Wavelet analysis for upscaling of two dimensional permeability fields

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We use a multiresolution analysis based on a wavelet transform to upscale two-dimensional heterogeneous permeability fields. Results are compared with conventional averaging upscaling schemes using arithmetic, geometric and harmonic averaging. It is found that the wavelet analysis is able to preserve the fine scale heterogeneous features with the increase of the coarse scale size significantly better than the averaging methods, while preserving information required for downsizing the properties of the system.

Keywords: Wavelet transform; upscaling; permeability field; heterogeneity.

Se presenta un esquema para el escalamiento de un campo de permeabilidad bidimensional heterogéneo basado en un método de análisis de multiresolución con transformada de ondícula. Los resultados obtenidos se comparan con los encontrados con esquemas convencionales de promedios aritméticos, geométricos y armónicos. Se encuentra que el análisis de ondícula es capaz de preservar las heterogeneidades a medida que se aumenta el tamaño de la escala, mucho mejor que los métodos de promediación, preservando al mismo tiempo la información necesaria para la reconstrucción de la permeabilidad a escala fina.

Descriptores: Transformada de ondículas; escalamiento campo de permeabilidad; heterogeneidad.

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1. Introduction

Natural porous formations are characterized by their intrinsic heterogeneity. This condition is manifested as rock and flow properties varying with length scale. For this reason an upscaling process is used to calculate effective property values at a larger scale, for its use in a field scale simulation starting from a lab measurements [1,2].

Even though there are different approaches for upscaling reservoir properties that include several types of averaging techniques, the calculation of pseudo properties, and renormalization group, they are not completely satisfactory in terms of the amount of information lost in the process, the sensitivity to the boundary conditions, and the computational requirements for their implementation.

Wavelet analysis is a multiresolution framework well suited for the upscaling of rock and flow properties in a multiscale heterogeneous formation. The mathematical properties of the wavelet transform provide many advantages such as improved computational efficiency, flexibility for its application, smoothness control, and also the down sizing reconstruction of the transformed property at a given scale [3].

In this study we use the wavelet transform to upscale the permeability field of a heterogeneous two-dimensional system. Permeability values affect flow properties under multiphase flow conditions. They measure the ability of the fluid to flow through the pore space [1-4]. Results are compared with those obtained with conventional averaging methods such as arithmetic, geometric and harmonic averages. It is shown how the multiresolution analysis with a wavelet transform significantly improves the accuracy of the upscaled permeability values under various geological conditions.

2. Wavelet Analysis

The wavelet transform is a general class of linear integral transforms, which gives a space-frequency representation of a function. It is defined as a convolution of a given function $f(x)$ with a kernel function $\Psi_{\lambda,x}(u) = (1/\lambda)\Psi[(u-x)/\lambda]$ given as

$$Wf(x) = \int_{-\infty}^{\infty} f(x)\frac{1}{\sqrt{\lambda}}\Psi\left(\frac{u-x}{\lambda}\right) \tag{1}$$

where λ is a scale parameter, x the location parameter, and the family of functions, $\Psi_{\lambda,x}(u)$ the wavelet functions. The support length of the wavelet changes proportionally with λ, *i.e.*, increasing λ will dilate it and decreasing it will contract the wavelet function [5].

A wavelet has some additional properties such as orthogonality, compact support, good localization in time and space, which are appropriate for a multiresolution analysis [6].

3. Proposed Upscaling Method and Statistical Averaging

In the oil industry, the upscaling process is typically done by the calculation of spatial averages over geological grid blocks which give effective values that can be used directly in the flow model. It involves the application of composition rules to a group of property values or "cells" selected on a given scale, to produce composite cells of larger dimensions. The process is applied repeatedly to the new generated composite cells until the desired coarse scale level. It is assumed that the upscaling rules are invariant, and do not change the physics of the process being described.

There are three general types of spatial averages commonly used. These are arithmetic, geometric and harmonic averages. The arithmetic average corresponds to an algebraic average of the local permeability values at a given scale, that is,

$$\overline{k} = \frac{1}{n}\sum_{i=1}^{n} k_i$$

where n is the number of cells at the fine grid which are grouped to form the composite cell. The geometric average is obtained form the $1/n$-root of the product of the local permeabilities $\overline{k} = (k_1 k_2 \ldots k_n)^{1/n}$, and the harmonic average is calculated as the average of the sum of the inverse local permeabilities

$$\left(\overline{k}\right)^{-1} = \frac{1}{n}\left(\frac{1}{k_1} + \frac{1}{k_2} + \ldots \frac{1}{k_n}\right).$$

In general, the required discretization level is difficult to predict since it depends on how heterogeneous the system is. It also depends on the correlations present in the porous formation, and the numerical scheme used to solve the flow equations.

On a fine scale, absolute permeability values could be generated from a probability distribution function representative of the system being modeled. We start with a permeability matrix on a fine scale. Equivalent properties at a coarse scale are calculated using a revised version of the Mallat algorithm [7], which consists in performing the convolution of the permeability value along a given direction with a low pass filter, followed by a convolution using the same filter of the matrix that represents the absolute permeability in the orthogonal direction.

To choose the wavelet function, the matrix filter could be generated from the coefficients of a base function. Here, b spline filters were used. B spline functions have compact support, a given symmetry according to their index, and they display oscillatory behavior with null moments, and recursivity properties.The next step consists in the building of the matrix which represents the low pass filter with four coeficients of the base function with a scale ratio of two. The dimension of the columns of this matrix is two times larger than that of the rows.

To avoid confusion, a matrix X of dimension Lxn at scale m is denoted by $X^m_{L,n}$. Thus, a decomposition of the permeability distribution at the scale level (o) along the j direction can be written as:

$$\left(k^l_{L/2,n}\right)^j = H_{L/2,L} k^0_{L,n}.$$

The number of permeability values along the j direction are reduced by a factor of two after the decompositon process [8]. The final permeability matrix at the scale level $l-th$ can be obtained by decomposition of the resulting intermediate permeability matrix along the i direction in a similar way.

$$k^l_{L/2,n/2} = \left\{H_{n/2,n}\left[\left(k^l_{L/2,n}\right)_j\right]^T\right\}^T \tag{2}$$

A recursive procedure for the generation of the equivalent absolute permeability values at a coarse arbitrary scale m is derived. This procedure uses the information derived from the finer scale $m-1$ as follows:

$$k^m_{L/2,n/2} = H_{L/2,L} k^{m-1}_{L/2,n}\left(H_{n/2,n}\right)^T \tag{3}$$

In the Eqs. (1) and (2), $H_{i,j}$ is the low-pass filter matrix constructed from the coeficients of the basis functions (b spline), and $k_{i,j}$ represents the absolute permeability matrix for the block. The process is repeated up to the desired coarsening level.

The density probability function used here to model the permeability field has a power law form characterized by a power-law exponent μ, $Z(\mu,k) = |\mu|\, k^{\mu-1}$, with $0 \le k \le 2$ and $0 < \mu \le 1$. According to the values of μ, the distribution has a different shape, allowing the modeling of a wide variety of heterogeneous systems with the variation of just one parameter. For $\mu = 1$, it gives the uniform distribution. As $\mu \to 0$, the distribution grows to lower permeability values.

4. Results

As an example of the proposed application a two-dimensional permeability field was generated using the power-law distribution function $Z(k)$ for $\mu = 0.5$. This permeability field was represented in the form of a nxn matrix with $n = 2^{n1}$, with $n1 = 17$. Once the initial permeability matrix was generated, the different upscaling schemes were applied using a schematic matrix reduction with a scaling factor of $n = 2^{n1}/2$.

The fine scale system is a two-dimensional multifacial system. Figures 1 to 4 display the results for the different upscaling methods. It can be seen that the conventional averaging methods cannot preserve the channel structure of the initial permeability field after the first few iterations. As seen in Fig. 4b, the wavelet method is able to preserve such structures in fairly good conditions. Note also that the range of values of the scaled property at each upscaling level are closer to the original multifacial system for the upscaling with a wavelet transform. However, all upscaled values tend to decrease with respect to their fine scale values.

The harmonic average tends to produce too low permeability values in some regions and anomalously large in others, giving rise to structures not present in the orginal property field. Averaging methods tend to smooth out fine scale heterogeneities. The recursive approach of this multiresolution method makes it possible to recover the original signal in a downsizing process.

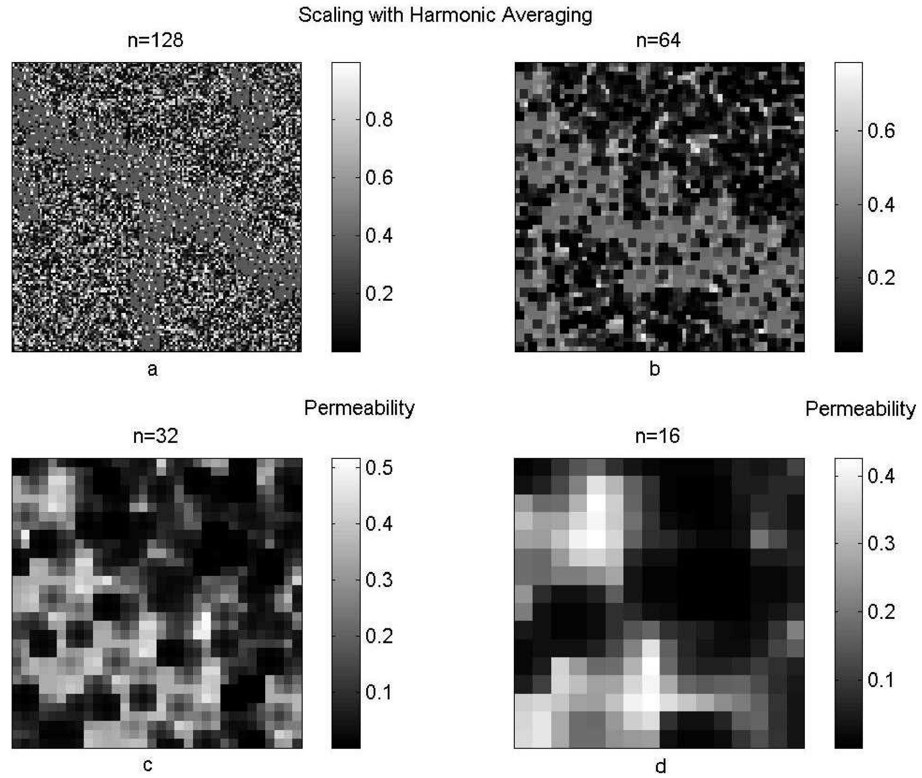

FIGURE 1. Harmonic averaging of a 2D heterogenous permeability field with a channel system. a), b), c), and d) show the upscaled field after the first, second, and third generations.

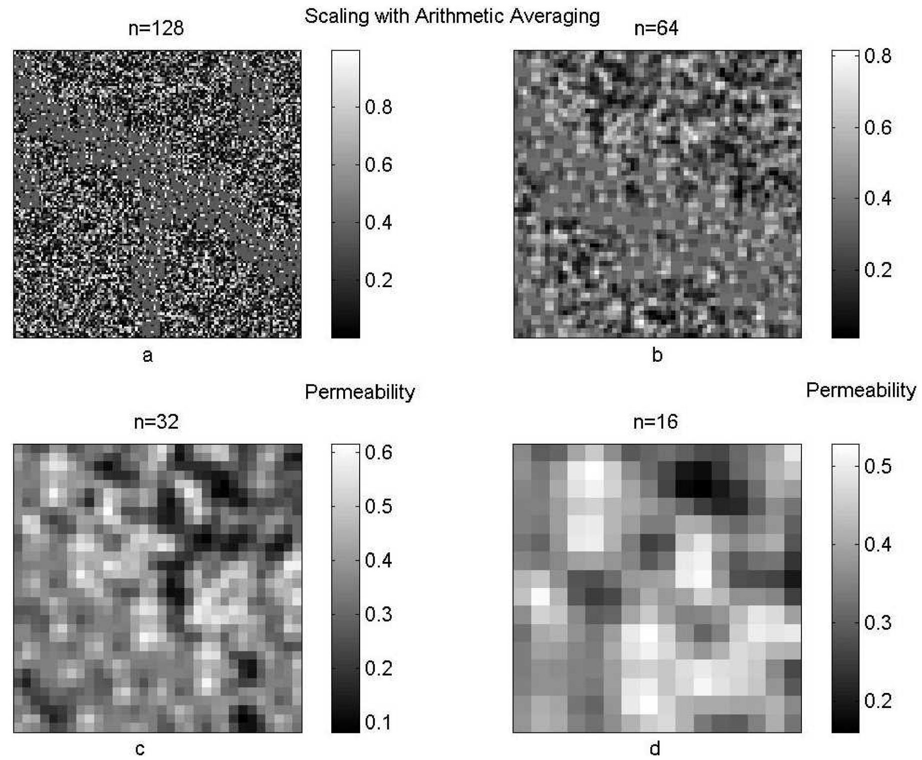

FIGURE 2. Arithmetic averaging. Upscaled results after the first, second and third generations are shown in b), c), and d) respectively.

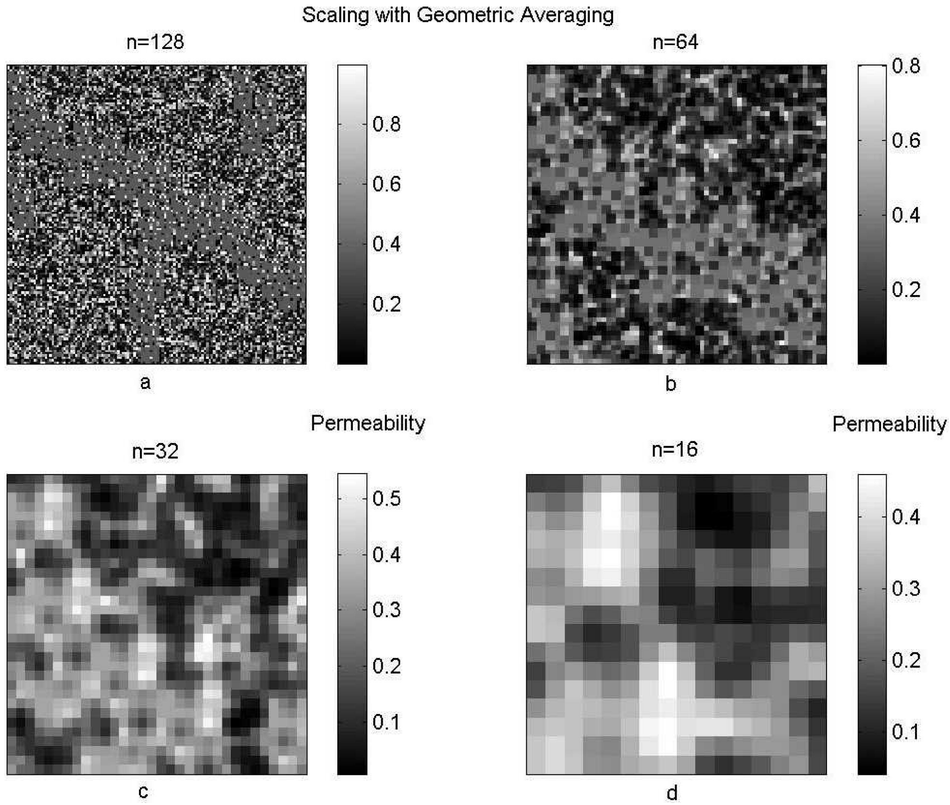

FIGURE 3. Geometric averaging. Upscaled results after the first, second and third generations are shown in b), c), and d) respectively.

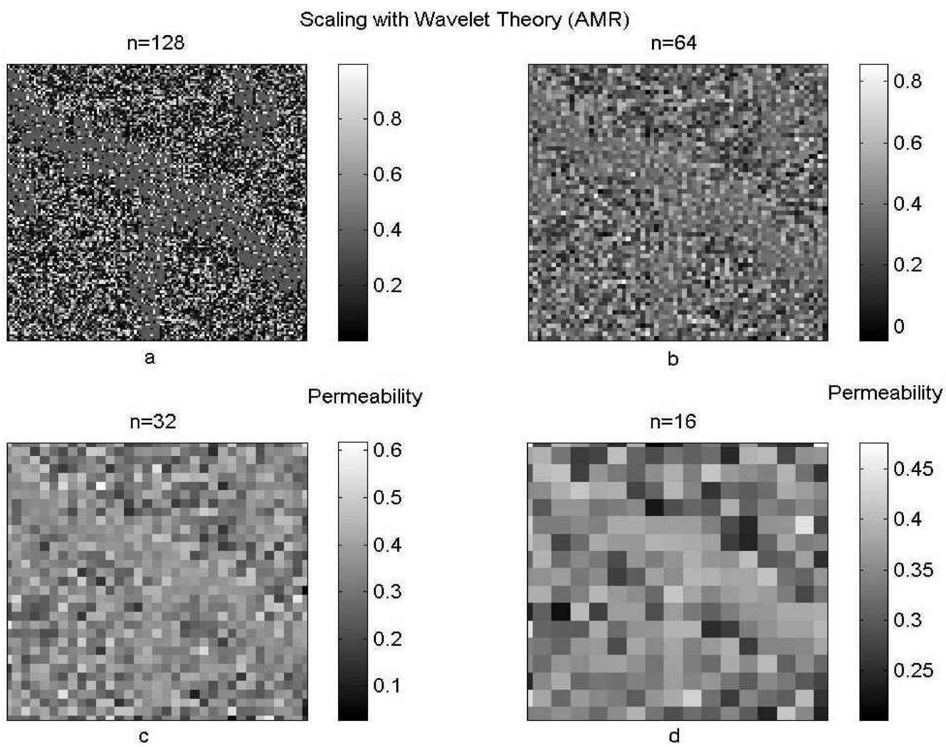

FIGURE 4. Multiresolution analysis. Upscaled resutls after the first, second and third generations are shown in b), c), and d) respectively.

In terms of the CPU time, there are no significant time differences when using the wavelet method as compared to the conventional averaging methods. However, this result combined with the simplicity of implementation of the multiresolution scheme makes the wavelet transform very promising as an upscaling method for reservoir properties.

5. Summary

From the results found it can be seen that the wavelet analysis is a convenient and efficient method for upscaling the two-dimensional permeability field of heterogeneous systems, with the advantage over conventional averaging systems of preserving the maximum information thus permitting the reconstruction of the original permeability field at any scale. The multiresolution property of the wavelet analysis permits a better handling of permeability contrasts, thus preserving the relevant heterogeneities initially present in the system.

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