Strange form factors of the nucleon

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The strangeness content of nucleon form factors is analyzed in a two-component model with a quark-like intrinsic structure surrounded by a meson cloud. A comparison with the available experimental data from the SAMPLE, PVA4, HAPPEX and G0 collaborations shows a good overall agreement for the strange form factors.

Keywords: Strange form factors; vector meson dominance.

Se analiza el contenido de extraña de los factores de forma del núcleon en un modelo de dos componentes que consiste en una estructura intrínseca de tres cuarks constituyentes rodeada de una nube mesónica. Una comparación con los datos experimentales disponibles de las colaboraciones SAMPLE, PVA4, HAPPEX y G0 muestra un buen ajuste para los factores de forma con extraña.

Descriptores: Factores de forma extraños; dominancia de mesones vectoriales.


1. Introduction

The contribution of the different quark flavors to the electromagnetic structure of the nucleon can be studied by combining the nucleon’s response to the electromagnetic and weak neutral vector currents [1]. Especially, the contribution of strange quarks to the nucleon structure is of interest because it is exclusively part of the quark-antiquark sea.

In recent experiments, parity-violating elastic electron-proton scattering has been used to probe the contribution of strange quarks to the structure of the nucleon [2]. The strange quark content of the form factors can be determined assuming charge symmetry and combining parity-violating asymmetries with measurements of the electric and magnetic form factors of the proton and neutron. The various experiments are sensitive to different combinations of the strange quark contributions to the charge distribution and the magnetization represented by the strange electric and magnetic form factors. There are various methods to disentangle the electric and magnetic contributions: by measuring parity-violating asymmetries at both forward and backward angles [3], by using different targets [4], or by combining parity-violating asymmetries with (anti)neutrino-proton scattering data [5].

The first experimental results from the SAMPLE [2, 3], PVA4 [6], HAPPEX [4, 7, 8] and G0 [9] collaborations have shown evidence for a nonvanishing strange quark contribution to the structure of the nucleon. In particular, the strangeness content of the proton magnetic moment was found to be positive [8], suggesting that the strange quarks reduce the proton’s magnetic moment. This is an unexpected and surprising finding, since a majority of theoretical studies favors a negative value [10].

The aim of this contribution is to analyze the available experimental data on strange form factors in a two-component model of the nucleon.

2. Two-component model of nucleon form factors

Electromagnetic and weak form factors contain the information about the distribution of electric charge and magnetization inside the nucleon. These form factors arise from matrix elements of the corresponding vector current operators

\[
\langle N | V_\mu | N \rangle = \bar{u}_N \left[ F_1(Q^2)\gamma_\mu + \frac{i}{2M_N} F_2(Q^2)\sigma_{\mu\nu} q^\nu \right] u_N. \tag{1}
\]

Here \( F_1 \) and \( F_2 \) are the Dirac and Pauli form factors which are functions of the squared momentum transfer \( Q^2 = -q^2 \). The electric and magnetic form factors, \( G_E \) and \( G_M \), are obtained from \( F_1 \) and \( F_2 \) by the relations \( G_E = F_1 - \tau F_2 \) and \( G_M = F_1 + F_2 \) with \( \tau = Q^2/4M_N^2 \).

Different models of the nucleon correspond to different assumptions about the Dirac and Pauli form factors. In this contribution, I consider the two-component model of [11, 12] in which the external photon couples both to an intrinsic three-quark structure described by the form factor \( g(Q^2) \), and to a meson cloud via vector-meson (\( \rho, \omega \) and \( \phi \)) dominance (VMD). In the original VMD calculation [11], the Dirac form factor was attributed to both the intrinsic structure and the meson cloud, and the Pauli form factor entirely to the meson cloud. In Ref. 12, it was shown that the addition of an intrinsic part to the isovector Pauli form factor as suggested by studies of relativistic constituent quark models in the light-front approach [13], improves the results for the neutron electric and magnetic form factors considerably.

In order to incorporate the contribution of the isoscalar (\( \omega \) and \( \phi \)) and isovector (\( \rho \)) vector mesons, it is convenient to introduce the isoscalar and isovector current operators

\[
V^{I=0}_\mu = \frac{1}{6} \left( \bar{u} \gamma_\mu u + d \gamma_\mu d - 2s \gamma_\mu s \right),
\]

\[
V^{I=I}_\mu = \frac{1}{2} \left( \bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d \right). \tag{2}
\]
The corresponding isoscalar Dirac and Pauli form factors depend on the couplings to the \( \omega \) and \( \phi \) mesons

\[
F_{1}^{I=0}(Q^2) = \frac{1}{2} g(Q^2) \left( 1 - \beta_\omega - \beta_\phi + \beta_\omega \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} \right),
\]

and the isovector ones on the coupling to the \( \rho \) meson

\[
F_{1}^{I=1}(Q^2) = \frac{1}{2} g(Q^2) \left( 1 - \beta_\rho + \beta_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2} \right),
\]

\[
F_{2}^{I=1}(Q^2) = \frac{1}{2} g(Q^2) \left[ \frac{\mu_p - \mu_n - 1 - \alpha_\rho}{1 + \gamma Q^2} + \alpha_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2} \right].
\]

The proton and neutron form factors correspond to the sum and difference of the isoscalar and isovector contributions. For the intrinsic form factor a dipole form \( g(Q^2) = (1 + \gamma Q^2)^{-2} \) is used whose asymptotic behavior is consistent with p-QCD [14] and which coincides with the form used in an algebraic treatment of the intrinsic three-quark structure [15].

The large width of the \( \rho \) meson plays an important role for the small \( Q^2 \) behavior of the form factors and is taken into account in the same way as in Refs. 11 and 12 by the replacement [16]

\[
\frac{m_\rho^2}{m_\rho^2 + Q^2} \rightarrow \frac{m_\rho^2 + 8 \Gamma_\rho m_\pi/\pi}{m_\rho^2 + Q^2 + (4m_\pi^2 + 2Q^2) \alpha(Q^2) \Gamma_\rho/m_\pi},
\]

with

\[
\alpha(Q^2) = \frac{2}{\pi} \left[ \frac{4m_\pi^2 + Q^2}{Q^2} \right]^{\frac{1}{2}} \ln \left( \frac{\sqrt{4m_\pi^2 + Q^2} + \sqrt{Q^2}}{2m_\pi} \right).
\]

The meson dynamics is important for small values of \( Q^2 \), whereas for large values the form factors satisfy the asymptotic behavior of p-QCD, \( F_1 \sim 1/Q^4 \) and \( F_2 \sim 1/Q^6 \) [14].

Since the intrinsic part is associated with the valence quarks of the nucleon, the strange quark content of the nucleon form factors arises from the meson wave functions

\[
|\omega\rangle = \cos \epsilon |\omega_0\rangle - \sin \epsilon |\phi_0\rangle,
\]

\[
|\phi\rangle = \sin \epsilon |\omega_0\rangle + \cos \epsilon |\phi_0\rangle,
\]

where \( |\omega_0\rangle = (u\bar{u} + d\bar{d})/\sqrt{2} \) and \( |\phi_0\rangle = s\bar{s} \) are the ideally mixed states. Under the assumption that the strange form factors have the same form as the isoscalar ones, the Dirac and Pauli form factors that correspond to the strange current

\[
V_\mu^s = s\gamma_\mu s,
\]

are expressed as the product of an intrinsic part \( g(Q^2) \) and a contribution from the meson cloud as

\[
F_1^s(Q^2) = \frac{1}{2} g(Q^2) \left[ \beta_\omega^s \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\phi^s \frac{m_\phi^2}{m_\phi^2 + Q^2} \right],
\]

\[
F_2^s(Q^2) = \frac{1}{2} g(Q^2) \left[ \alpha_\omega^s \frac{m_\omega^2}{m_\omega^2 + Q^2} + \alpha_\phi^s \frac{m_\phi^2}{m_\phi^2 + Q^2} \right].
\]

The \( \beta^s \)’s and \( \alpha^s \)’s in Eqs. (4) and (9) are not independent of one another. The coefficients appearing in the isoscalar and strange form factors depend on the same nucleon-meson and current-meson couplings [17]. In addition, they are constrained by the electric charges and magnetic moments of the nucleon which leads to two independent isoscalar couplings

\[
\alpha_\omega = \mu_p + \mu_n - 1 - \alpha_\phi,
\]

\[
\beta_\omega = -\beta_\phi \tan(\theta_0 + \epsilon)/\tan \epsilon.
\]

The strange couplings are then given by

\[
\beta_\omega^s/\beta_\omega = \alpha_\omega^s/\omega = -\sqrt{6} \sin \epsilon/\sin(\theta_0 + \epsilon),
\]

\[
\beta_\phi^s/\beta_\phi = \alpha_\phi^s/\phi = -\sqrt{6} \cos \epsilon/\cos(\theta_0 + \epsilon).
\]

where \( \tan \theta_0 = 1/\sqrt{2} \). The mixing angle \( \epsilon \) can be determined from the decay properties of the \( \omega \) and \( \phi \) mesons. Here I take the value \( \epsilon = 0.053 \) rad obtained in Ref. 18.

3. Results

In order to calculate the nucleon form factors in the two-component model the five coefficients, \( \gamma \) from the intrinsic form factor, \( \beta_\phi \) and \( \alpha_\phi \) from the isoscalar couplings, and \( \beta_\rho \) and \( \alpha_\rho \) from the isovector couplings, are determined in a least-square fit to the electric and magnetic form factors of the proton and the neutron using the same data set as in Ref. 12. The electromagnetic form factor of the proton and neutron are found to be in good agreement with experimental data [19]. According to Eq. (11), the strange couplings can be determined from the fitted values of the isoscalar couplings to be \( \beta_\omega^s = -\beta_\phi^s = 0.202, \alpha_\omega^s = 0.648 \) and \( \alpha_\phi^s = -0.018 \) [19].

Figures 1 and 2 show the strange electric and magnetic form factors as a function of \( Q^2 \). The theoretical values of \( G_E^s = F_1^s - \tau F_2^s \) are small and negative. The Dirac form factor \( F_1^s \) is small due to canceling contributions of the \( \omega \) and \( \phi \) couplings which arose as a consequence of the fact that the strange (anti)quarks do not contribute to the electric charge \( G_E^s(0) = F_1^s(0) = \beta_\omega^s + \beta_\phi^s = 0 \). Moreover, for this range of momentum transfer the contribution from the Pauli form factor \( F_2^s \) is suppressed by the factor \( \tau = Q^2/4M_N^2 \). The theoretical values are in good agreement with the recent experimental result of the HAPPEX Collaboration in which \( G_E^s \) was determined for the first time in parity-violating electron scattering from \(^4\)He [4]. The experimental value \( G_E^s = -0.038 \pm 0.042 \pm 0.010 \) (circle) measured at \( Q^2 = 0.091 \) (GeV/c)^2 is consistent with zero.
The strange magnetic form factor $G_s^M = F_s^1 + F_s^2$ is positive, since it is dominated by the contribution from the Pauli form factor. The SAMPLE experiment measured the parity-violating asymmetry at backward angles, which allowed to determine the strange magnetic form factor at $Q^2 = 0.1$ (GeV/c)$^2$ as $G_s^M = 0.37 \pm 0.20 \pm 0.26 \pm 0.07$. The other experimental values of $G_s^E$ and $G_s^M$ in Figs. 1 and 2 were obtained [5, 20] by combining the (anti)neutrino data from E734 [21] with the parity-violating asymmetries from HAPPEX [7] and G0 [9]. The theoretical values are in good overall agreement with the experimental ones for the entire range $0 < Q^2 < 1.0$ (GeV/c)$^2$.

The strange magnetic moment is calculated to be positive
\[ \mu_s = G_s^M(0) = \frac{1}{2}(\alpha_s^+ + \alpha_s^-) = 0.315 \mu_N \quad (12) \]
in units of the nuclear magneton, $\mu_N = e\hbar/2M_Nc$. This value is in agreement with recent experimental evidence from the SAMPLE collaboration [3] and an analysis of the world data $G_s^M = 0.55 \pm 0.28$ at $Q^2 = 0.1$ (GeV/c)$^2$ [8]. By convention, the definition of the strange magnetic moment $\mu_s$ does not involve the electric charge of the strange quark.

Theoretical calculations of the strange magnetic moment show a large variation, although most QCD-inspired models seem to favor a negative value in the range $-0.6 \lesssim \mu_s \lesssim 0.0 \mu_N$ [10]. There are relatively few calculations that give a sizeable positive strange magnetic moment, ranging from 0.074 – 0.115 $\mu_N$ in the $SU(3)$ chiral quark soliton model [22], 0.16 ± 0.03 $\mu_N$ in a group theoretical approach with flavor $SU(3)$ breaking [23] to 0.37 $\mu_N$ in the $SU(3)$ chiral bag model [24]. Recent quenched lattice-QCD calculations give a small value, e.g. 0.05 ± 0.06 [25] and $-0.046 \pm 0.019$ [26].

The strange form factors determined in the PVA4 [6], HAPPEX [7, 8] and G0 [9] experiments correspond to a linear combination of electric and magnetic form factors. Fig. 3 shows the results for the strange form factor combination $G_s^E + \eta G_s^M$ measured recently by the G0 Collaboration at forward angles [9]. A comparison of the calculated values in the two-component model with the PVA4 and HAPPEX data shows a similar good agreement as for the G0 data shown in Fig. 3 [19].

### 4. Summary and conclusions

In summary, in this contribution it was shown that the recent experimental data on the strange nucleon form factor can be explained very well in a two-component model of the nucleon consisting of an intrinsic three-quark structure with a spatial extent of $\sim 0.49$ fm surrounded by a meson cloud. The present approach is a combination of the two-component

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**Figure 1.** Comparison between theoretical and experimental values of the strange electric form factor. The experimental values are taken from [4] (circle) and [20] (triangle).

**Figure 2.** Comparison between theoretical and experimental values of the strange magnetic form factor. The experimental values are taken from [3] (circle) and [20] (triangle).

**Figure 3.** Comparison between theoretical and experimental values of strange form factors $G_s^E + \eta G_s^M$. The experimental values were measured by the G0 Collaboration [9].
model of [12] with the treatment of the strange quark content of the vector mesons according to [17]. The parameters in the model are completely determined by the electric and magnetic form factors of the proton and neutron. It is noted, that the strange couplings do not involve any new parameters. On the contrary, the condition that the strange quarks do not contribute to the electric charge of the nucleon, leads to an extra constraint relating $\beta_{\omega}$ and $\beta_{\phi}$, thus reducing the number of independent coefficients of the two-component model of Ref. 12 by one.

The good overall agreement between the theoretical and experimental values for the electromagnetic form factors of the proton and neutron and their strange quark content shows that the two-component model provides a simultaneous and consistent description of the electromagnetic and weak vector form factors of the nucleon.

The first results from the SAMPLE, PVA4, HAPPEX and G0 collaborations have shown evidence for a nonvanishing strange quark contribution to the charge and magnetization distributions of the nucleon. Future experiments on parity-violating electron scattering at backward angles (PVA4 and G0 [27]) and neutrino scattering (FINeSSE [28]) will make it possible to disentangle the contributions of the different quark flavors to the electric, magnetic and axial form factors, and thus to provide new insight into the complex internal structure of the nucleon.

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