Selection rule and energetic stability. Complementary aspects of nuclear clusterization

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The deformation-dependence of clusterization in atomic nuclei is investigated. In particular, allowed and forbidden cluster-configurations are determined for the ground, superdeformed, and hyperdeformed states of some light and heavy nuclei, based on a microscopic (real and effective $SU(3)$) selection rule. For light nuclei the real $U(3)$ symmetry is used to characterize the parent and cluster nuclei. In the case of heavy nuclei our study is based on the application of the effective $U(3)$ symmetry, which was first introduced for nuclei with large prolate deformation [1]. The stability of the different cluster configurations from the viewpoint of the binding energy [2] is also investigated for comparison.

Keywords: Binary and ternary clusterization; Pauli-principle; $U(3)$ symmetry; Energetic preference.

La dependencia de la clusterización de la deformación nuclear ha sido estudiada. En particular las configuraciones de cúmulos permitidas y prohibidas de algunos núcleos ligeros y pesados ha sido estudiada utilizando una regla de selección microscópica ($U(3)$ real y efectiva). En el caso de los núcleos ligeros, la simetria $U(3)$ real ha sido utilizada para caracterizar el nucleo padre y los cúmulos. En el caso de los núcleos pesados, nuestro estudio se basa en la aplicación de la simetria $U(3)$ efectiva, que fue introducida por primera vez para núcleos con gran deformación prolata [1]. La estabilidad de las distintas configuraciones de cúmulos también ha sido estudiada desde el punto de vista de la preferencia energetica [2] a efectos comparativos.

Descriptores: Clusterizacion binaria y ternaria; Principio de Pauli; Simetria $U(3)$; preferencia energetica.

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1. Introduction

The main objective of this contribution is to present our results on the study of the dependence of clusterization on the deformation. Our approach is based on the assumption that the preference of clusterization depends on two basic principles. One is related to maximizing the sum of the binding energies of the clusters (called *energy-minimum principle* [2,3]) and the other is determined by the *Pauli-exclusion principle*. They represent complementary constraints in the formation of clusters. The Pauli-exclusion principle is in general difficult to implement microscopically for complex systems, it gives large computational difficulties. In Ref. 4 an approximate treatment to implement the Pauli-exclusion principle was introduced for binary systems. Considering both principles is important because even if one clusterization is preferred from the energetical point of view, it might be forbidden due to the exclusion principle, and vice versa. An overlap of preferred clusterizations using both principles might therefore give information on the possible observable clusterizations.

In this contribution we present our results related to the dependence of binary clusterizations on the deformation. We will apply our method to light and heavy systems. In Sec. 2 we introduce the technique for determining the possible clusterizations for binary channels and discuss a possible extension to the ternary case. We also present a short review on how to obtain the preferred clusterizations from the energetic point of view. In Sec. 3 we present our results and finally in Sec. 4 conclusions are drawn.

2. Selection of clusterizations

The microscopic part of our study is based on the application of the $U(3)$ selection rule. The structural selection rule is based on the $U(3)$ symmetry, what is known to be an approximate good symmetry of light nuclei [5].

The $U(3)$ selection rule reads [6]:

$$[n_1, n_2, n_3] = [n_1^{(1)}, n_2^{(1)}, n_3^{(1)}] \otimes [n_1^{(2)}, n_2^{(2)}, n_3^{(2)}] \otimes [n^R, 0, 0] \quad (1)$$

where $[n_1, n_2, n_3]$ is the set of $U(3)$ quantum numbers of the parent nucleus, the superscript (i) stands for the ith cluster, and R indicates relative motion.

The SU(3) symmetry is well realized in light nuclei. However, in medium and heavy nuclei the SU(3) symmetry is broken due to the spin-orbit interaction. In Ref. 7 it was shown that in spite of the breaking of the symmetry one can still define an *effective* SU(3) irreducible representation (irrep) related to an SU(3) group, called *effective* SU(3). In practical terms, this effective irrep is a mixture of the SU(3) irreps in such a way that observables like the quadrupole-quadrupole interaction have matrix elements similar to a real SU(3). For more details, please see Ref. 7. In Ref. 1 this idea was applied to heavy nuclei with large prolate deformation and a procedure to obtain effective quantum numbers was outlined. Because we were interested in a systematic study of clustering effects in heavy nuclei, where systems also can have oblate deformation as well as small deformations (both oblate and prolate) the procedure outlined in Ref. 1 was extended to oblate and to small deformations in Ref. 8. The generalization for small deformations is based on an expansion of single particle orbitals in terms of asymptotic Nilsson states and then the application of the procedures of Refs. 1 and 8 for prolate and oblate systems respectively. To validate our procedure we carried out two types of consistency-checks. In the first one we compared the effective quantum numbers obtained from our procedure with the real $U(3)$ quantum numbers in a series of light nuclei. In the case of heavy nuclei there are no "real" $U(3)$ quantum numbers of reference. In this case the following self-consistency criterium was applied: starting from a given deformation (β_i), taken from tables published in Ref. 9 we can determine the effective SU(3) irrep that characterize the ground state of a given nucleus. Based on Ref. 10, it is possible to deduce a deformation (β_f) associated to the effective quantum numbers. Self consistency is obtained, if the two deformation values coincide ($\beta_i \sim \beta_f$). Our results showed that both in light and in heavy nuclei the procedure gives reliable results [8]. The number of quanta of the relative motion can be determined using the Harvey prescription [11]. The prescription says that in order to satisfy the *Pauli-exclusion principle* when the nucleons are rearranged to form the compound system, the number of oscillator quanta can be increased only in one direction (see for more details an example in Ref. 12).

In the application of the selection rule it is important to have a quantitative measure of how far a given cluster irrep (including the relative motion) is from the effective SU(3) irrep of the parent nucleus. For that reason the quantity of *reciprocal forbiddenness* is defined [13], *i.e.*

$$S = \frac{1}{1 + min(\sqrt{(\Delta n_1)^2 + (\Delta n_2)^2 + (\Delta n_3)^2})}, \quad (2)$$

where $\Delta n_i = |n_i - n^c_{i,k}|$. Here n_i refers to the $U(3)$ representation of the parent nucleus, while $n^c_{i,k}$ stands for the $U(3)$ representation of channel c, obtained from the right-hand-side of Eq. (1), with the k index distinguishing the different product-representations. Then S approaches zero for forbidden clusterizations, and becomes $S = 1$ for allowed clusterizations.

The consideration outlined before concerned the microscopic structure part of the problem. However, as was shown in Ref. 2, the energy-minimum principle is also an important consideration. The criterium of maximal stability requires maximizing the value of the summed differences of the measured binding energies and the corresponding liquid drop values:

$$D(1,2) = [B(1) - B_L(1)] + [B(2) - B_L(2)] \quad (3)$$

where $B(i)$ is the experimental binding energy of the i*th* cluster and $B_L(i)$ stands for the liquid drop value [2].

The individual clusters were determined using also the so-called *dipole constraint* [2] which exploits the fact that the dipole transition strengths in heavy nuclei are small. When Z_k (A_k) is the charge (mass) of the k'th cluster and Z_T (A_T) is the total charge (mass) then we have the additional condition

$$\frac{Z_T}{A_T} \approx \frac{Z_1}{A_1} \approx \frac{Z_2}{A_2} \quad (4)$$

The procedures outlined here for binary clusterizations can be generalized to ternary or multicluster configurations.

3. Applications

In what follows, we present some results for the case of a light nucleus (^{36}Ar) and of two heavy nuclei (^{252}Cf, ^{232}Th). Our interest in the ^{36}Ar case is related to recent results where the existence of a superdeformed state was confirmed experimentally [14]. States of large deformation were studied theoretically in this nucleus using different approaches. Superdeformed states were predicted in [14, 15] within cranked Nilsson-Strutinsky calculations and in large-scale shell model calculations. They were interpreted as a configuration of two protons and two neutrons that moved to the pf major shell. There is also prediction for a hyperdeformed state in Ref. 16.

The ^{252}Cf nucleus attracted much attention due to recent experiments published in Ref. 17. The ^{232}Th case is of interest because theoretical calculations [18] have showed that the structure of the third minimum in this nucleus resembles a binuclear configuration involving a spherical heavy fragment around ^{132}Sn and a well deformed lighter fragment around ^{100}Zr.

3.1 ^{36}Ar

The states of ^{36}Ar, we have investigated are shown in Table I together with their quantum numbers.

In determining the $U(3)$ symmetry of the superdeformed states we have followed two different methods. The largest prolate deformation what can be obtained from 4-nucleon excitation to the pf major shell belongs to the [32,12,12] representation. It is worth mentioning that the same representation corresponds to the $\beta_2 = 0.6$ deformation parameter,

which is generally considered to characterize the superdeformed shape. On the other hand one can derive the effective, or quasi-dynamical $U(3)$ symmetry quantum numbers corresponding to the $\beta_2 \approx 0.45$ deformation (experimental value, [14]), as it was proposed in [1]. In this way one gets the [32,14,10] $U(3)$ representation, which also corresponds to $4\hbar\omega$ excitation.

TABLE I. The quantum numbers of the ground, superdeformed, and hyperdeformed states of the ^{36}Ar nucleus.

State	$\hbar\omega$	$[n_1, n_2, n_3]$
Ground	0	[20,20,12]
Superd.(a)	4	[32,12,12]
Superd.(b)	4	[32,14,10]
Hyperd.(a)	12	[48, 8, 8]
Hyperd.(b)	6	[36,12,10]

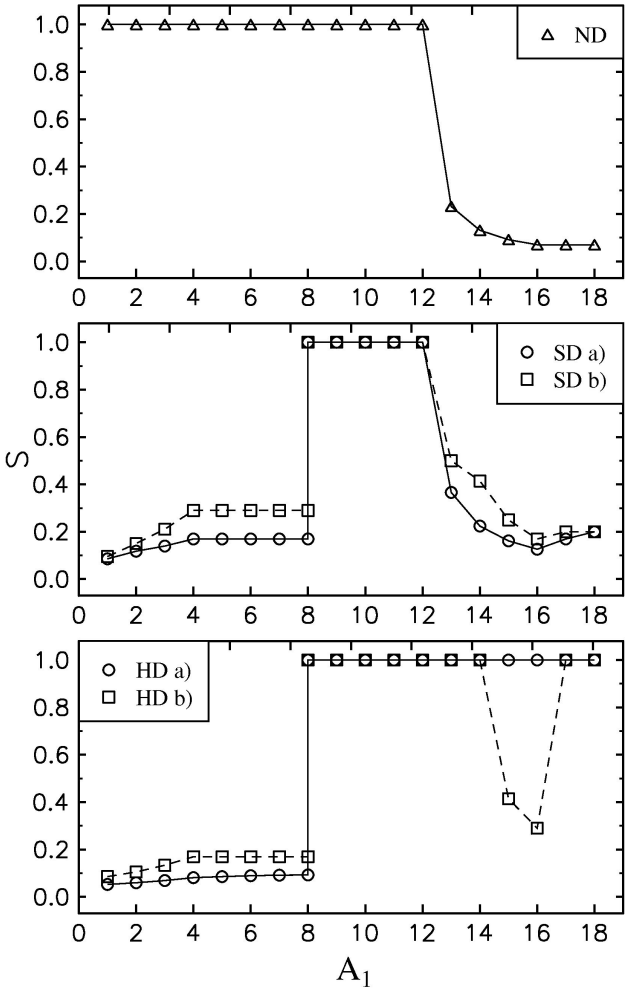

FIGURE 1. Reciprocal forbiddenness for the ^{36}Ar binary clusterizations. ND stands for the normal deformed (ground) state, while SD and HD indicate super and hyperdeformed states, respectively.

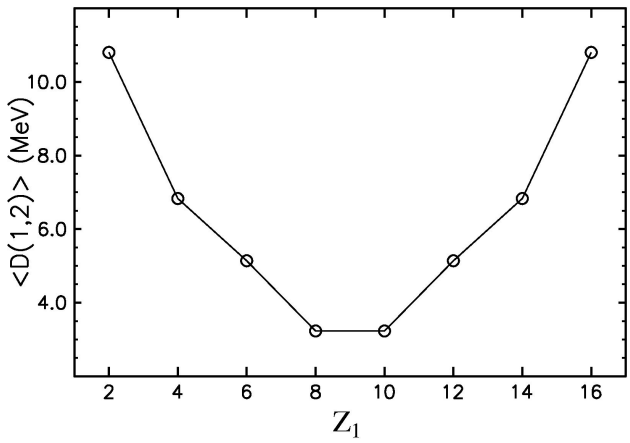

FIGURE 2. Energetic stability of binary clusterizations in ^{36}Ar.

In relation to the possible hyperdeformed state in ^{36}Ar, it was predicted from cranked Bloch-Brink α-cluster model calculation with ratio of major to minor axis 3:1 [16], and a correspondence to some heavy-ion resonances was conjuctioned. This state has a $U(3)$ symmetry [48,8,8], noted as "Hyperd.(a)" in Table I, and corresponds to $12\hbar\omega$ excitation. The effective $U(3)$ quantum numbers, determined from the Nilsson-scheme for the $\beta_2 \approx 0.86$ deformation are [36,12,10] ("Hyperd.(b)"), corresponding to merely $6\hbar\omega$ excitation. The differences in the $U(3)$ quantum numbers of the super and hyperdeformed states reflect the uncertainty of the applied theoretical methods for the prediction of these extremely deformed states.

Our results for the possible binary clusterizations of ^{36}Ar are shown in Figs. 1 and 2. An interesting finding is that certain allowed cluster combinations appear in all the normal, super and hyperdeformed state (see Fig. 1). Our preliminary results show that also some ternary cluster combinations have the same property and that these are strongly related to the binary configuration. For example ^{24}Mg+^{8}Be+^{4}He, ^{20}Ne+^{4}He+^{12}C, and ^{16}O+^{8}Be+^{12}C satisfy this property. (In the first case if the two last clusters are united we obtain the ^{12}C nucleus and similarly when the first two clusters are united in the last two cases we obtain the ^{24}Mg. In this sense, these ternary cluster configurations resemble the binary case ^{24}Mg+^{12}C).

3.2 ^{252}Cf

The results are depicted in Table II, Figs 3, and 4. In this case too, all possible binary clusterizations were studied.

$$^{252}_{98}Cf \rightarrow ^{A}_{Z}X + ^{252-A}_{98-Z}Y. \quad (5)$$

The procedure applied here is based on the use of effective U(3) quantum numbers to characterize both the parent nucleus and the clusters. We have studied all possible binary clusterizations (308 fission channels) with ($8 \leq Z \leq 90$). The starting point is always the deformation of the nucleus. Then we fill in the Nilsson orbitals from below at that deformation value and determine the effective (λ, μ) using the

TABLE II. The quantum numbers of the ground [9], superdeformed, and hyperdeformed states of the ^{252}Cf nucleus.

State	β_2	$[n_1, n_2, n_3]$
Ground	0.24	[414,321,303]
Superd.	0.60	[520,285,267]
Hyperd.	0.86	[600,260,245]

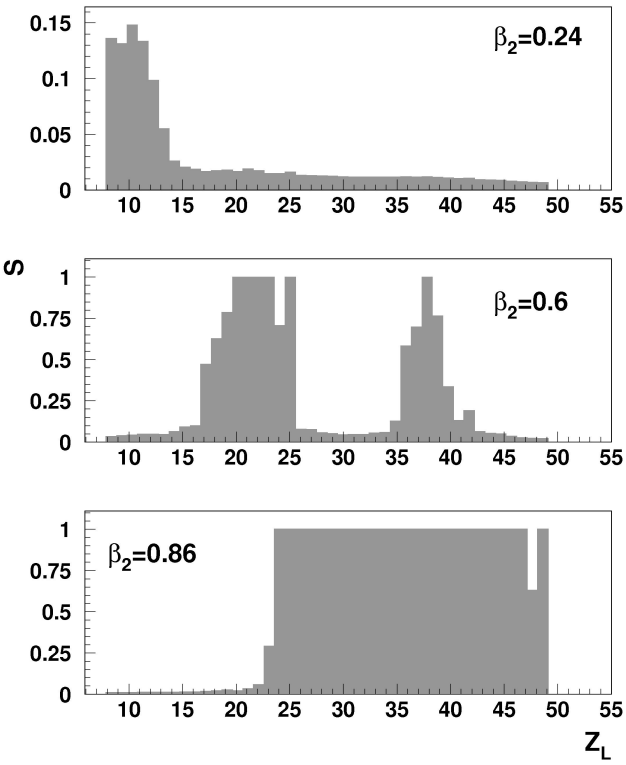

FIGURE 3. Reciprocal forbiddenness S versus the Z_{light} of the studied binary cluster configurations for the ^{252}Cf case. The values of S correspond to mean values over channels that have the same Z_{light} and different A_{light}.

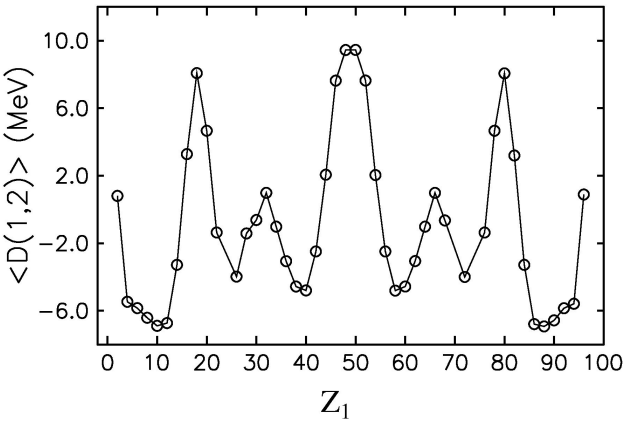

FIGURE 4. Energetic stability of binary clusterizations in ^{252}Cf.

relations of [1, 8]. The parameters for the Nilsson Hamiltonian where taken from [15].

TABLE III. Some selected ternary clusterizations of ^{252}Cf (for details see the text). D(1,2,3) is a generalization of (3).

$C_1 + C_2 + C_3$	S			D(1,2,3)	
	ND	SD	HD		
^{208}Pb+^{40}Si+^{4}He	0.0412	1.0000	0.0186	24	.78
^{208}Pb+^{22}O+^{22}O	0.0343	1.0000	0.0222	24	.66
^{132}Sn+^{70}Ni+^{50}Ca	0.0099	0.0178	0.0343	23	.80
^{78}Ni+^{126}Sn+^{48}Ca	0.0100	0.0178	0.0343	23	.78
^{78}Ni+^{90}Kr+^{84}Se	0.0099	0.0174	0.0329	20	.65
^{48}Ca+^{132}Sn+^{72}Ni	0.0102	0.0186	0.0374	24	.77

Using ground state deformations for the parent and daughter nuclei [9], all studied cluster configurations in question turn out to be forbidden (Fig. 3, upper part). A clear tendency towards cluster radioactivity (or very asymmetric fission) can be inferred from this figure. We have also addressed the question if there are allowed clusterizations in case when we change the deformation of the parent nucleus to superdeformation ($\beta_2 \sim 0.6$) or hyperdeformation ($\beta_2 \sim 0.86$). The obtained results are presented in the lower parts of Fig. 3. As in the earlier calculations the clusters are considered to have ground state deformations. It is interesting to see that in these cases there are allowed clusterizations as well. In the case of a superdeformed ^{252}Cf the regions of allowed clusterizations correspond mainly to two particular regions in which:

a) both clusters have large prolate quadrupole deformation (region with $Z_{light} \sim 36$),

b) one cluster with prolate quadrupole deformation and the other with oblate deformation (region with $Z_{light} \sim 22$).

For the hyperdeformed ^{252}Cf case more channels are open, and from Fig. 3 (lower panel) a clear tendency to symmetric clusterization can be inferred.

As in the ^{36}Ar case, we have also studied the criterium of maximum stability for comparison. The results of our study are presented in Fig. 4. It is clear from this figure that the preferred clusterizations of the ground state of ^{252}Cf concentrate mainly in three regions around $Z = 2$, $Z \sim 18$ and $Z \sim 50$. This result agrees only with our U(3) procedure for the $Z = 2$ case (the α clusterization is allowed in the U(3) framework). The other two regions are not favored in the U(3) approach if we consider ^{252}Cf in its ground state deformation (Fig. 3, upper panel), but $Z \sim 18$ is preferred if we assume ^{252}Cf in a superdeformed state and the $Z \sim 50$ is allowed in the case of the hyperdeformed state.

As in the case of ^{36}Ar, we have just started to study possible ternary clusterizations of ^{252}Cf. Table III shows some preliminary results of our study in the case when one cluster is fixed as a double magic nucleus, and the other two clusters are selected using the clusterization dictated by (3) (criterium of maximum stability applied to the residual nucleus once the

double magic nucleus is subtracted from the ^{252}Cf). In this particular case our results showed that only clusterizations related to ^{208}Pb double magic nucleus are allowed, and only when ^{252}Cf is assumed in a SD state. A more systematic study is underway [19].

3.3 ^{232}Th

One question of particular interest that can be addressed in this framework is if the ^{100}Zr + ^{132}Sn clusterization is an allowed one. This clusterization is located at the maximum of the mass distribution of the fission of ^{232}Th. Another reason for studying this particular clusterization is that in [18] it was shown that the structure of the third minimum in ^{232}Th corresponds to a bi-nuclear configuration involving a spherical heavy fragment around ^{132}Sn and a well deformed lighter fragment around ^{100}Zr.

In a first step we have studied if this clusterization is allowed assuming that the parent nucleus and the clusters have ground state deformations (^{232}Th ($\beta_2 \sim 0.2$), ^{100}Zr ($\beta_2 \sim 0.36$), ^{132}Sn ($\beta_2 \sim 0.0$)). The U(3) selection rule shows that this clusterization is not allowed. The studied clusterization remains forbidden even if we assume that ^{232}Th is in a hypothetical superdeformed (SD, $\beta_2 \sim 0.6$) or hyperdeformed (HD, $\beta_2 \sim 0.86$) state, keeping the clusters in their ground state deformations.

In a second step we have studied if the ^{100}Zr + ^{132}Sn clusterization is allowed when we change the deformation of the clusters. Considering the high stability against deformation of the double magic nucleus ^{132}Sn, we have changed the deformation of ^{100}Zr. ^{100}Zr can be considered soft against deformation, and it is located in a region of shape coexistence [20]. Assuming ground state deformation for ^{232}Th, the clusterization remains forbidden when we change the deformation of ^{100}Zr. A change occurs when we assume that the ^{232}Th is in a SD or in a HD state. In these cases the clusterizations are allowed when the ^{100}Zr has large oblate deformation ($\beta_2 \leq -0.4$).

4. Conclusions

In this contribution we have shown that the two basic principles which govern the nuclear clusterization, namely the energy-minimum and the Pauli-exclusion principles play complementary roles. Therefore, they do not necessarily result in the same preference of cluster configurations. The most likely clusterizations are those, which are preferred from both aspects.

Concerning the deformation-dependence of clusterizations, an interesting finding is that the same cluster-configuration can be present in the ground state, as well as the superdeformed and hyperdeformed states of a nucleus, if one takes into account the deformation of the clusterizations properly, *i.e.* without oversimplifying constraints *e.g.* on the spherical or cylindrical symmetry in the description.

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