Can spectroscopic informations be extracted from transfer reactions?

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We revise the standard method of extracting spectroscopic factors from transfer reactions and show the strong dependence on the single particle parameters. We propose an alternative method applicable whenever there is a significant contribution from the interior to the reaction amplitude while still being well described within Distorted Wave Born Approximation (DWBA). The energies for such reactions should be well above the Coulomb barrier. The alternative method is based on fixing the contribution of the peripheral part of the reaction amplitude, depending on the Asymptotic Normalisation Coefficient (ANC), through another independent totally peripheral reaction. Then, by combining this information with the transfer data above the barrier, one can determine the spectroscopic factor and control the uncertainty coming from the single particle parameters.

Keywords: Spectroscopic factors; ANC; DWBA analysis; transfer reactions.

Se revisa el metodo standard para extraer factores espectroscopicos de reacciones de transferencia y se muestra la fuerte dependencia que el metodo tiene en los parametros de particula independiente. Proponemos un metodo alternativo aplicable siempre que exista una contribucion significante del interior de la amplitud de reaccion, ademas de que el fenomeno pueda describirse adecuadamente con el metodo aproximado de la onda distorsionada de Born (DWBA). Las energias para esta reaccion deben ser dependientes del coeficiente de normalizacion asintotico (ANC) a traves de otra reaccion periferica totalmente independiente. De esta froma, al combinar esta informacion con los datos de transferencia sobre la barrera, es posible determinar el factor espectroscopico y controlar la incertidumbre originada en los parametros de particula independiente.

Descriptores: Factores espectroscopicos; ANC; analisis DWBA; Reacciones de transferencia.

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1. Introduction

The shell model has been extremely successful in nuclear structure, with all its modern developments. Spectroscopic Factors (SF) were introduced by the shell model formalism and are related to the shell occupancy of a state n in one nucleus relative to a state m in a nearby nucleus. In particular, the one particle SFs are defined as the norm of the overlap function of a nucleus (A+1) in state n with nucleus A in state m, where the valence nucleon is in an orbital with orbital and total angular momentum (l, j). Spectroscopic factors ($S_{lj}^{n,m}$) predicted by shell model have been extensively compared with those extracted from reactions. At present ab-initio calculations are improving the accuracy of the calculated spectroscopic factors and one can, in some cases, find surprises and disagreements, especially when moving toward the driplines. It is timely to think of an accurate probe that will test the predictions of these models and will disentangle the relevant elements of the NN force that are still missing (e.g. [1]).

Spectroscopic factors (SF) of stable nuclei were traditionally measured through either knock-out (e,e'p) reactions or transfer reactions. Interestingly, they probe very different regions of the wavefunction. The (e,e'p) cross section is sensitive to the nuclear structure all the way to the inside, whereas the transfer is typically peripheral and surface peaked. These methods have been proven to give identical results if the appropriate finite-range and non-local effects are included in the transfer reaction model [2]. The spectroscopic factor is the norm of the overlap function, and peaks well inside the nuclear radius R_N. Therefore, in general, the (e,e'p) reaction is far more adequate to extract a SF. In fact, there are also some problems with the high momentum transfer component, associated with the probe inside the nucleus, since then the Born approximation is not very good. We will leave that discussion aside as it is not relevant for this work. Undoubtedly, (e,e'p) is limited to stable targets and a smaller combination of initial and final states, when compared to transfer reactions.

A systematic study of spectroscopy on a variety of nuclei, ranging from the stability valley to the proton dripline, have been performed at NSCL [3] using a new technique: knockout with radioactive beams. It is found that the measured spectroscopic factors suffer from a reduction relative to the Shell Model predictions which changed with binding energy [4]. It is not yet clear where this quenching [5] comes from, but it is believed to be associated either with long-range or very short-range correlations which are not included in the present day non-ab-initio shell model. It should be mentioned that the knock-out reactions with radioactive beams are performed at sufficiently high energy for the eikonal approximation to be valid, simplifying the reaction theory tremendously, as well as reducing reaction uncertainties [6]. Consequently, these reactions are extremely peripheral, more so than the standard transfer reactions of the sixties and seven-

ties which are surface peaked, and do not contain direct information from the interior. One extrapolates based on the tail of the overlap function. The transfer technique sits between the electromagnetic knockout and the radioactive beam knockout, and offers great potential as a spectroscopic tool of rare isotopes.

Experimental spectroscopic factors are defined as the ratio between the experimental cross section and the theoretical cross sections predicted by Distorted Wave Born Approximation (DWBA) [7]. The use of transfer reactions to extract nuclear spectroscopic information started in the sixties and had a large peak during the seventies. Only now, with the new intense radioactive beams at several facilities around the world, can one start to realistically use transfer reactions in inverse kinematics to learn more about exotic nuclei. Before that, several lingering issues related to the analysis of transfer reaction on stable beams need to be resolved before the method can be adapted to the transfer of loosely bound nucleons.

In the next two sections we will briefly describe the standard transfer analysis and derive an alternative method for transfer reactions. In section 4. we apply the new method to a ^{7}Li(d,p)^{8}Li at 22 MeV. Finally, in section 5. we present a summary of the work and discuss a few open issues.

2. The standard method

The standard procedure for extracting spectroscopic factors from transfer reactions involves fitting the differential transfer cross section within the DWBA [8]. We will here consider, for simplicity, the reaction A(d,p)B. The matrix element for the process $M =< \psi_f^{(-)} I_{An(\alpha)} | \Delta V | I_{pn}\, \psi_i^{(+)} >$ depends on the initial and final distorted waves $\psi_i^{(+)}$ and $\psi_f^{(-)}$, as well as the overlap functions for $d = p + n$ (referred to as I_{np}) and $B = A + n$ (here called $I_{An(\alpha)}$ where α contains all relevant quantum numbers). The transfer operator ΔV contains V_{np} as well as a remnant term. In the past, the remnant term was often neglected and the zero range approximation was performed on the np interaction. Today, there is no need to perform these approximations as most codes enable finite range with full remnant.

One important issue concerning DWBA analysis is associated with the optical potentials. Typically the distorted waves are calculated within the optical model framework and the optical potentials are determined from elastic scattering studies. Recent work [7] on the uncertainty of the optical potential has proved that a systematic analysis over a wide energy range can reduce the large uncertainties associated with this crucial element. In many (d,p) reactions, deuteron breakup is important and should be included. One simple and effective way to include it, is through an adiabatic distorted wave approximation (ADWA) prescription [9].

Another equally important issue concerns the radial dependence of the overlap functions. The short range behaviour of the overlap function is of many body nature and unknown. On the other hand, the asymptotic behaviour is simply given by:

$$I^B_{An(\alpha)}(r) \overset{r>R_N}{\approx} C_\alpha\, i\, \kappa\, h_l(i\,\kappa\, r), \tag{1}$$

where C_α is the asymptotic normalisation coefficient and R_N defines the radius at which the nuclear interaction is no longer appreciable. It is standard practice to approximate the short range part of the overlap function to a single particle function $\varphi_{An(\alpha)}(r)$ calculated within a Woods-Saxon well, with a specific radius and diffuseness (r_0, a). In general it also contains a spin orbit term which we will neglect for the purpose of this derivation. The overlap function can then be written explicitly as:

$$I^B_{An(\alpha)}(r) = K_\alpha\, \varphi_{An(\alpha)}(r). \tag{2}$$

The spectroscopic factor is then simply $S_\alpha = K^2_\alpha$.

The single particle wavefunction has the same asymptotic dependence as the overlap function

$$\varphi_{An(\alpha)}(r) \overset{r>R_N}{\approx} b_\alpha\, i\, \kappa\, h_l(i\,\kappa\, r)$$

but the normalisation constant is now associated with the single particle well and obviously depends on the single particle parameters. It is trivial to show that $C_\alpha = K_\alpha\, b_\alpha$. While a small change in these parameters does not affect the normalisation of the interior considerably, it can change the asymptotic normalisation by an order of magnitude. A set of $2g_{9/2}$ single particle wavefunctions simulating the valence orbital of the ground state (g.s.) of ^{209}Pb is shown in Fig. 1 and illustrates the sensitivity of b_α to the single particle parameters. We show in Fig. 2 the transfer cross section calculated in ADWA for ^{208}Pb(d,p)^{209}Pb at 22 MeV for the set of single particles wavefunctions of Fig. 1 [10]. The normalisations of cross sections change enormously and could produce spectroscopic factors differing by a factor of four. However, for

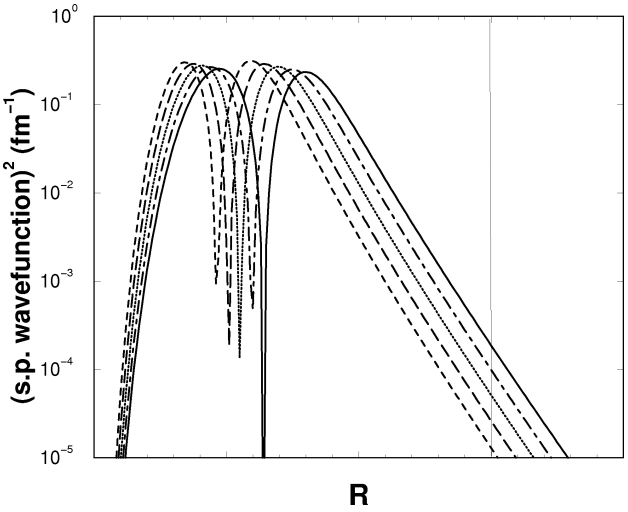

FIGURE 1. Single particle wavefunctions for $2g_{9/2}$ orbitals bound by 3.937 MeV, for a range of radial parameters r_0.

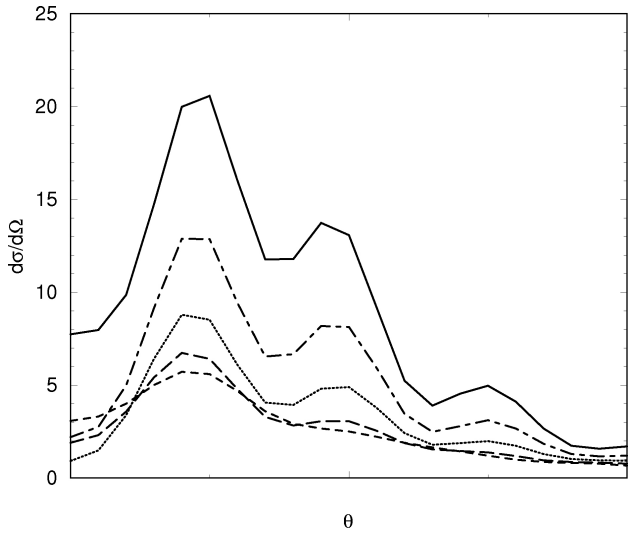

FIGURE 2. Cross sections resulting for the range of single particle wavefunctions depicted in Fig. 1.

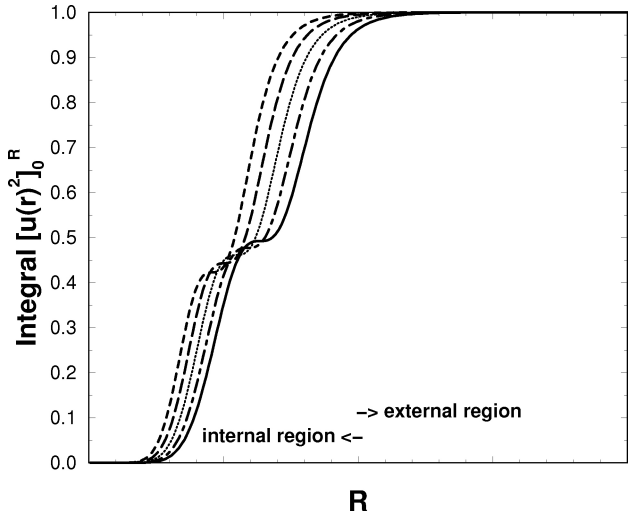

FIGURE 3. Integrated wavefunctions for the same variety of radial parameters r_0.

most single particle radial parameters, the shape of the angular distribution does not change considerably. The real spectroscopic factor is defined as the norm of the overlap function, and is mostly sensitive to the interior part. This is depicted in Fig. 3 by the cumulative sum of the single particle wavefunction, which reaches over 80% of its total value well inside the nuclear well.

Aware of the uncertainty due to single particle parameters, Hartree Fock densities have often been used as a constraint (e.g. [7]). However, specially as moving away from the stability line, Hartree Fock is not adequate. Alternatively, rms radii of the nucleus have also been suggested [11] as a means of pinning down these parameters, but, again, different approaches result in different preferred parameters. We believe that the best way to pin down this uncertainty is through a direct independent measurement of this normalisation constant. This is the basis of the method we discuss in the next section.

3. The proposed method

Following [12], we rewrite the transfer amplitude as:

$$M = K_\alpha \, (\tilde{M}_{int}[b_\alpha] + b_\alpha \, \tilde{M}_{ext}), \tag{3}$$

where subscripts *int* and *ext* refer to contribution from the interior region ($R < R_N$) and the exterior region ($R > R_N$) respectively. We use this equation for illustration purposes only. The interior part contains the dependence on b_α through the bound state wavefunction $\varphi_{An(\alpha)}$, while the external part $\tilde{M}_{ext}$ does not depend on b_α, as the normalisation of the single particle wavefunction was already factored out. We now rewrite the transfer cross section as

$$\frac{\mathrm{d}\,\sigma^{th}}{\mathrm{d}\Omega} = C_\alpha^2 \, \frac{\sigma^{th}}{b_\alpha^2}, \tag{4}$$

and instead of comparing the cross sections we compare the ratios of the cross section and the square of the asymptotic normalisation coefficient:

$$R^{exp} = \frac{\sigma^{exp}(peak)}{C_{exp}^2}$$

and

$$R^{th}(b_\alpha) = \frac{\sigma^{th}(peak)}{b_\alpha^2}.$$

The later depends on the choice of the single particle parameters. One can then generate $R^{th}(b_\alpha)$ for a range of b_α values. An illustration of these curves are plotted in Fig. 4 for a few of cases:

1. The dashed line corresponds to R^{th} when the reaction is completely peripheral. As there is no contribution from the interior, the dependence in b_α factors out from the transition amplitude such that R^{th} becomes independent of b_α.

2. The typical case is shown by the dot-dashed line, where there is some contribution from the interior such that R^{th} becomes sensitive to b_α.

3. We also show by the dotted line, the $1/b_\alpha^2$ dependence to guide the eye. This would be the result if we only had contributions from the interior and the interior matrix element were independent of b_α, which will never be the case.

The normalisation is arbitrary. More details can be found in [13].

If one can now determine R^{exp} with a narrow error bar, it is possible from a graph such as that illustrated in Fig. 4 to determine the range of b_α that are permissible and from it the range of spectroscopic factors $S_\alpha = C_\alpha^2/b_\alpha^2$. With present day technology, and if all other uncertainties are well under control, this alternative method can bring down the error of the SF to $< 15\%$. In the standard method, this error is not controlled at all.

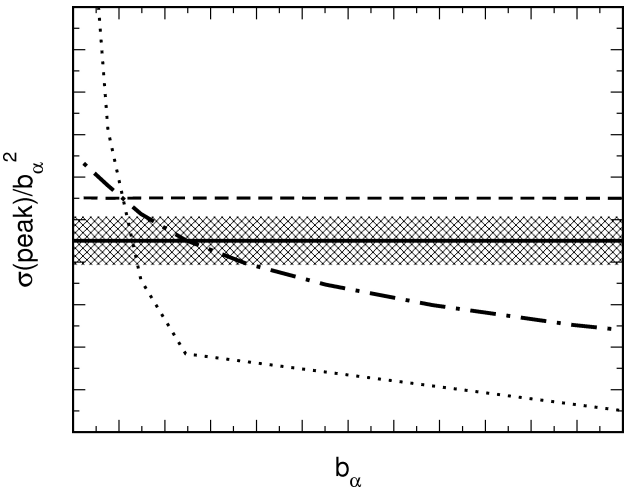

FIGURE 4. An illustration of the ratio $R^{th}(b_\alpha)$ for a reaction which is completely peripheral (dashed line), interior dominated (dotted line) and a typical realist case (dot-dashed line). The solid line represents the data and the error bar.

It is important to keep in mind that this method uses the experimental value of the cross section at the first peak and not the whole angular distribution. This only makes sense if the angular distribution remains essentially the same over the relevant range of single particle parameters and the optical potentials used provide an adequate description of that same distribution. In general this may not be the case. Also, the proposed method is based on the DWBA or ADWA framework. Although we do not discuss it in this work, the actual validity of the framework needs to be addressed case by case. In many situations coupling to collective states (vibrational or rotational) or single particle excitation spoil the simple picture. Then, the whole reaction should be analysed within the coupled channel framework [14].

4. Application

In Fig. 5 we show an analysis for ^{7}Li(d,p)^{8}Li at 12 MeV. A series of finite range DWBA ^{7}Li(d,p)^{8}Li calculations were performed for $E_d = 12$ MeV, varying the single particle parameters (r_0, a) to span a range of $b_{1p3/2}$ values. We neglect the spin orbit of the ^{7}Li$-n$ and calculate the differential cross section at the peak of the distribution as a function of the single particle ANC b_{1p}. Optical potentials are taken from [15]. We compare the theoretical predictions for

$$R^{th} = \frac{\sigma_{peak}(b_{1p})}{b_{1p}^2}$$

(dashed line) with

$$R^{exp} = \frac{\sigma(peak)}{C_{1,3/2}^2 + C_{1,1/2}^2}$$

. For R^{exp} we use the cross section data from [15] at the most forward angle, and the ANC from [16]. We obtain $R^{exp} = 43.0(4.1)$ fm mb/srad (represented by the solid line and the shaded area in Fig. 5). Taking the average value for R^{exp} would be consistent with $b_{1p} \approx 0.65$ fm$^{-1/2}$ and would imply $S_{g.s.} \approx 1.0$. The standard DWBA analysis in [15] provides $S_{g.s.} = 0.87$. Both results are consistent with the predictions from Variational Monte Carlo calculations [17] $S_1 = S_{p1/2} + S_{p3/2} = 0.923$. However, given the error bar for R_{exp} and the weak dependence of R^{th} on b_{1p}, the only safe conclusion is that $S_{g.s.} > 0.8$. As mentioned above, the dependence on optical potentials and any higher order effects should also be considered.

Applications to other reactions can be found in [13].

5. Summary and Concluding remarks

In summary, one of the most important ingredients in a reaction $A(d,p)B$ is the overlap function $I(r) = \langle A|B\rangle$. This many body overlap function is usually approximated to a single-particle wave function. In this approximation, the asymptotic normalisation of the tail of the overlap relates directly to the asymptotic normalisation of the single particle wavefunction through $C^2 = Sb^2$. If the shape of the overlap function were well known (if the single particle parameters were determined uniquely) then knowing the normalisation of the tail, would provide the spectroscopic factor without further uncertainties. However, the single particle parameters are not known and b depends strongly on them. This is one of the main reasons why, in the past, it was impossible to extract *accurate* spectroscopic factors from transfer reactions. In fact, the situation was very unsatisfying. We looked back at a few sets of old transfer data and determined the actual ANCs that were used in the DWBA analysis through $C^2 = Sb^2$. We compared them with ANCs extracted from sub-Coulomb heavy-ion reaction measurements, which are interior-free. In several cases we have found serious disagreements, well beyond any acceptable error bar. This reflects the

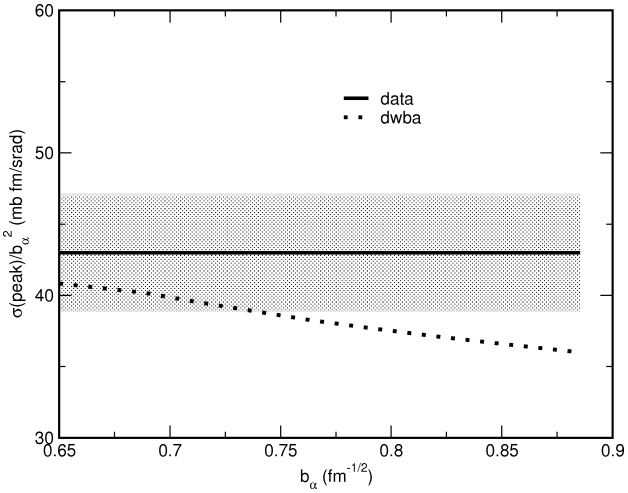

FIGURE 5. Cross section for ^{7}Li(d,p)^{8}Li(g.s.) at 12 MeV and the dependence on the single particle parameters: experimental value (solid line), experimental error bar (shaded area) and the DWBA prediction (dotted).

essential problem: in the standard analysis, the exterior contribution can be adjusted arbitrarily.

In the alternative method here discussed, the exterior part is fixed independently, by introducing the measured ANC. This not only reduces the SF uncertainty, but detects inconsistencies with other ingredients in the DWBA analysis, such as wrong optical potentials or coupling effects. When all other uncertainties are controlled, the proposed method has the potential of providing reliable SFs within 15%.

One of the main drawbacks of the combined method here proposed is the fact that only the peak of the transfer cross section enters into the solution. This means that a good description of the angular distribution is not ensured. A better approach, presently under study, is to jointly constraint the SF by both the ANC and the transfer angular distribution. This may require further adjustments to the optical potentials.

Finally, ADWA calculations for ^{40}Ca(d,p)^{41}Ca and ^{208}Pb(d,p)^{209}Pb, both involving the transfer onto a closed shell nucleus, predict SFs much smaller than unity. This is an aspect that has been addressed before [18] and calls for improvements in the description of the reaction in the nuclear interior. One effective way to include effects beyond DWBA is by non localities in the optical potentials [20]. An alternative to CCBA calculations are polarization potentials. These possibilities need to be studied, to provide a reliable *calibration* of the combined method.

Acknowledgments

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