# Deformation quantization for fermionic fields

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Recibido el 1 de mayo de 2006; aceptado el 1 de noviembre de 2006

The deformation quantization of any Grassmann free field, or fermionic free field, in particular, the Dirac free field is discussed. Stratonovich-Weyl quantizer, Moyal product and Wigner functionals are obtained for this field by deforming suitable Fermi oscillator variables. In addition the propagator of the Dirac field is computed in this context.

*Keywords:* Deformation quantization; Weyl-Wigner-Moyal formalism; Dirac field.

Se discute la cuantización por deformación de un campo libre de Grassmann, o campo fermiónico libre; en particular, el formalismo es aplicado al campo libre de Dirac. El cuantizador de Stratonovich-Weyl, el producto de Moyal y las funcionales de Wigner son obtenidos para este campo, deformando las variables apropiadas del oscilador de Fermi. Además, el propagador del campo de Dirac es calculado en este contexto.

*Descriptores:* Cuantización por deformación; formalismo de Weyl-Wigner-Moyal; campo de Dirac.

PACS: 03.70.+k; 11.10.-z

## 1. Introduction

The deformation quantization is an alternative and independent formulation to the canonical quantization and the path integral quantization in quantum mechanics. In this formalism, the quantization is understood as a deformation of the structure of the algebra of classical observables instead of a radical change in the nature of them, said quantization is originated from the deformation of the usual product and therefore, as a deformation of the Lie algebra determined by the Poisson bracket. The product deformed is called star product, particularly we work with the Moyal product.

Since the mathematical point of view, the deformation quantization is very well posed, nevertheless its application to physical systems presents large difficulties.

The deformation quantization has been extensively studied for systems with a finite number degrees of freedom, and is natural to be asked if is possible quantizer systems with an infinite number of degrees of freedom, that besides be consistent with the Lorentz invariance and with the gauge invariance.

In this work we present the formalism of Weyl-Wigner-Moyal for fermionic fields, and it is applied to Dirac field as an example.

## 2. Deformation Quantization of Grassmann Scalar Field

Consider a real scalar Grassmann field on the Minkowski spacetime $M^{d+1}$ of signature $(+,-,-,\dots,-)$. By a Grassmann scalar field we will understand an smooth function $\Theta$ over $M^{d+1}$ and which takes values in the *field* of (anticommuting) Grassmann numbers $\mathbb{G}$, *i.e.*, $\Theta$ is the map $\Theta : M^{d+1} \to \mathbb{G}$. Canonical variables of this classical Grassmann field will be denoted by $\Theta(x,t)$ and $\pi_\Theta(x,t)$ with $(x,t) \in M^{d+1} = \mathbb{R}^d \times \mathbb{R}$.

We deal with fields at the instant $t = 0$ and we denote $\Theta(x,0) \equiv \Theta(x)$ and $\pi_\Theta(x,0) \equiv \pi_\Theta(x)$. It is worth to mention that some of the functional formulas and their manipulations are formal. It is also important to notice that since we will deal with Grassmann variables all the computations and results obtained are valid under the specified conventions and ordering of factors given in this section. In this section we study the deformation quantization of these Grassmann fields, including: Stratonovich-Weyl quantizer, Moyal product and Wigner functional [1–5].

### 2.1. The Stratonovich-Weyl Quantizer

Let $F[\pi_\Theta,\Theta]$ be a functional on the phase space $\mathcal{Z}_\mathbb{G} \equiv \{(\pi_\Theta,\Theta)\}$. By analogy to quantum mechanics we can establish the Weyl quantization rule as follows

$$\widehat{F} = W\left(F[\pi_\Theta,\Theta]\right) = \int \mathcal{D}\left(\frac{\pi_\Theta}{2\pi\hbar}\right)\mathcal{D}\Theta F[\pi_\Theta,\Theta]\widehat{\Omega}[\pi_\Theta,\Theta], \qquad (1)$$

where $\widehat{\Omega}$ is the Stratonovich-Weyl quantizer (see Refs. [1]) and it is given by:

$$\widehat{\Omega}[\pi_\Theta,\Theta] = \int \mathcal{D}\eta \exp\left\{-\frac{i}{\hbar}\int dx\eta(x)\pi_\Theta(x)\right\} \times \left|\Theta + \frac{\eta}{2}\right\rangle\left\langle\Theta - \frac{\eta}{2}\right|. \qquad (2)$$

One can check that this operator satisfies the following properties:

$$\mathrm{Tr}\left\{\widehat{\Omega}[\pi_\Theta,\Theta]\right\} = 1,$$

$$\mathrm{Tr}\left\{\widehat{\Omega}[\pi_\Theta,\Theta]\widehat{\Omega}[\pi'_\Theta,\Theta']\right\} = \delta[\Theta-\Theta']\delta[\pi_\Theta-\pi'_\Theta]. \qquad (3)$$

If one multiplies Eq. 1 by $\widehat{\Omega}[\pi_\Theta,\Theta]$ and takes into account the Eq. 3 one easily gets

$$F[\pi_\Theta,\Theta]=\mathrm{Tr}\Big\{\widehat{\Omega}[\pi_\Theta,\Theta]\widehat{F}\Big\}. \quad (4)$$

### 2.2. The Moyal Product

Now we are in a good position to define the Moyal product [5–7] in a theory involving Grassmann scalar fields. Let $F=F[\pi_\Theta,\Theta]$ and $G=G[\pi_\Theta,\Theta]$ be some functionals on $\mathcal{Z}_{\mathbb{G}}$ that correspond to the field operators $\widehat{F}$ and $\widehat{G}$ respectively, *i.e.* $F[\pi_\Theta,\Theta]=W^{-1}(\widehat{F})=\mathrm{Tr}\big(\widehat{\Omega}[\pi_\Theta,\Theta]\widehat{F}\big)$ and $G[\pi_\Theta,\Theta]=W^{-1}(\widehat{G})=\mathrm{Tr}\big(\widehat{\Omega}[\pi_\Theta,\Theta]\widehat{G}\big)$. We want to find the functional which corresponds to the operator product $\widehat{F}\widehat{G}$ will be denoted by $(F\star G)[\pi_\Theta,\Theta]$. So we have

$$(F\star G)[\pi_\Theta,\Theta]:=W^{-1}(\widehat{F}\widehat{G})=\mathrm{Tr}\Big\{\widehat{\Omega}[\pi_\Theta,\Theta]\widehat{F}\widehat{G}\Big\}. \quad (5)$$

Using Eqs. 1 and 5 and performing some simple calculations one gets

$$\begin{aligned}(F\star G)[\pi_\Theta,\Theta]=&\int\mathcal{D}\Big(\frac{\pi'_\Theta}{2\pi\hbar}\Big)\mathcal{D}\Big(\frac{\pi''_\Theta}{2\pi\hbar}\Big)\mathcal{D}\Theta'\mathcal{D}\Theta''\,F[\pi'_\Theta,\Theta']\\ &\times\exp\left\{\frac{2i}{\hbar}\int dx\Big((\Theta-\Theta')(\pi_\Theta-\pi''_\Theta)\right.\\ &\left.-(\Theta-\Theta'')(\pi_\Theta-\pi'_\Theta)\Big)\right\}G[\pi''_\Theta,\Theta''].\end{aligned} \quad (6)$$

For future convenience let's introduce new variables $\Psi'{=}\Theta'{-}\Theta$, $\Psi''{=}\Theta''{-}\Theta$, $\Pi'{=}\pi'_\Theta{-}\pi_\Theta$, $\Pi''{=}\pi''_\Theta{-}\pi_\Theta$. Using the expansion of $F[\pi'_\Theta,\Theta']=F[\pi_\Theta+\Pi',\Theta+\Psi']$ and $G[\pi''_\Theta,\Theta'']=G[\pi_\Theta+\Pi'',\Theta+\Psi'']$ in Taylor series and after some manipulations we obtain

$$\big(F\star G\big)[\pi_\Theta,\Theta]=F[\pi_\Theta,\Theta]\exp\left\{\frac{i\hbar}{2}\overleftrightarrow{\mathcal{P}}_{\mathbb{G}}\right\}G[\pi_\Theta,\Theta], \quad (7)$$

where

$$\begin{aligned}\overleftrightarrow{\mathcal{P}}_{\mathbb{G}}:=-(-1)^{\varepsilon_F\varepsilon_G}\int dx\Bigg(&\frac{\overleftarrow{\delta}}{\delta\Theta(x)}\frac{\overrightarrow{\delta}}{\delta\pi_\Theta(x)}\\ &+(-1)^{\varepsilon_F\varepsilon_G}\frac{\overleftarrow{\delta}}{\delta\pi_\Theta(x)}\frac{\overrightarrow{\delta}}{\delta\Theta(x)}\Bigg).\end{aligned} \quad (8)$$

is the Poisson operator, which define the Poisson bracket for two functionals $F$ and $G$ given by

$$\begin{aligned}F\overleftrightarrow{\mathcal{P}}_{\mathbb{G}}\,G:=&\{F,G\}_P\\ =&-(-1)^{\varepsilon_F\varepsilon_G}\int d^3x\bigg\{\frac{\delta F[\pi_\Theta,\Theta]}{\delta\Theta(x)}\frac{\delta G[\pi_\Theta,\Theta]}{\delta\pi_\Theta(x)}\\ &+(-1)^{\varepsilon_F\varepsilon_G}\frac{\delta G[\pi_\Theta,\Theta]}{\delta\Theta(x)}\frac{\delta F[\pi_\Theta,\Theta]}{\delta\pi_\Theta(x)}\bigg\},\end{aligned} \quad (9)$$

with $\varepsilon=1,0$ depending if the corresponding functionals $F$ and $G$ are even or odd. This is precisely the super-Poisson bracket reported in the literature [8–12].

This Poisson bracket is associated to the symplectic structure

$$\omega_{\mathbb{G}}=\int_\Sigma dx\delta\pi_\Theta(x)\wedge\delta\Theta(x),$$

which gives to $\mathcal{Z}_{\mathbb{G}}$ the structure of a symplectic manifold.

### 2.3. The Wigner Functional

Let $\widehat{\rho}=|\Phi\rangle\langle\Phi|$ be the density operator of a quantum state. Then the Wigner functional $\rho_W[\pi_\Theta,\Theta]$ corresponding to this state according to 4, is given by [2–4, 13]

$$\begin{aligned}\rho_W[\pi_\Theta,\Theta]=\int\mathcal{D}\eta\exp\Big\{-\frac{i}{\hbar}\int dx\,\eta(x)\pi_\Theta(x)\Big\}\\ \times\Phi^*[\Theta-\frac{\eta}{2}]\Phi[\Theta+\frac{\eta}{2}].\end{aligned} \quad (10)$$

## 3. Deformation Quantization of the Dirac Free Field

The aim of this section is to provide an example of the application of the deformation quantization of Grassmann fields to the Dirac free field. In addition we compute the propagator of the Dirac field in this context. We stress the uses of the oscillator variables $\mathbf{b}^*$ and $\mathbf{b}$ which allowed us to perform the construction (for details, see for instance Ref. 14).

### 3.1. Dirac Free Field

In this section we discuss the Dirac free field $\psi(x)$ over Minkowski spacetime $M=\mathbb{R}^3\times\mathbb{R}$ with signature $(+,-,-,-)$ and $x=(\vec{x},t)\in M$. The action is given by

$$\begin{aligned}I_D[\psi]&=\int d^3xdt\mathcal{L}(\psi(\vec{x},t),\partial_\mu\psi(\vec{x},t))\\ &=\int d^3xdt\overline{\psi}(\vec{x},t)\big(i\,\partial\!\!\!/-m\big)\psi(\vec{x},t),\end{aligned} \quad (11)$$

where $\partial\!\!\!/=\gamma^\mu\partial_\mu$, $\gamma^\mu$ are the Dirac matrices ($\mu=0,\ldots,3$), $\overline{\psi}(x)\equiv\psi^\dagger(x)\gamma^0$ and $m$ is the mass parameter. Thus, the field $\psi(x)$ fulfills the Dirac equation

$$\big(i\,\partial\!\!\!/-m\big)\psi(\vec{x},t)=0. \quad (12)$$

Its conjugate momentum is given by

$$\pi_\psi(\vec{x},t)=\frac{\partial\mathcal{L}}{\partial(\partial\dot{\psi})}=i\psi^\dagger(\vec{x},t),$$

where $\dot{\psi}(\vec{x},t)\equiv\partial\psi(\vec{x},t)/\partial t$. Then the hamiltonian can be written as

$$H_D[\pi_\psi,\psi]=\frac{1}{2}\int d^3x\,\psi^\dagger(\vec{x},t)i\frac{\partial}{\partial t}\psi(\vec{x},t), \quad (13)$$

where $\gamma^i = \beta\alpha^i$, $\gamma^0 = \beta$ and $j$ runs over the spatial coordinates $j = 1, 2, 3$.

According to the definition of the Poisson bracket for Grassmann fields given by Eq. 9, the Poisson bracket for $\psi_\alpha(\vec{x}, t)$ and $\pi_{\psi_\alpha}(\vec{x}, t)$ is given by:

$$\{\psi_\alpha(\vec{x},t), \pi_{\psi_\beta}(\vec{y},t)\}_P = -\delta(\vec{x}-\vec{y})\delta_{\alpha\beta},$$
$$\{\psi_\alpha(\vec{x},t), \psi_\beta(\vec{y},t)\}_P = 0,$$
$$\pi_{\psi_\beta}(\vec{x},t), \pi_{\psi_\beta}(\vec{y},t)\}_P = 0, \quad (14)$$

where $\alpha = 1, 2, 3, 4$ runs over the components of the Dirac spinor.

As usual, the field variable $\psi_\alpha(\vec{x}, t)$ can be expanded as plane waves

$$\psi_\alpha(\vec{x},t) = \sum_{r=1}^{4}\int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{p}}}}\mathbf{b}(\vec{p},t,r)w_\alpha(\vec{p},r) \times \exp\big(i\vec{p}\cdot\vec{x}\big), \quad (15)$$

where $\mathbf{b}(\vec{p}, t, r) = \mathbf{b}(\vec{p}, r)\exp\big\{-i\varepsilon_r E_{\vec{p}} t\big\}$, where $\varepsilon_r = 1$ for $r = 1, 2$ and $\varepsilon_r = -1$ for $r = 3, 4$. The functions $w$'s satisfy the following relations,

$$w^\dagger_\alpha(\vec{p},r)w_{\alpha'}(\vec{p},r') = \frac{E_{\vec{p}}}{m}\delta_{rr'}\delta_{\alpha\alpha'},$$
$$\bar{w}_\alpha(\varepsilon_r\vec{p},r)w_{\alpha'}(\varepsilon'_r\vec{p},r') = \varepsilon_r\delta_{rr'}\delta_{\alpha\alpha'},$$
$$\sum_{r=1}^{4} w_\alpha(\vec{p},r)w^\dagger_{\alpha'}(\vec{p},r) = \frac{E_{\vec{p}}}{m}\delta_{\alpha\alpha'}. \quad (16)$$

Substituting Eq. 15 (and the corresponding to $\pi_{\psi_\alpha}(\vec{x}, t) = i\psi^\dagger(\vec{x}, t)$) into Poisson bracket 14 we find that the variables $\mathbf{b}$ y $\mathbf{b}^*$, must satisfy the following Poisson brackets

$$\{\mathbf{b}(\vec{p},r), i\mathbf{b}^*(\vec{p}\,',r')\}_P = -\delta(\vec{p}-\vec{p}\,')\delta_{rr'},$$
$$\{\mathbf{b}(\vec{p},r), \mathbf{b}(\vec{p}\,',r')\}_P = 0,$$
$$\{\mathbf{b}^*(\vec{p},r), \mathbf{b}^*(\vec{p}\,',r')\}_P = 0. \quad (17)$$

Thus the Grassmann variables $\mathbf{b}$ and $\mathbf{b}^*$, determine precisely the canonical conjugate variables which we will use to describe the Dirac field in the Weyl-Wigner-Moyal formalism.

Now if one substitute the expansions for the field variables of Eq. 15 in the hamiltonian in Eq. 13 one gets

$$H_D[\mathbf{b}^*,\mathbf{b}] = \sum_{r=1}^{4}\int d^3p\varepsilon_r E_{\vec{p}}\mathbf{b}^*(\vec{p},t,r)\mathbf{b}(\vec{p},t,r), \quad (18)$$

### 3.2. Stratonovich-Weyl Quantizer

Proceeding like in the previous section, we find that the Weyl-Wigner-Moyal correspondence for the Dirac field in terms of the canonical field variables take the following form

$$\widehat{F} = W(F[\mathbf{b}^*,\mathbf{b}]) = \int \mathcal{D}\Big(\frac{\mathbf{b}^*}{2\pi\hbar}\Big)\mathcal{D}\mathbf{b}F[\mathbf{b}^*,\mathbf{b}]\widehat{\Omega}[\mathbf{b}^*,\mathbf{b}], \quad (19)$$

and the Stratonovich-Weyl quantizer take the follow form

$$\widehat{\Omega}[\mathbf{b}^*,\mathbf{b}] = \int \mathcal{D}\xi \exp\left\{\frac{1}{\hbar}\sum_{r=1}^{4}\int d^3p\xi(\vec{p},r)\mathbf{b}^*(\vec{p},r)\right\} \times |\mathbf{b}+\frac{\xi}{2}\rangle\langle\mathbf{b}-\frac{\xi}{2}|, \quad (20)$$

where $\chi$ and $\xi$ are Dirac spinors.

### 3.3. The Moyal Product

The Moyal product in this case can be defined similarly as Eqs. 7 and 8. Let $F_1[\mathbf{b}^*, \mathbf{b}]$ and $F_2[\mathbf{b}^*, \mathbf{b}]$ be functionals over the Dirac phase space defined by:

$$\mathcal{Z}_D = \{(\pi_{\psi_\alpha}(\vec{x}), \psi_\alpha(\vec{x}))_{\vec{x}\in\Sigma}\}$$
$$= \{(\mathbf{b}^*(\vec{p},r)), (\mathbf{b}(\vec{p},r))_{r=1,\cdots,4}\}$$

and let $\widehat{F}_1$ and $\widehat{F}_2$ be their corresponding operators. Then by a similar computation to that done in the second part of the previous section, we finally get

$$\big(F_1 \star F_2\big)[\mathbf{b}^*,\mathbf{b}] = F_1[\mathbf{b}^*,\mathbf{b}]\exp\left(\frac{i\hbar}{2}\overleftrightarrow{\mathcal{P}}_D\right)F_2[\mathbf{b}^*,\mathbf{b}], \quad (21)$$

where

$$\overleftrightarrow{\mathcal{P}}_D := -(-1)^{\varepsilon_1\varepsilon_2}\sum_{r=1}^{4}\int d^3p\Big(\frac{\overleftarrow{\delta}}{\delta\mathbf{b}(\vec{p},r)}\frac{\overrightarrow{\delta}}{\delta\mathbf{b}^*(\vec{p},r)} + (-1)^{\varepsilon_1\varepsilon_2}\frac{\overleftarrow{\delta}}{\delta\mathbf{b}^*(\vec{p},r)}\frac{\overrightarrow{\delta}}{\delta\mathbf{b}(\vec{p},r)}\Big), \quad (22)$$

is the Poisson operator for any functionals $F_1$ and $F_2$. The operator $\overleftrightarrow{\mathcal{P}}_D$ determines the Poisson bracket $\{F, G\}_P = F\overleftrightarrow{\mathcal{P}}_D G$ defined by the symplectic structure

$$\omega_D = \int d^3x\sum_\alpha \delta\psi_\alpha(\vec{x})\wedge\delta\pi_\psi(\vec{x})$$
$$= i\int d^3x\sum_\alpha \delta\psi_\alpha(\vec{x})\wedge\delta\psi^\dagger_\alpha(\vec{x}).$$

This symplectic structure defines the symplectic manifold structure $(\mathcal{Z}_D, \omega_D)$ for the phase space.

### 3.4. Wigner Functional

The Wigner functional for the Dirac free field is defined in analogy to the scalar field case. Let $\widehat{\rho}^{phys}$ be the density operator corresponding to the quantum physical state of the Dirac field. Then, the corresponding Wigner functional to this state is given by

$$\rho_W[\mathbf{b}^*,\mathbf{b}] = \int \mathcal{D}(\frac{\xi}{2\pi\hbar})\exp\Big\{-\frac{i}{\hbar}\sum_{r=1}^{4}\int d^3p\,\xi(\vec{p},r) \times \mathbf{b}^*(\vec{p},r)\Big\}\langle\mathbf{b}+\frac{\xi}{2}|\widehat{\rho}^{phys}|\mathbf{b}-\frac{\xi}{2}\rangle. \quad (23)$$

In the case when $\widehat{\rho}^{phys} = |\Phi\rangle\langle\Phi|$, the above equation turns out into

$$\rho_W[\mathbf{b}^*,\mathbf{b}] = \int \mathcal{D}\left(\frac{\xi}{2\pi\hbar}\right) \times \exp\left\{\frac{-i}{\hbar}\sum_{r=1}^{4}\int d^3p\,\xi(\vec{p},r)\mathbf{b}^*(\vec{p},r)\right\} \times\ \Phi^*[\mathbf{b}-\frac{\xi}{2}]\Phi[\mathbf{b}+\frac{\xi}{2}], \quad (24)$$

where $\Phi^*[\mathbf{b}] = \langle\mathbf{b}|\Phi\rangle$.

For the ground state of the Dirac free field, the Wigner functional is given by

$$\rho_{W0}[\mathbf{b}^*,\mathbf{b}] = \mathcal{N} \times \exp\left\{\frac{-2i}{\hbar}\int d^3p\sum_{r=1}^{4}\varepsilon_r\mathbf{b}^*(\vec{p},r)\mathbf{b}(\vec{p},r)\right\}. \quad (25)$$

### 3.5. Dirac Propagator

To compute the propagator of the Dirac field we need to find

$$iS_F(\vec{x}-\vec{y}) = \langle 0|\psi_\alpha(\vec{x})\overline{\psi}_\beta(\vec{y})|0\rangle\cdot\theta(t-t') - \langle 0|\overline{\psi}_\beta(\vec{y})\psi_\alpha(\vec{x})|0\rangle\cdot\theta(t'-t). \quad (26)$$

Thus we first compute the quantities $\langle 0|\psi_\alpha(\vec{x})\overline{\psi}_\beta(\vec{y})|0\rangle$ and $\langle 0|\overline{\psi}_\beta(\vec{y})\psi_\alpha(\vec{x})|0\rangle$. These expectation values, in terms of deformation quantization are:

$$\langle 0|\psi_\alpha(\vec{x})\overline{\psi}_\beta(\vec{y})|0\rangle = \frac{\int\mathcal{D}\mathbf{b}^*\mathcal{D}\mathbf{b}\ \psi_\alpha(\vec{x})\star\overline{\psi}_\beta(\vec{y})\ \rho_{W_0}[\mathbf{b}^*,\mathbf{b}]}{\int\mathcal{D}\mathbf{b}^*\mathcal{D}\mathbf{b}\ \rho_{W_0}[\mathbf{b}^*,\mathbf{b}]}, \quad (27)$$

where $\rho_{W_0}[\mathbf{b}^*,\mathbf{b}]$ is the Wigner functional of the ground state [see Eq. (25)].

Using the following equation:

$$\int\mathcal{D}\Theta^*\mathcal{D}\Theta\exp\left(-\int dxdy\Theta^*(x)M(x,y)\Theta(y)\right) = \det(M)\cdot(M^{-1}(x,y)), \quad (28)$$

where $\Theta$ and $\Theta^*$ are Grassmann fields, and the relations

$$\sum_{r=1}^{2}w_\alpha(\vec{p},r)\overline{w}_\beta(\vec{p},r) = \frac{(\not{p}+m)_{\alpha\beta}}{2m},$$

$$\sum_{r=3}^{4}\overline{w}_\alpha(\vec{p},r)w_\beta(\vec{p},r) = \frac{(\not{p}-m)_{\alpha\beta}}{2m}.$$

After some straightforward calculations we get

$$\langle 0|\psi_\alpha(\vec{x})\overline{\psi}_\beta(\vec{y})|0\rangle = -i\hbar\int\frac{d^3p}{(2\pi)^3}\frac{(\not{p}+m)_{\alpha\beta}}{2E_{\vec{p}}}\exp\big(ip\cdot(y-x)\big)$$

$$\langle 0|\overline{\psi}_\beta(\vec{y})\psi_\alpha(\vec{x})|0\rangle = -i\hbar\int\frac{d^3p}{(2\pi)^3}\frac{(\not{p}-m)_{\beta\alpha}}{2E_{\vec{p}}}\exp\big(ip\cdot(x-y)\big). \quad (29)$$

Substituting this results in Eq. 26 reproduces exactly the propagator of the Dirac field obtained by canonical quantization.

## 4. Final Remarks

We did the deformation quantization for fermionic systems with an number infinite of degrees of freedom, *i.e.* fermionic fields as an extension of the formalism for Fermi classical systems [15]. As an example, we applied this theory for the quantization of the Dirac free field. Besides we compute the propagator of the Dirac field [16].
Once we have the quantization of the Dirac field, we will couple the Dirac field to the Maxwell field in this context as a deformation of the algebraic structure. In order to perform the deformation quantization of electrodynamics [17]. We will be able to show that many well known results of deformation quantization in quantum mechanics could be extended to the case of quantum field theory.

## Acknowledgements

I am indebted with G. Dito, H. García-Compeán, C. Maldonado-Mercado, M. Przanowski and F. Turrubiantes for intensive discussions and useful suggestions. This review was partially supported by the CONACyT graduate fellowship and the CONACyT grant 45713-F. I want to thank the CINVESTAV-Monterrey for their warm hospitality.

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