An analysis of a displacement sensor based on optical fibers

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In this work, the results of the theoretical analysis, computational simulation, and experimental study of a displacement sensor transfer function are presented. This intensity-based sensor is composed of two elements (two aligned multimode fibers or one fiber and one mirror) which move longitudinally. With the use of a geometrical approximation, we have managed to express the sensor's transference as a function of the distance with only one analytical equation, valid for both near-field and far-field zones. The results of this theoretical analysis coincide with the computational simulation and agree with experimental data.

Keywords: Displacement optical fiber sensor; transfer function.

En este trabajo se presentan los resultados del análisis teórico, simulación de cómputo y estudio experimental de la función de transferencia de un sensor de desplazamiento. Este sensor basado en intensidad está compuesto de dos elementos (dos fibras multimodo alineadas o bien una fibra y un espejo) que se desplazan longitudinalmente. Con el uso de una aproximación geométrica, hemos logrado expresar la transferencia del sensor en función de la distancia con una sola ecuación analítica, válida tanto para el campo cercano como para el campo lejano. Los resultados de este análisis teórico coinciden con la simulación de cómputo y concuerdan con datos experimentales.

Descriptores: Sensor de desplazamiento en fibra óptica; función de transferencia.

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1. Introduction

In an optical fiber sensor, a change of an external physical or chemical factor is manifested as a variation in the transmitted light parameters. Those sensors which use a direct transformation of the external factor to intensity of the light caught by the receiving fiber are defined as intensity-based sensors.

Displacement sensors based on optical fibers have many applications in positioning and distance control, linear and angular movement, pressure, temperature, etc. In comparison with interferometric displacement sensors, intensity-based sensors have certain advantages: they require a common non-coherent source, multimode optical fibers, and a simple system of detection. Furthermore, the sensors mentioned above offer good sensitivity and resolution for short distances [1,2].

Commonly, these sensors work on the distance between the ends of the transmitting and receiving fibers which is very short and approximately as long as the core diameter. The analysis of the distribution of light intensity in a near-field zone, necessary for modeling displacement sensors, is rather difficult. In accordance with the required accuracy, more or less advanced models are used [3–5]. The most advanced models do not have any analytical solution for the transfer function of the sensor in a near-field zone. In these cases, numerical calculation is applied.

In this work, the theoretical analysis of the transfer function of an intensity-based displacement sensor was carried out, for both the near-field and the far-field zones, using a geometrical approximation.

2. Theoretical analysis

The sensor is composed of two parts facing each other (two multimode fibers or a fiber and a mirror), which displace along a common axis. The transfer function of the sensor is defined as the ratio between the optical power P_r received by a fiber and the optical power P_e emitted by the other one. Received power is expressed by means of an integral of the illumination function in the receiver plane vs. the radial variable, while the emitted power (emittance E) is constant.

In the model of a sensor of distance L, between two multimode optical fibers (Fig. 1), it is considered that the intensity distribution of the light emitted by all the core surface of the transmitting fiber is homogeneous; moreover, the intensity distribution inside the light cone, emitted by each point of the surface, is also homogeneous at the receiving plane [5,6].

The illumination received by a point inside the receiving plane S_r is affected by elementary emission surfaces dS_e which are contained in the emission plane S_e inside a circle

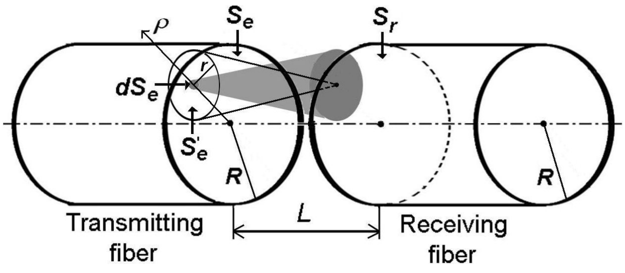

FIGURE 1. Intensity-based sensor for displacement detection, composed of two aligned multimode optical fibers.

(more correctly an overlapping area S'_e) with its center situated opposite to the receiving point (Fig. 1). Therefore, the illumination function is established as:

$$I(\rho) = \frac{\int\limits_{S'_e} E dS_e}{S_r} = E\frac{S'_e(\rho)}{S_r}, \tag{1}$$

where ρ is the radial coordinate of a cylindrical polar coordinate system with the axis coincident with the common axis of the fibers.

Therefore, it is possible to express the transfer function of a displacement sensor in terms of the illumination function as:

$$T = \frac{P_r}{P_e} = \frac{2\pi \int\limits_0^R I(\rho)\rho d\rho}{\pi E R^2}, \tag{2}$$

where R is the radius of the fiber core.

The analysis of the illumination function is based on calculating the overlapping area S'_e, between the circle of the transmitting fiber core, with radius R, and the circle of the projection of the acceptance cone at the transmitting plane, with radius r, and with its center situated on the point ρ_c (Fig. 1). The radius r of the illumination area will depend on the distance L and on the numerical aperture *NA*. To generalize expressions, we use a variable of relative (dimensionless) distance between fibers z:

$$z = \frac{r}{R} = \frac{\tan[\arcsin(NA_{eff})] \cdot L}{R}, \tag{3}$$

where NA_{eff} is the effective numerical aperture of fibers ($NA_{eff} \cong NA$ for a homogeneous intensity distribution).

By integrating the illumination function in (2), the transfer function vs. the relative distance z is expressed as:

$$T(z) = Re\left[1 + \frac{1}{2z^2} - \frac{1}{2\pi}\left(\frac{1}{2}z + \frac{1}{z}\right)\sqrt{4 - z^2} - \frac{1}{z^2\pi}\arcsin\left(1 - \frac{1}{2}z^2\right) - \frac{2}{\pi}\arcsin\left(\frac{1}{2}z\right)\right]. \tag{4}$$

The equation (4) is generalized for all interval z>0. Furthermore for the interval $z > 2$ it becomes the well-known expression $T(z) = 1/z^2$, true for a far-field zone.

Figure 2a is the result of calculations performed using the analytical expression obtained (4) and the results of the numerical calculation (points) obtained with the same model using an adapted program of simulation of refractometric sensors [7]. The maximum difference between the results of analytical and numerical calculations is less than 0.5% in the interval $0 < z < 3$.

There are models [3–5] that describe the illumination function with Gaussian beam approximations which are believed, by their authors to be better than the geometric approximation. Taking into account our definition of transfer function $T(z)$, we are able to compare the results obtained with several models. In accordance with Wang [4] we have:

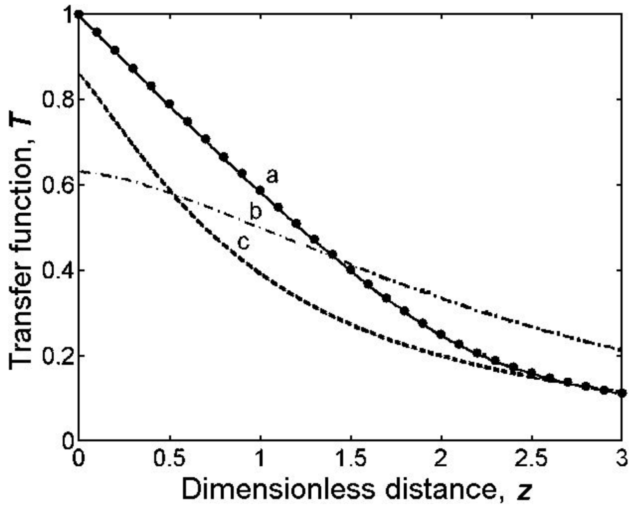

FIGURE 2. Transfer function obtained with different models: Geometric approximation proposed in this work (a), Gaussian beam approximation by Wang and Faria (b) and by Libo (c).

$$T(z) = 1 - \exp\left[-\frac{A}{(z+1)^2}\right], \tag{5}$$

where A is a constant related to the distribution of modal power. In a parallel way, Faria [5] shows the same Gaussian approximation expression, with the value of constant $A = 2$ (Fig. 2b). Libo [3], using a model similar to the ones mentioned above, shows an expression, which, in our case, turns into (Fig. 2c):

$$T = 1 - \exp\left[-\frac{1}{\left(1 + kz^{3/2}\right)^2}\right], \tag{6}$$

where k depends on the type of the source (for white light $k = 0.2$).

3. Experimental results

The experiments were conducted using a sensor of the type shown in Fig. 1, where several types of fiber have been used for a better characterization of our experiment. Two different multimode step-index fibers were used: a silica fiber (core diameter $d = 105$ μm, numerical aperture $NA = 0.21$, length $l = 1.5$ m) and a polymer fiber ($d = 1000$ μm, $NA = 0.47$, $l = 1$ m). For each type of fiber, the intensity distribution within the emission cone under excitation from a LED source ($\lambda = 665$ nm, $\Delta\lambda = 20$ nm, $d_{eff} = 1$ mm, $P = 0.5$ mW) was determined (Fig. 3).

In expression (3), the numerical aperture NA_{eff} appears. In the case of a non-homogeneous intensity distribution, it is necessary to find an adequate method to determine it. In accordance with the most exact of the methods considered, $NA_{eff} = \sin(\alpha_{eff})$, where α_{eff} is half of the angle of the

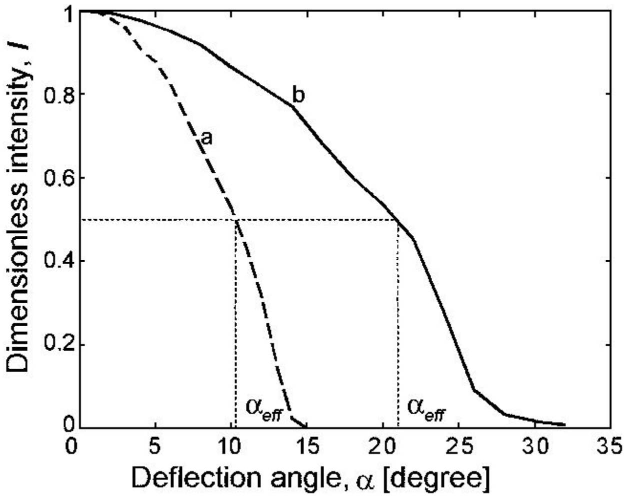

FIGURE 3. Far-field intensity distribution to the transmitting fiber, considering the emission pattern belonging to a LED source, for two types of fiber: silica fiber (a) and polymer fiber (b).

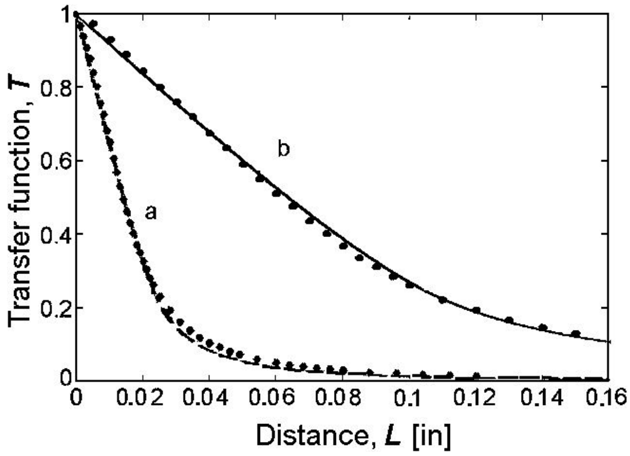

FIGURE 4. Experimental results (dots) and analytic transfer function (lines) considering the value of effective numeric aperture for two types of fiber: silica fiber (a) and polymer fiber (b).

transmitting fiber emission cone, which corresponds to the 0.5 level of the light intensity distribution (Fig. 3).

Figure 4 shows a plot of the transfer function $T(z)$, obtained through our analytical expression (lines), considering the effective value of the numerical aperture and experimental values (points) for the two types of step-index multimode optical fibers: silica (a) and polymer (b).

4. Conclusions

Generally, both fiber and source types play an important role in the characteristics of optical-fiber-based displacement sensors. Even though the model presented in this article is prepared for the ideal case of the homogeneous distribution of intensity in step-index multimode fibers, it offers very good results in the simulation of the transfer function of real sensors with non-homogeneous distribution. Using the effective value of the numerical aperture obtained by characterization of the radiation pattern of the emitting fiber, it is easy to apply this model in practice. From the results obtained we can say that our model is closer to realty than those based on the Gaussian beam approximation.

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