of a black hole. The (ADM) tension of the NS5-brane can be computed as usual from the asymptotic form of the metric; it is found to be

$$T_{\mathrm{NS5}} \equiv \frac{M}{V_5} = \frac{N}{(2\pi)^5 g_s^2 l_s^6} \ . \tag{1}$$

The dependence on the string coupling g_s shows clearly that this is a *non-perturbative* object: it is heavy when the theory is weakly-coupled.

Another class of solutions is that of the Rp-branes [10], with $p = 0, 2, \ldots, 8$ in IIA string theory, $p = 1, 3, \ldots, 9$ in IIB, $p = 1, 5, 9$ in I, and no possible values in HO and HE. These solutions are similar to the NS5-brane, but differ from it in two respects (beyond their dimensionality). First, they carry charge $Q^{(C)} = N$ not under $B_{\mu\nu}$, but under the antisymmetric tensor field $C_{\mu_1\ldots\mu_{p+1}}$— the various string theories are thus seen to have a source for every gauge field. Second, their tension is

$$T_{\mathrm{R}p} \equiv \frac{M}{V_p} = \frac{N}{(2\pi)^p g_s l_s^{p+1}} \ , \tag{2}$$

and so they are heavy at weak coupling, but lighter than the NS5. For all cases but $p = 3$ there is a non-trivial dilaton field. The geometry of these solutions is also asymptotically flat at large r (where $e^\varphi \to$ const. $\equiv g_s$), and has a throat that starts at $r \sim (g_s N)^{1/(7-p)} l_s$ and ends on a horizon at $r = 0$ (where $e^\varphi \to 0$ for $p > 3$ and $\to \infty$ for $p < 3$). We should emphasize that Rp-branes and NS5-branes are fully dynamical objects; to understand their excitations one ought to consider closed strings (or, at low energies, supergravity modes) that propagate on the relevant curved background.

Much of the progress of recent years originated from the discovery that Rp-branes admit an alternative description: they can be understood as a stack of N coincident Dp-branes [11]. A Dp-brane is an object extended in p spatial dimensions, whose presence allows the strings (normally closed) to break open, as long as the resulting endpoints remain attached to the D-brane [12]. This property confers dynamical attributes to the D-brane, and in particular determines the way in which it interacts with closed strings. Notice that an open string can form on the D-brane, and its endpoints join together to yield a closed string, which is then free to wander off the brane. In other words, D-branes can emit closed strings, and in particular, gravitons and 'photons' of the $C_{\mu_1\cdots\mu_{p+1}}$ field. As a result, a D-brane has a calculable tension and charge, which turn out to be [11]

$$T_{\mathrm{D}p} \equiv \frac{M}{V_p} = \frac{1}{(2\pi)^p g_s l_s^{p+1}} \tag{3}$$

and $Q^{(C)} = 1$, respectively. (The brane with the same mass but with $Q^{(C)} = -1$ is known as an *anti*-D-brane.) So, even though they are defined in completely different ways, it is clear that the Rp-brane and a stack of N parallel Dp-branes have at least the same charge and mass[iv]. They can also be seen to agree on all other calculable properties[v].

Open strings are only allowed to exist by the presence of D-branes, and so they must be understood to describe excitations of the branes. In other words, just like, when we poke the stringy generalization of spacetime, we find fluctuations described by closed strings, when we poke a D-brane we obtain fluctuations represented by open strings. Remarkably, then, even though D-branes are clearly, judging from (3), non-perturbative objects, their dynamics can be partially understood by quantizing open strings, which is comparatively a very simple task. As always, upon quantizing these strings one gets an infinite tower of states; for the open string the free mass spectrum is $m = 0, \sqrt{1}/l_s, \sqrt{2}/l_s, \ldots$ Since the open string can only slide along the directions parallel to the D-brane, the fields associated with these states live in $p+1$ dimensions. An interesting property is that, for a stack of N D-branes, one must consider N^2 types of open strings, because each string can start and end on any one of the D-branes, and so the states naturally assemble themselves into $N \times N$ matrices. At the massless level, the bosonic states one obtains correspond to a $p+1$-dimensional gauge field $(A_\alpha)_{IJ}$, with $\alpha = 0, \ldots, p$ and $I, J = 1, \ldots, N$, and, for each direction transverse to the branes, a scalar field $(\Phi^i)_{IJ}$, (the trace of) which describes transverse displacements of the brane along the given direction. Together with the corresponding fermions, one thus obtains the field content of $(p+1)$-dimensional supersymmetric Yang-Mills theory, with gauge group $U(N)$. As promised, then, we see that non-Abelian gauge theories, the basic building blocks of the Standard Model, emerge naturally from open strings, i.e., from D-branes.

This leads in particular to a potential phenomenological application of D-branes, which provides an alternative mechanism to hide the additional dimensions. The idea, known as the *braneworld* scenario, is to propose that the world we see around us is simply a collection of (effectively) D3-branes, embedded in a universe with nine spatial dimensions. All of the particles of the Standard Model, and as we have seen, non-Abelian gauge fields in particular, can be obtained from open strings, and so in this scenario they would naturally be confined to the three spatial dimensions along the branes. Only the modes arising from closed strings, and gravity in particular, would be able to feel the other six dimensions. Since we have probed gravity at the microscopic level much less sensitively than the Standard Model interactions, this would allow the hidden dimensions to be much larger than in the standard compactification (where all fields propagate in nine spatial dimensions). In a straightforward realization of this scenario [13], experimental bounds are found not to be able to rule out hidden dimensions even as large as $\sim 10^{-2}$cm. In a more contrived realization [14], it turns out that, as long as they are suitably 'warped', the additional dimensions could even be of infinite extent!

3. Dualities

The I, IIA, IIB, HO and HE string theories, previously thought to be distinct, were understood in the mid-nineties to be in fact connected to one another through various *dualities*. The concept of duality refers precisely to the non-trivial equivalence between two theories, and pre-dates its use in string theory.

The simplest example of such a connection (so simple that it is not ordinarily referred to as a duality) is the fact that type I string theory has been understood to correspond simply to a specific class of states of the IIB theory [11, 12]. These two theories are at first sight rather different: as mentioned in Sec. 1, in type I there are open and closed non-oriented strings, while in IIB there are only closed oriented strings. Contact is made between the two by considering states in IIB with D9-branes. Open strings are then allowed, and, since the branes in question fill all space, such strings can move freely in all directions. The resulting strings would still be oriented, however, so this should not be the end of the story. And indeed, D9-branes carry a charge under a ten-index antisymmetric tensor gauge field, whose equation of motion requires the total charge to vanish, so we cannot simply add an arbitrary number of D9-branes. It is in fact possible to obtain $Q^{(C10)} = 0$ by taking sixteen D9-branes together with a nine-dimensional object known as an *orientifold*, which can be thought of as a space-filling 'mirror' that removes the orientation of the strings (and gives a gauge group $SO(32)$ instead of $U(16)$). In this way one obtains precisely I string theory.

A second example is an equivalence between theories with compact directions of different sizes. Let us consider for simplicity the case of a single compact direction x^9 of radius R, i.e., such that $x^9 \simeq x^9 + 2\pi R$. Recall from Section 1. that a closed string can sense this dimension not only through its discretized momentum $p_9 = n/R$, but also by winding around the circle $w \in \mathbf{Z}$ times. Both n and w contribute to the nine-dimensional mass, according to

$$m^2_{(n,w)} = \left(\frac{n}{R}\right)^2 + \left(\frac{wR}{l_s^2}\right)^2 + \frac{4\mathbf{N}}{l_s^2}\,, \tag{4}$$

where $\mathbf{N} = 0, 1, \ldots$ denotes the level of excitation of the string. The first term on the right is simply the expected p_9^2 contribution; the second is just the string tension times w times the length of the compact dimension. According to (4), the spectrum coincides with that of a second string theory with a circle of radius $R' = l_s^2/R$, under the identification $(n', w') = (w, n)$. So far this is a statement of equivalence between the spectra of two free string theories, but in fact, the interacting theories are found to be identical as long as the couplings are related through $g_s' = g_s l_s/R$. This type of equivalence is known as *T-duality* [15]. When considering open strings, one finds that a fixed endpoint along x^9 in one theory corresponds to a free endpoint in the second theory. This in turn means that the associated D-brane loses a dimension if it was originally extended along x^9, and gains a dimension if it was not. Recall now that in IIA string theory there exist Dp-branes with even p, while in IIB p must be odd. Under T-duality, then, the IIA set of D-branes is mapped onto the IIB set. And in fact, looking at the details one finds that these two theories are T-dual images of one another, *i.e.*, IIA defined on a space with a circle of radius R (with coupling g_s) is equivalent to IIB defined on a circle of radius R' (with coupling g_s') [12, 16]. Notice that when $R \to 0$ we have $R' \to \infty$! This is a clear indication that an extended object such as the string senses space in a way that is drastically different from a point particle, and so by considering strings we are in effect shifting to a generalized notion of geometry[vi], which in particular leads to the phenomenon of topology change [17]. In a similar but more complicated manner, HO string theory turns out to be equivalent to HE under T-duality [18].

Thus far, we have reduced the number of independent string theories to two: I/IIA/IIB and HO/HE. Remarkably, these two can also be related to one another, through a type of equivalence known as *S-duality*. To understand this connection, let us recall first that in type I string theory (just like in IIB) there is a second kind of string, the D1-brane. As seen from (3) with $p = 1$, the D-string is heavy at weak coupling, $g_s \ll 1$, but it in fact becomes light when $g_s \gg 1$. An important point here is that, even though formula (3) is deduced at weak coupling, it can be trusted also at strong coupling because the unexcited D1-brane has the special property that it is the state with the lowest possible mass for its given charge, $Q^{(C2)} = 1$. The mass of such (necessarily stable) states (known as Bogomolnyi-Prasad-Sommerfield, or BPS, states [19]) turns out to be directly determined by their charges, and is therefore known even at strong coupling [20]. And in fact, among the excited states of the D-string (determined by quantizing open strings with both endpoints on the D1-brane, or with one end on the D1-brane and the other on

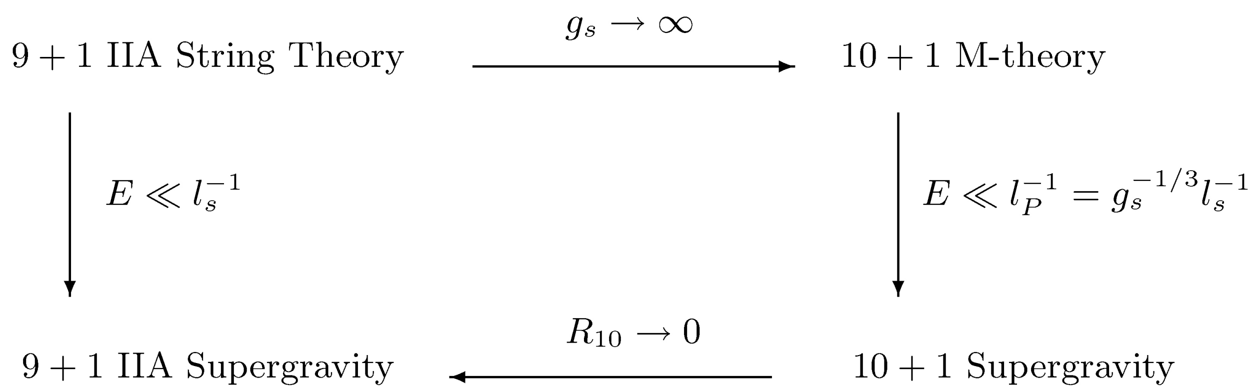

FIGURE 1. Summary of the connections between the ten- and eleven-dimensional theories discussed in the main text. The most important point is that the strong-coupling limit of ten-dimensional IIA string theory is in fact a theory in *eleven* dimensions.

one of the D9-branes), one can find an infinite family of states of this type, which must then exist also in the strongly-coupled limit. Remarkably, all of these states are found to be present in the spectrum of the *weakly*-coupled HO string theory! It is then natural to conjecture that the D-string of the strongly-coupled Type I string theory is precisely the same object as the *fundamental* string of the weakly-coupled HO theory. More generally, the conjecture [21, 22] is that the two theories are in fact *equivalent* (S-dual), with their couplings related through $g_s^{(I)} = 1/g_s^{(HO)}$, a relation that is visible also in the low-energy (supergravity) description. Over the years additional evidence has accumulated in support of this S-duality conjecture, which is by now widely believed to be correct.

It is extraordinarily difficult to conclusively *prove* a statement of this kind, because we do not have tools to directly compute arbitrary quantities in a strongly-coupled string theory. For this same reason, S-duality is a very powerful statement, because it allows us to understand the behavior of a strongly-coupled theory in terms of a weakly-coupled one. In particular, armed with S-duality, we are confident that we understand I and HO at $g_s \gg 1$![vii]. Remarkably, the behavior we deduce in this regime is not as exotic as one might have thought, given that we are considering theories with strong gravitational interactions. In a similar way, IIB is believed to be S-dual to *itself* [23].

By means of the dualities described above we have as promised connected all of the string theories previously thought to be distinct, and have moreover been able to bring the strongly-coupled I, IIB and HO theories under our calculational control. And yet another major surprise awaits us, when we consider the strong-coupling behavior of the remaining two theories.

Consider IIA, where there is a D0-brane (a point-like object) with charge $Q^{(C1)} = 1$ and mass $m = 1/g_s l_s$ (even at strong coupling). For $g_s \gg 1$, the D0 mass is the smallest energy scale in the theory. Moreover, the dynamics of D-particles (encoded in a $(0+1)$-dimensional gauge theory) are such that n D0-branes can form a (marginal) bound state, with mass $m_n = n/g_s l_s$ and charge $Q^{(C1)} = n$. This evenly-spaced tower of states gives rise to a continuum as $g_s \to \infty$, a phenomenon that is reminiscent of the Kaluza-Klein story discussed in Sec. 1, but for the case of a *single* compact direction. Indeed, if we consider an *eleven*-dimensional theory in which the x^{10} direction is a circle of radius R_{10}, then a massless eleven-dimensional field ϕ yields an infinite tower of ten-dimensional fields ϕ_n, with masses $m_n = |n|/R_{10}$. This would precisely match the D0-brane bound state spectrum if it turned out to be the case that

$$R_{10} = g_s l_s \; . \tag{5}$$

In fact, ten-dimensional IIA supergravity, which as mentioned before is the low-energy approximation to IIA string theory, has been known for many years to be directly related to supergravity in *eleven* dimensions, with the additional dimension a circle of radius R_{10} [24]. More precisely, IIA supergravity can be obtained by restricting the fields of eleven-dimensional supergravity (a metric g_{MN}, a gauge field A_{MNP}, and a gravitino Ψ_α^M) to be constant along x^{10}, *i.e.*, truncating their Kaluza-Klein expansions down to the $p_{10} = 0$ modes. When the circle is small, these modes have masses much lower than all the rest, and so the truncation is physically justified.

The D0-brane bound state spectrum discussed above is evidence that, by passing to IIA string theory, the correspondence at the level of IIA supergravity can be extended to the regime where R_{10} is not small, where it necessarily involves the $p_{10} = n/R_{10} \neq 0$ modes. Indeed, the D0-branes are charged under the gauge field C_μ of the IIA theory, which arises from the μ-10 component of the eleven-dimensional metric, and so the D-particle bound states have precisely the right properties to match the *full* Kaluza-Klein tower of eleven-dimensional supergravity. Moreover, the 10-10 component of the metric, which controls the size of the eleventh dimension, translates into the IIA dilaton field φ, which determines the string coupling constant. The precise relation is in fact (5). Finally, the IIA gauge field $B_{\mu\nu}$, which couples to the fundamental string, descends from the eleven-dimensional gauge field $A_{\mu\nu\{10\}}$, which would naturally couple to a *membrane*.

The conclusion [21, 25] is then that IIA string theory is secretly eleven-dimensional, and its fundamental degree of freedom, the string, is in fact a membrane (the 'M2-brane') wrapped around the hidden dimension! The well-known connection at the level of supergravity extends to the full string theory, which is understood then to be a special (small $R_{10} \longleftrightarrow g_s \ll 1$) limit of an eleven-dimensional theory. This larger theory is *not* a string theory; it has been provisionally baptized M-theory (with 'mystery' one of the intended meanings). From the preceding discussion we know that eleven-dimensional supergravity gives its effective low-energy description (at $E \ll 1/l_P$, with l_P the Planck length in eleven dimensions). The situation is summarized in Fig. 1.

M-theory can be shown to englobe not only IIA but also all of the other known string theories, which are thus understood to be part of a single underlying framework. Our pre-1994 understanding of the five known string theories and eleven-dimensional supergravity was comparable to that of the six blind men of the Indian fable: upon approaching an elephant, each of them touched a different part, and consequently perceived the animal in a way completely different from the other five— see Fig. 2. Just like theirs, our six formulations of *the* theory were at the same time all right, and all wrong! Various proposals have been made regarding the precise nature of the entire elephant, M-theory (some would have us believe that 'M' is for matrix [26], others, for membrane [27]), but the final word is yet to be spoken.

4. A Word on Phenomenology

The theoretical developments that we have reviewed in the previous sections are remarkable. Various apparently dissimilar pieces have all assembled together in a very non-trivial manner, leaving us with a glimpse of a unified theoretical structure, M-theory, that embodies what is regarded by a large portion of the particle physics community as our most promising attempt to give a complete description of the fundamental structure of our universe. Nevertheless, over the years, string theory has been harshly criticized for its lack of definite experimental predictions. The goal of this section is certainly not to provide an overview of what has been accomplished in the large subfield of string phenomenology, but merely to attempt to clarify the sense in which string theory provides predictions that can be compared with experiment, a point that is sometimes sorely misunderstood.

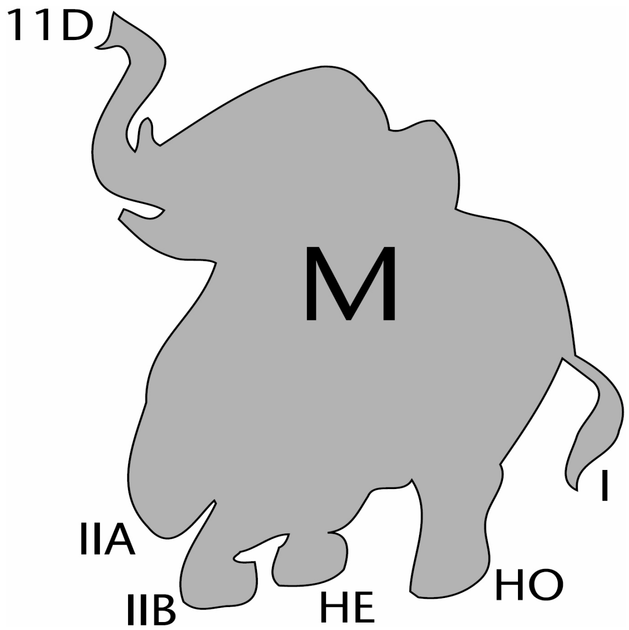

FIGURE 2. The M elephant. The five previously known string theories (I, IIA, IIB, HO, and HE), together with an eleven-dimensional theory (11D) whose low-energy dynamics are captured by supergravity, are all just special limits of a single underlying structure, provisionally known as M-theory.

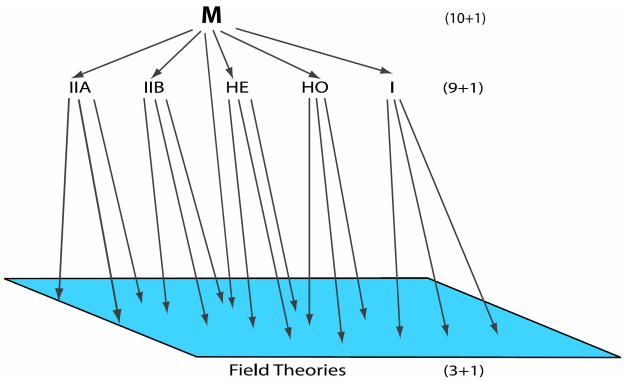

FIGURE 3. String/M-theory is *unique*, but it has a large number of *solutions*, each of which corresponds to an allowed universe. A given solution reduces at low energies to a *specific* field theory, with definite experimental predictions.

The simple point we wish to make is summarized in Fig. 3. At the bottom of the figure we have schematically depicted the space of all possible (particle) field theories. There is a large arbitrariness in defining a field theory: one must choose the dimension of spacetime, the field content, the symmetry groups (if any), the representations of these symmetry groups under which the fields transform, the masses, the coupling constants, etc. In stark contrast with this, at the top of the figure we have string/M-theory, where as we have seen, all evidence points to the existence of a *single* consistent theory, defined in eleven dimensions.

To the best of our knowledge, however, the equations of motion of this unique theory have an astronomical number of (at least approximate) *solutions*. Each of these represents a possible universe, and as such, leads (after sufficient calculation) to *specific* predictions that *can* be compared with experiment. A large number of these solutions are in fact discarded by such a comparison even with little calculation. For instance, most of the solutions correspond to universes where all interactions propagate in more than three macroscopic spatial dimensions, a possibility that is ruled out simply by looking around us. There is however a (still huge) number of solutions where six of the dimensions are hidden in some way, for instance by being compact and small. At the energies that we are able to probe in current particle accelerators, each specific choice of a size and shape for these hidden dimensions leaves us with a concrete four-dimensional *field theory*, a connection indicated by the vertical arrows in the figure^{viii}.

For practical purposes, then, string phenomenology can be seen as a subfield of particle phenomenology; from this perspective string theory should be viewed as a useful set of rules for generating field theories that have a built-in consistent coupling with quantum gravity. There is a significant number of string phenomenologists, who dedicate most of their efforts to working out the implications of those solutions of string theory that seem most promising for describing the world around us. An important question is how many of the known solutions to the approximately known equations remain as solutions of the *exact* equations, i.e., how many different universes are allowed by string/M-theory. Before 1994 the hope was often expressed (and, sometimes, not in the humblest terms!) that non-perturbative effects would rule out all but one solution (or at most a small number of them), and consequently yield a unique set of predictions regarding the fundamental structure of our universe. Since then, we have uncovered a fair portion of the non-perturbative framework of the theory, and the indications in this direction are not particularly encouraging (see, *e.g.*, Ref. 28).

Almost certainly, then, there is a huge number of exact solutions and, correspondingly, possible predictions [29]. Notice however, that, contrary to what has been sometimes asserted (see, for instance, [30]), this does not mean that string theory is 'not a good physical theory' because it 'cannot be falsified'. The point is that, at least at our current level of understanding, trying to falsify the entire framework of string theory is very much like trying to rule out all possible field theories in one go. It would be possible only if we could identify some prediction that holds for all allowed solutions (a more restrictive analog of CPT symmetry or the spin-statistics theorem for field theory)^{ix}. For recent progress in this direction, see Ref. 31, and references therein.

In lieu of this, the most important question is of course whether at least one of the allowed solutions reproduces *precisely* the Standard Model, plus additional effects that are not in conflict with any known experimental results (but could be detected in future experiments). Notice that this is a rather tall order: the required solution must not only yield the expected gauge group and three generations of chiral fermions, but also get the ~ 20 parameters of the Standard Model right on the nose. Approximate solutions are known that achieve the first task and make some progress on the second (see, *e.g.*, Ref. 32), as a result of which there is reasonable confidence in the string community that the goal is attainable. If one could establish that there is no such solution, then of course string theory will have proven to be completely useless for ambitious, 'theory-of-everything' type phenomenology. Conversely, the discovery of at least one such solution would be a momentous achievement: even before the all-important confirmation of the *new* effects predicted by it, we would for the first time have at our disposal a framework that unifies all of the known building blocks and interactions of our universe starting from an exceedingly simple set of principles^{x}.

5. Black Hole Entropy

Thirty years ago, Bekenstein and Hawking established that black holes possess thermodynamic properties, and in particular an entropy that is proportional to the area of their event horizon, $S_{\rm BH} = A_{\rm h}/4G_N$. Since then, the problem of finding, through explicit state-counting, a statistical-mechanical interpretation of this formula has been regarded as a crucial test for any theory that aims at describing gravity microscopically. Starting with [33], string theory has in recent years given strong indications that it can successfully overcome this test^{xi}.

The literature on this subject is enormous [2], and we will only be able here to comment briefly on some of the main ideas. The cases where the most *quantitative* progress has been made rely on the insight, reviewed in Sec. 2, that R-branes have an alternative description in terms of a collection of D-branes. On one side of this equivalence we have a black p-brane solution (or, if we wrap the p spatial dimensions of the brane along some hidden dimensions, a black hole in $9-p$ dimensions), and in particular, an event horizon. On the other side, we have the open string description of the D-brane dynamics, which reduces at substringy energies to a non-Abelian gauge theory, where an explicit state-counting may be performed. The only problem is that these two descriptions are under calculational control in mutually exclusive regimes: the black brane solution is reliable as long as its radius of curvature is much larger than the string length, which translates into the requirement that $g_s N \gg 1$, where N is (any one of) the charge(s) of the black brane; the gauge theory, on the other hand, is weakly-coupled only if $g_s N \ll 1$.

For $g_s N \gg 1$, then, we can reliably determine the area of the horizon, but we cannot in general directly compute the number of states in the strongly-coupled gauge theory. However, as mentioned in Sec. 3, in the special case that the black brane carries the minimum allowed mass for the given (set of) charge(s), the number of states does not depend on the coupling, and so can be safely determined in the weakly-coupled gauge theory. The corresponding black branes are said to be *extremal*; it is for these branes that we can most confidently put the Bekenstein-Hawking relation to the test. The simplest examples are the extremal black Rp-branes discussed in Sec. 2, but the agreement there is trivial: the horizon area of the solutions vanishes, corresponding to the fact that the entropy of their microscopic counterpart, an unexcited N Dp-brane system, is zero (there is only one such state).

The first successful account of black brane entropy was obtained by comparing certain extremal (yet finite-horizon-area) black holes in five dimensions against microscopic state counting in a D5-brane/D1-brane system that carries momentum along the direction of the D-strings [33]. Perfect agreement is achieved, despite the fact that in the two alternative descriptions one is determining the entropy in drastically different ways. Over the years, many other successful quantitative comparisons have been made, for dozens of different types of black holes, most of them extremal or near-extremal. In some cases it has been possible to reproduce additional properties of the black hole, such as the rate of Hawking radiation. In the far-from-extremal regime, which includes in particular the neutral (Schwarzschild) black hole, gaining enough control over the relevant calculations to be able to provide a fully quantitative account of the black hole thermodynamics has proven to be more challenging. Various proposals have been made (see, *e.g.*, Ref. 36 and references therein), but the issue is not completely settled yet.

In some cases, it has been possible to peform these comparisons even beyond the leading order contribution (in an expansion in large charges), where an impressive agreement was again found between corrections to the microscopic and black hole entropies, first at subleading order [37], and more recently at all orders, utilizing and interesting conjectured relation between the state-counting for a four-dimensional BPS black hole and the partition function for the topological string [38].

In addition to the quantitative tests, there is an important qualitative consistency criterion, known as the correspondence principle [39], that can be put to the test in situations more general than those involving D-branes. The idea is that, when considering a specific black brane with a fixed mass and a given set of charges, there will generically be two possible descriptions of the system that, as in the D-brane cases discussed above, are valid in mutually exclusive regimes. One can pass from one regime to the other by progressively increasing the value of the string coupling g_s. Letting $\bar{g}_s$ denote the value of the coupling at the (only approximately defined) cross-over point, the correspondence principle asserts that, if we equate the masses of the system in the two alternative descriptions at $g_s = \bar{g}_s$, we should find that the corresponding entropies roughly agree. This prediction has been verified in a wide range of cases, a result that is certainly non-trivial. There is no *a priori* reason to expect even rough agreement between the entropies in the two descriptions, unless they both refer to the *same* physical system.

The significant progress that has been made so far on the black hole entropy front strongly suggests that string theory provides a sensible description of quantum gravity even at the non-perturbative level. Needless to say, many questions remain to be addressed. One would, for instance, like to be able to analyze more general classes of black holes, where the overall spacetime geometry is not necessarily stationary, or understand how the geometric and causal structure of the black hole is encoded in the gauge theory description (for recent progress in this direction, see, *e.g.*, Ref. 40, and references therein).

6. Gauge/Gravity Duality

A remarkable by-product of the work on black hole entropy was the discovery of the Maldacena, or gauge/gravity, duality [41, 42], a surprising equivalence between string theory defined on certain backgrounds and ordinary non-Abelian gauge theories.

The starting point for this development is again the equivalence between Rp-branes and Dp-branes. To be specific, let us consider the best understood example, $p = 3$. We begin then with two alternative descriptions of the same physical system. On the one hand, we have the black R3-brane solution with charge $Q^{(C4)} = N$, which involves a constant dilaton $e^{\varphi} = g_s$ and a curved, asymptotically-flat geometry. In this description, closed strings are the only allowed excitations, and they may propagate out in the flat region or somewhere down the throat. Perturbative calculations may be carried out as long as $g_s N \gg 1$ (and $g_s \ll 1$). On the other hand, we have a stack of N coincident D3-branes in *flat* ten-dimensional spacetime. The excitations here are either closed strings moving about the nine spatial dimensions or open strings that slide along the worldvolume of the branes. A perturbative analysis of the dynamics is then possible only if $g_s N \ll 1$. It is important to emphasize that, despite the fact that the two *perturbative* descriptions are mutually exclusive, their non-perturbative completions are believed to exist and be equivalent to one another. In other words, already at the string theory level there is a *duality*, a 'worldvolume/geometry' correspondence (see, *e.g.*, Ref. 43, and references therein).

Maldacena's observation [41] is that, if we consider this system at extremely low energies ($E \ll 1/(g_s N)^{1/4} l_s \ll 1/l_s$), two interesting things happen. First, there is a decoupling between the closed strings that propagate on (asymptotically) flat space and the degrees of freedom of the branes, described on the worldvolume side by open strings, and on the geometry side by closed strings that live in the immediate vicinity of the horizon. Second, the descriptions on both sides simplify drastically. On the geometry side, to follow the brane degrees of freedom we are forced to zoom in on the near-horizon region, and are then left with a much simpler geometry: the product of a five-dimensional spacetime with *constant negative* curvature (known as anti-de Sitter, or AdS, space) and a five-dimensional sphere. On the worldvolume side, the low-energy limit leaves us only with the non-Abelian gauge theory describing the massless open string modes (*i.e.*, the strings are effectively reduced to point particles), namely, $(3+1)$-dimensional super-Yang-Mills with gauge group $SU(N)$ (and $\mathcal{N} = 4$ supersymmetries, in the four-dimensional sense), which happens to be a conformally-invariant field theory (CFT)^{xii}. On the geometry side we would at first sight expect the low-energy limit to induce a similar reduction of the closed strings to point particles (in which case we would be left with supergravity), but the presence of a gravitational redshift factor can be seen to imply that, in fact, the whole tower of string states is retained. Maldacena's astonishing conclusion is then that IIB *string* theory defined on the *ten*-dimensional spacetime $\mathrm{AdS}_5 \times \mathbf{S}^5$ is equivalent to $SU(N)$ SYM, a *four*-dimensional *field* theory! In the regime where the geometric description is under control, the gauge theory is strongly-coupled.

By means of this correspondence one obtains, in one direction, a prediction for the behavior of strongly-coupled gauge theories, and in the opposite direction, a potential resolution to some of the mysteries of quantum gravity, and in particular, a non-perturbative definition of string theory on specific classes of fixed backgrounds. It is not surprising therefore that the Maldacena duality has elicited so much excitement[xiii]. It is a concrete incarnation of the concept of *holography* [44], which (based on the black hole entropy formula) asserts that quantum gravity in some region of space should be describable in terms of non-gravitational degrees of freedom that, morally speaking, live on the boundary of that region. It is perhaps worth emphasizing that the dynamics captured by the gauge theory description is *not* restricted to just small fluctuations about the AdS background, but includes arbitrarily large deformations of the geometry (*e.g.*, black holes). $\mathcal{N} = 4$ SYM can therefore be viewed as a complete, non-perturbative definition of type IIB string theory on asymptotically $\mathrm{AdS}_5 \times \mathbf{S}^5$ spacetime. On the other hand, the choice of a gauge theory does restrict the *asymptotic* behavior of the spacetime, so the description is not completely background-independent.

The original AdS/CFT correspondence has been subjected to a large number of non-trivial tests, and generalized to many other (non-AdS) background geometries and, correspondingly, many different (non-CFT) gauge theories [2]. In particular, geometries that are only asymptotically AdS and have boundary conditions modified by turning on non-normalizable modes are understood to be dual to gauge theories obtained by deforming the CFT by the addition of infrared-relevant (but ultraviolet-irrelevant) operators. This seems to suggest that, if one wanted to obtain a formulation that is able to describe in one package a wider class of background geometries, one would have to somehow consider a superposition of gauge theories[xiv].

A very important line of development in recent years has been the construction of string theory backgrounds that are dual to more realistic, QCD-like, gauge theories (see, *e.g.*, Ref. 47, and references therein), which raises the hope that in the not-too-distant future the gauge/gravity machinery could be brought to bear on the analysis of real-world QCD physics. Notice that this potential application is substantially more modest than, and completely orthogonal to, the more familiar goal of establishing string theory as a 'theory of everything': whether or not we can eventually understand the entire universe we see around us (QCD plus electroweak theory plus gravity plus dark matter plus dark energy plus corrections) in terms of a particular string theory that lives on a specific choice of background, there are well-grounded hopes that a string theory on a *different* background will one day be found to be *equivalent* to QCD alone, an equivalence that would provide us with a much-needed analytic handle on the strong-coupling physics of the strong interaction[v]. In fact, recent work [48] has shown that, even at its present stage of development, the AdS/CFT correspondence could soon allow us to make quantitative predictions on the properties (entropy, viscosity, jet-quenching, charmonium suppression, etc.) of the strongly-coupled quark-gluon-plasma produced at RHIC[xvi] and ALICE[xvii]!

Another outstanding recent result was the derivation of the entire spectrum of the IIB closed string in flat space (and more generally, on certain plane-wave geometries) directly from the SYM description [49], which has led to significant advances in the direction of determining the underlying degrees of freedom in non-perturbative string/M theory [50].

7. And More…

As we have seen in the previous sections, quite independently of its eventual success or failure on the phenomenology front (either as a 'theory of everything' or as a tool for QCD computations), string theory has led to important advances on several *theoretical* questions. These include not only the various 'internal' developments that have vastly improved our understanding of the theory (like the discovery of M-theory) or our confidence in it (like the black hole entropy story), but also 'external' results that bear on other problems of interest in modern theoretical physics (like understanding the behavior of strongly-coupled gauge theories).

There have in fact been many more results in the past decade than we have had space to review here [2]. Among internal developments, probably our most important omission is Matrix theory [26], a proposal for the non-perturbative definition of M-theory (in a specific kinematic setup) which, just like the Maldacena proposal [41], is based on matrices of large rank ($N \to \infty$). Loosely speaking, Matrix theory proposes that D0-branes are the fundamental degrees of freedom of M-theory. Other internal developments include constructions of string theory backgrounds with positive cosmological constant and statistical analyses of the physical properties of large classes of string vacua [51], results on the decay of unstable D-branes [52] which have led to a resurgence of string field theory [53] as an alternative candidate for a non-perturbative formulation of the theory, the discovery of various open [54] or wrapped [55] brane theories that do not incorporate gravity in the usual sense and therefore constitute simplified versions of string/M-theory, and the reaffirmation of certain two-dimensional string theories as toy models that can teach us valuable lessons on the nature of non-perturbative string theory [56]. As for external developments, besides braneworld models and the gauge/gravity correspondence one should include the advances in supersymmetric gauge theories (see in particular Ref. 57), the connection between D-branes with magnetic fields and noncommutative field theories [58], the development of simplifying techniques for certain perturbative field-theoretic computations (see in particular Ref. 59), and the formulation of various novel cosmological models (see, *e.g.*, Ref. 60), as well as several points of contact with mathematics (see, for instance, Ref. 61).

As we have noted throughout this article, there is of course much that remains to be done. On the theoretical front, the main open problem is to ascertain the precise nature of M-theory, ideally in the context of a background-independent formulation (which should in addition be manageable for practical calculations!) On the phenomenology front, the main outstanding tasks are to find the string theory dual of QCD and use it to obtain firm experimental predictions (*e.g.*, for the quark-gluon-plasma), and/or to establish string theory as a 'theory of everything' by making complete quantitative contact with the Standard Model, and then predict new effects that could establish conclusively the viability (or lack thereof) of string/M-theory for describing the world around us.

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i. The above discussion is merely schematic, most importantly because each of the variables θ^A is a ten-dimensional *spinor* with sixteen independent components, which implies that many more terms should be included in the Taylor expansion of Φ. For instance, at the massless level of the $\mathcal{N}=2$ string theories one obtains 128 bosonic and 128 fermionic states.

ii. Non-supersymmetric ($\mathcal{N}=0$) string theories generically have *tachyons*. As in the case of the Standard Model Higgs field, this in itself is merely a sign of instability (one is expanding about a *maximum* of the relevant potential) and not of inconsistency; but the question of whether (as in the Higgs case) a stable vacuum exists remains to be answered. A non-supersymmetric *and* tachyon-free theory was in fact known [3] already in 1987; it is closely related to the HO and HE theories [4]. In any event, the post-1994 developments allow us to consider many consistent non-supersymmetric models, each of which would have been considered a separate 'theory' in the old days. The restriction on the bosonic dimension must also be qualified. The simplest theory of strings one can write down has $\mathcal{N}=0$ and lives in $D=26$ dimensions; but its spectrum includes a tachyon and no fermions. (This theory might, however, be connected to the $\mathcal{N}>0$ theories in a subtle way [5].) In addition, one can have $D\leq 10$ (or $D\leq 26$) if the string propagates on backgrounds whose scale of variation (in space or time) is of order the string length [6]. It has been suggested that it could be possible to evade the restriction on the dimension altogether if one quantizes the string in a non-standard manner [7], but it remains to be seen whether such alternative 'quantizations' are in fact physically relevant.

iii. More generally, a p-dimensional object naturally carries an electric charge under an antisymmetric tensor gauge field with $p+1$ indices, and, in D spacetime dimensions, a $(D-4-p)$-dimensional object carries magnetic charge. The case that is familiar to us is of course a vector potential in $D=4$, for which both the electric and magnetic sources are point-like ($p=0$).

iv. Parallel Dp-branes do not interact, and as a consequence their masses just add linearly.

v. Notice that this identification would imply in particular that D-branes can be viewed as closed string solitons, *i.e.*, coherent excitations of the stringy background.

vi. An even more radical indication is provided by *mirror symmetry*.

vii. Notice, however, that we still do not understand the regime $g_s\sim 1$.

$viii$. The advances of the past decade have in particular allowed us to consider choices/arrows that originate directly from M-theory, without passing through one of its perturbative string limits.

ix. Note that, contrary to widespread belief, low-energy supersymmetry is *not* a truly generic prediction of string theory (see, *e.g.*, [28, 29]). On the other hand, if supersymmetry were to be discovered, it could potentially give important clues regarding the nature of the desired solution.

x. We emphasize that it is simplicity of *principles*, and not of results, that should be sought. For instance, people sometimes complain that string theory is too complicated because it predicts a large number of additional dimensions and an infinite number of particles that have so far escaped detection; but this is the output, not the input, of the theory.

xi. As has the main alternative approach to quantizing gravity, loop quantum gravity (the original Rer. 34; for a review, see, *e.g.*, Ref. 35). In both approaches, however, there are technical as well as conceptual issues that remain to be grappled with.

xii. The conformal group ($SO(4,2)$ in $3+1$ dimensions) is obtained by enlarging the Poincaré algebra with the generator of rigid scale transformations, together with the so-called special conformal transformations that are necessary to close the algebra. QCD with massless quarks is conformally invariant at the classical level (because it has no intrinsic mass scale), but the scale invariance is broken at the quantum level by the running of the gauge coupling (which is associated with the emergence of Λ_{QCD} as an intrinsic mass scale.

$xiii$. In the nine years that have elapsed since its publication, the original work [41] has accumulated close to 4200 citations!

xiv. The perturbative description of the string in terms of two-dimensional conformal field theories suggests a similar 'theory of theories' background-independent formulation [45, 46].

xv. This was in fact the original goal of string theory from the time of its inception and up to the mid-seventies, when it was overtaken by QCD.

xvi. The acronym stands for Relativistic Heavy Ion Collider, an accelerator at Brookhaven, U.S.A.

$xvii$. A Large Ion Collider Experiment, currently under construction at the LHC in CERN.

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