Quantification and differentiation of nuclear tracks in solid state detectors by simulation of their diffraction pattern applying Fourier optics

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The proposed method to count and differentiate nuclear tracks in Solid State Detectors is based on digital simulation and analysis of the Fraunhofer diffraction pattern, formed when coherent light passes through tracks in an etched detector. Analytical and numerical models were developed using, as transformation element, an optical system and a digital procedure of the Fourier Transform, respectively. Different components of developed software are described, and depending on the kind of detector used, variants of optical microscopy are suggested. The proposed method allows to calculate real track density and to differentiate tracks by their diameters.

Keywords: Track counting and discrimination; passive detectors; Fraunhofer diffraction pattern; Fourier transform.

El método propuesto para contar y diferenciar trazas nucleares en detectores de estado sólido se basa en la simulación digital y el análisis del patrón de difracción de Fraunhofer, formado cuando un haz de luz coherente pasa a través de trazas en un detector revelado. Se desarrollaron modelos analíticos y numéricos usando como elemento de transformación un sistema óptico y un procedimiento digital de Transformada de Fourier, respectivamente. Se describen los diferentes componentes del software desarrollado y se sugieren variantes técnicas de microscopía óptica. El método propuesto permite calcular la densidad de trazas y diferenciarlas por sus diámetros.

Descriptores: Conteo y discriminación de trazas; detectores pasivos; patrón de difracción de Fraunhofer; transformada de Fourier.

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1. Introduction

Nuclear Track Detectors provide the most common method to measure charged particles [1]. For track counting a great number of semiautomatic or automatic methods have been developed [1-3], but problems persist when high resolution to discriminate track diameters or high-speed in track images processing are required. The aim of this work are to develop the theoretical bases of an alternative method to analyze nuclear tracks in plastic detectors [4], to detail aspects of developed software, and to illustrate examples of its application.

2. Model of diffracted light when crossing etched tracks applying Fourier Optics

An etched track is assumed to be like an object that causes small phase changes in the transmitted light and that can be considered as a two-dimensional object. The amplitude distribution in the Fraunhofer diffraction field can be determined from the $2-D$ Fourier Transform ($2-D$ FT) [5] of complementary optical density profile $D(r, \varphi)$, according to Babbinet’s Principle. Considering the track binarized image as a circularly symmetrical object, and using polar coordinates $(r, \varphi)$ and $(R, \phi)$ in the object and Fourier planes, respectively, the $2-D$ FT of density profile is:

$$FT[D(r, \varphi)] = \frac{ja}{\pi} \exp [ja(2f^2 + R^2)] \int_0^\infty rdrD(r) \times \int_0^{2\pi} d\varphi \exp [-j2aRr \cos(\phi - \varphi)], \quad (1)$$

where $a = \pi/\lambda f$, $\lambda$-wavelength of illuminating light and $f$-focal length of transform lens used. Using the Bessel identity function, the amplitude distribution can be expressed as:

$$A(R) = FT[D(r)] = (ja) \exp [ja(2f^2 + R^2)] \times \int_0^\infty rD(r)J_0(2aRr)dr. \quad (2)$$

If $D(r) = D$ for $0 \leq r \leq d/2$, and $D(r) = 0$ for $d/2 < r$, where $D$-track density and $d$- its diameter, and using the approach given by Tüke et al. [6], the intensity distribution is:

$$I(R) = A(R)A^*(R) = \frac{D^2d}{4\pi aR^2} \left[1 + \cos \left(2aRd - \frac{3\pi}{2}\right)\right] \quad (3)$$
For an image with $N_T$ tracks, the principle based on a subclass of diffraction effects can be considered [7]. Applying Parseval’s Theorem [5] the equation for total intensity can be written as:

$$I_{\text{tot}}(R) = \sum_{i=1}^{N_T} \left[ \frac{D_i^2 d_i}{4 \pi a R^3} \left( 1 + \cos \left( \frac{2 a R d_i - 3 \pi}{2} \right) \right) \right]$$  \hspace{1cm} (4)

Spectral analysis of this signal should show different components. Mathematically, a further Fourier Transform (FT) to Eq. (4) describes this analysis:

$$\text{FT} \left[ I(R) R^3 \right] = \sum_{i=1}^{N_T} \frac{D_i^2 d_i}{4 a} \left[ 2 \delta(0) + \delta(2 a d_i) \right].$$  \hspace{1cm} (5)

### 2.1. Numerical approach

If transformation is carried out with a digital procedure of the Fourier Transform, then Eqs. (4) and (5) can be expressed as:

$$I_{\text{tot}} R^3 = \sum_{i=1}^{N_T} \sum_{j=1}^{N_p} \left[ M_{n_i, d_i, R^k} \left[ 1 + \frac{1}{F_i} \cos \left( 2 \pi d_i \frac{R}{N_p} - \frac{3 \pi}{2} \right) \right] \right],$$  \hspace{1cm} (6)

where $n_i$-number of tracks with diameters $d_i$, $N_p$-number of points of the image, $N_T$-number of different diameters, $M$-constant, $K_i$-background component, and $F_i$-track form parameter.

$$\text{FT} \left[ I(R) R^3 \right] = \sum_{i=1}^{N_T} \left[ I_0^N \delta(0) + I_p^{N_i} \delta(d_i) \right],$$  \hspace{1cm} (7)

where $I_0^N$-amplitude of zero order harmonic, and $I_p^{N_i}$-amplitude of secondary harmonic of order $i$.

Equation (6) shows that the intensity is formed by different harmonics of frequencies whose values coincide with track diameters. Spectral analysis of the radial intensity function, represented by Eq. (7), determines the spectrum, which is formed by the overlapping of different harmonics of frequencies, and is represented by an array of peaks in an $x-y$ system.

### 2.2. Determination of the number of tracks

According to Eqs. (6) and (7), there is a linear relationship between the total number of tracks of a given diameter and the peak amplitude ($I_0^N$). When there are two groups of tracks with different diameters (Fig. 1), the calculation of the number of tracks, corresponding to each diameter is:

$$N_1 = \frac{I_0^{N_1}}{I_0^{d_1}}, \hspace{1cm} N_2 = \frac{I_0^{N_2}}{I_0^{d_2}}, \hspace{1cm}$$  \hspace{1cm} (8)

where $I_0^{N_1}$ and $I_0^{N_2}$ - “corrected” value of the peak amplitude at diameters $d_1$ and $d_2$, respectively. Assuming each peak amplitude as a consequence of the lineal contribution of all the tracks, then:

$$I_0^{N_1} = N_1 I_1^{d_1, 1} + N_2 I_2^{d_2, 1}, \hspace{1cm} I_0^{N_2} = N_1 I_1^{d_1, 2} + N_2 I_2^{d_2, 1}.$$  \hspace{1cm} (9)

The $I_0^{N_i}$ values are calculated from the spectrum of individual tracks. Generalizing, in an image of tracks grouped by $n$ different diameters, each peak of the spectrum satisfies:

$$N_1 I_1^{d_1, i} + \sum_{j=1}^{n-1} N_j I_1^{d_2, i} = I_0^{N_i}.$$  \hspace{1cm} (10)

This system can be solved applying the Gauss method, so that $N_i$ values can be obtained.

### 3. Software “TRACKS”

Simulation was carried out by means of a software elaborated with version III of the Borland Delphi application. The software presents three groups of general operations:

1. **Block 1**: open an image, create a new image (for simulation), graphic operations in the image (copy, cut, and paste), save it in a disk active window, print and save the active window.

2. **Block 2**: treatments of images, calculation of 2-D FT, radial intensity curve and spectrum, quantification process (determination of number of tracks according to their diameters).

3. **Block 3**: tools to manipulate graphics: localization of peaks in the spectrum, background subtraction, scale change in the axes, change of the graph type, and work with regions of interest.

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The comparison between model, Eq. (6), and the simulated radial intensity distribution gave satisfactory correspondence [4]. Appropriate qualitative and quantitative similarity between results using Eq. (7), and by simulation was also achieved.

4. Techniques of optical microscopy to improve the analysis of nuclear tracks

To obtain images of etched tracks with optical density profile similar to the theoretical one, it is necessary a microscopic technique that enables high contrast between the etched track and the means around it. Since the formation of a track consists in the chemical dissolution of material along the particle track, the etched tracks have a refraction index different to the not revealed part of detector. Dark background microscopy allows to achieve high contrast in track images of CR-39 detectors (Fig. 2c), while configuration of brilliant field can be used for LR-115 detectors (Fig. 2b).

5. Application of proposed method to count and to differentiate tracks in CR-39 detectors

Results of count and discrimination of the etched tracks in CR-39 detectors are shown in Table I. The comparison with ocular evaluation demonstrates that on average, the error in total track calculations doesn’t overcome 7.0%. Using the method proposed here, Palacios et al. [4] obtained track diameter discriminations with a resolution ranged between 8 and 25%. These values are better than the reported by Wong and Tommasino [8]. Analysis of track images using software “TRACKS” allowed to differentiate $^7$Li ion energies [4]. Once obtained binarized track images from 10 fields of view, the mean time to obtain differential counts is only about 2 to 3 minutes.

6. Conclusion

An alternative method to count and to differentiate nuclear tracks in passive detectors was established. The method is based on the simulation and analysis of Fraunhofer diffraction pattern that should be formed when the detector is crossed by a beam of coherent light.

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