REVISTA MEXICANA DE FÍSICA S **52** (2), 86–88 FEBRERO 2006

Simulation of bessel beam transducers using impulse response technique

H. Calas[1], E. Moreno[1], J.A. Eiras[2], Y. Crespo[1], L. Leija[3], and G. González[3]
[1]*ICIMAF – Cuba*
[2]*UFSCar – Brasil*
[3]*CINVESTAV – México*

Recibido el 27 de octubre de 2004; aceptado el 3 de junio de 2005

In this paper, a new method for designing a Bessel transducer is described. The method relies on the comparison between the transducer surface excitation and the theoretical Bessel function. This make it posible to select the right kerf between successive rings and obtaining the best characteristics in the radiation pattern. The field simulation is accomplished by means of the impulse response technique. The proposed method is applied to a three-ring transducer and compared with experimental results.

Keywords: Ultrasonic; Bessel transducer; nondiffracting beams.

En el presente trabajo se presenta un nuevo método para diseñar un transductor de Bessel. El método consiste en la comparación entre la excitación sobre la cara del transductor y la función de Bessel. Esto permite seleccionar el espacio entre anillos optimo para así obtener las mejores características del campo de radiación. La simulación del campo fue llevada a cabo por medio del uso de la respuesta al impulso. El método propuesto es aplicado a un transductor de tres anillos y los resultados de la simulación son comparados con resultados experimentales.

Descriptores: Ultrasonido; transductores de Bessel; haz de difracción limitada.

PACS: 43.20.Px; 43.20.Fn; 43.20.Bi; 43.88.Ar

1. Introduction

The problem of obtaining good lateral resolution and a large depth of field is quite an important problem in medical and industrial ultrasound imaging applications. Solving this problem using conventional device is practically impossible. However developing transducers with limited diffraction [1–3] is a good alternative in achieving this goal.

In order to produce a Bessel beam, a Bessel excitation over the transducer surface is necesary [4]. But in this case, we need a discrete excitation function because a finite annular array transducer requires this implementation.

In this paper we introduce a new method for the design of a Bessel transducer. This method is based on the estimate of the optimum inter ring distance, by means of the comparison between the excitation function over the transducer surface and the theoretical Bessel function, using the least-square error criterion. The use of this method for designing a Bessel transducer makes it possible to obtain good characteristics in the radiation field and expedites the process of a transducer construction. To simulate the radiation pattern, the technique of impulse response was used. The proposed method is applied to a three-ring transducer and compared with experimental results.

2. Theoretical preliminaries

2.1. Bessel transducer

For obtaining a Bessel beam, a Bessel excitation is necessary over the transducer face, but in practice when these transducers are constructed, the excitation function should be quantized under the circular array of construction possibilities.

We considering a Bessel transducer placed at $z = 0$. This is an N-ring transducer whose ring edges are located at the first N zeros of $J_0(x_n) = 0$, $(n = 1, 2, \ldots, N)$. The transducer rings radii are in the following positions:

$$r_1^- = 0, \quad r_1^+ = x_1/\alpha - d/2,$$
$$r_2^- = x_1/\alpha + d/2, \quad \ldots, \quad r_N^+ = x_N/\alpha = D/2, \tag{1}$$

where r_n^- and r_n^+ indicate the internal and external radii of the nth-ring, D is the diameter of the system, and d is the kerf width as shown in Fig. 1.

The parameter α is obtained as $\alpha = 2x_n/D$ and the quantized value over the rings is taken as a fraction of the maximum value of $J_0(\alpha r)$ in each annulus. The velocity on the transducer surface can be mathematically expressed as

$$\begin{aligned} v(r,d) =& v_1 \Theta(r)\Theta(r_1^+ - r) \\ &+ \sum_{n=2}^{N-1} v_n \Theta(r - r_n^-)\Theta(r_n^+ - r) \\ &+ v_N \Theta(r - r_N^-)\Theta(r_N^+ - r), \end{aligned} \tag{2}$$

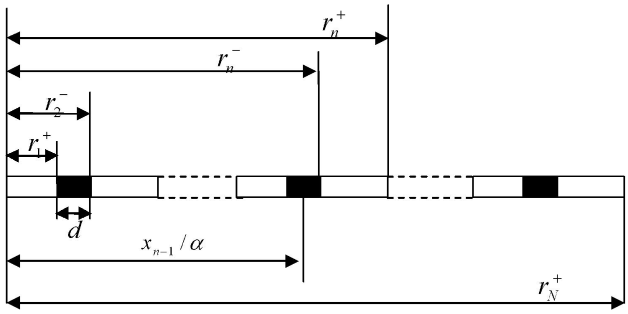

FIGURE 1. N-ring array transducer.

where $v_n = pJ_0(x_n^1)$ is the velocity amplitude in the nth-ring, p, is a real number ($0 \leq p \leq 1$),$\Theta(\cdot)$is the Heaviside function and x_n^1 are the zeros of the first order Bessel function $[J_1(x_n^1) = 0]$.

2.2. Application of impulse response technique

Impulse response technique is used for simulating the behavior of the radiation field assuming each ring is a planar piston. The impulse response function of the complete system has the following relationship:

$$h_T(x,y,z,t) = \sum_{n=1}^{N} v_n \iint_{S_n} \frac{\delta(t - R/c)}{2\pi R} dS \qquad (3)$$

where R is the distance to the integration point, S_n is the nth-ring surface. Then, the pressure $\vec{P}(x,y,z,t)$ can be obtained from the impulse response function as:

$$\vec{P}(x,y,z,t) = \rho_0 v(t) \underset{t}{*} \frac{\partial h_T(x,y,z,t)}{\partial t} \qquad (4)$$

where the symbol $\underset{t}{*}$ is used to denote the temporal convolution operation and ρ_0 is the density of the medium.

3. Estimation of optimum kerf

In order to find the optimal distance between rings (d), we will compare the excitation function on the transducer surface (2) with a theoretical Bessel excitation. For this purpose we select the $L2$ error criteria.

The best characteristic in the radiation field should be found when real excitation and the theoretical Bessel function have the maximum proximity. This will be true when the $L2$ criteria have a minimum. In our case the $L2$ criteria can be formulated mathematically as follows:

$$L2(d) = 20 \log \sqrt{\frac{\sum_{j=1}^{M} (J_0(\alpha r_j) - v(r_j,d))^2}{\sum_{j=1}^{M} J_0(\alpha r_j)^2}} \qquad (5)$$

where M is the number function sample.

We consider a three-ring Bessel transducer, with diameter $D = 24.4$ mm and parameter value of $\alpha = 709$ m^{-1}. The calculus has been done: assuming a tone bur emission of 5 cycles with frequency $f = 2$ MHz. and wave velocity $c = 1500$ m/s. The optimum d using equation (5) (see Fig. 2) was estimated as 1.5 mm. In Fig. 3, the radiation pattern generated by three transducers is shown, with $d = 0$ mm, $d = 1.5$ mm and $d = 2$ mm. In this figure, a limited diffracting beam appears as contour labels in -6 dB, -10 dB and -12 dB. Note that best characteristics are obtained for $d = 1.5$ mm, as expected.

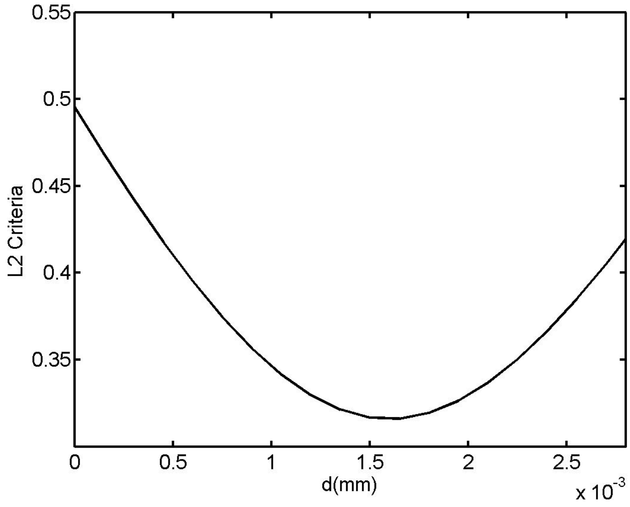

FIGURE 2. Determination of optimum cut between rings.

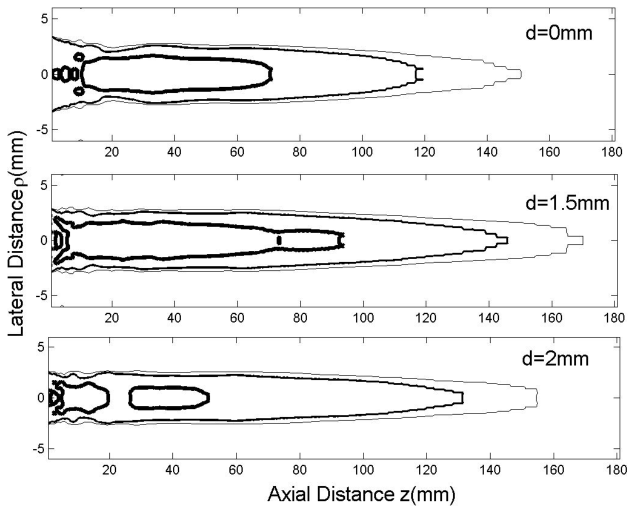

FIGURE 3. Field image for 3-ring Bessel transducer.

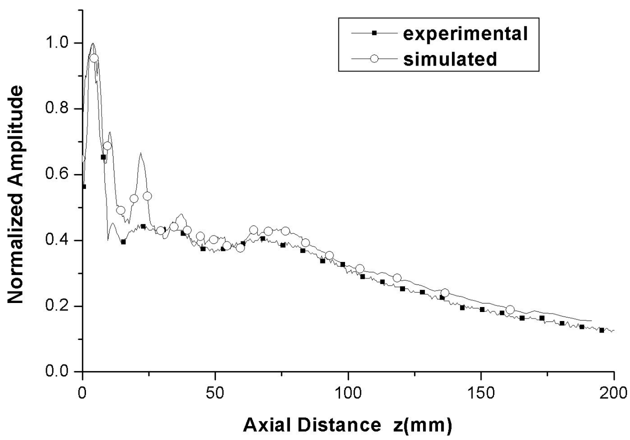

FIGURE 4. Experimental and simulated field in the center axis.

A physical transducer was constructed using the previous parameters and $d = 1.5$ mm. The comparison between simulated and experimental results for axial behavior is shown in Fig. 4. The experimentally measured values matched theoretical predictions. The real transducer has a large depth of field and narrow beamwidth.

4. Conclusions

This paper presents a new method for obtaining the optimum cut between rings needed for designing a Bessel transducer. This method was applied to a three-ring Bessel transducer, with the result that the maximum approximation to the theoretical excitation, and the best field characteristics, were obtained for $d = 1.5$ mm. The impulse response technique was used to simulate the radiation field. The comparison between simulations and experimental results showed a good agreement.

1. J. Lu and J.F. Greenleaf: *IEEE Trans. Ultrason. Ferroelect. Freq. Contl*. **37** (1990) 438.
2. P.D. Fox and S. Holm, *IEEE Trans. Ultrason. Ferroelect. Freq. Contl.* **49** (2002) 85.
3. J. Lu, *IEEE Trans. Ultrason. Ferroelect. Freq. Contl.* **45** (1998) 84.
4. O.K. Hsu, F.J. Margetan, and D.O. Thompson, *Appl. Phys. Lett.* **55** (1989) 2066.