

# Band structure for the cladding of a hollow core photonic crystal fibre

J. Arriaga and B. Meneses

*Instituto de Física, Universidad Autónoma de Puebla,  
Apartado Postal J-48, 72570, Puebla, México*

Recibido el 18 de octubre de 2002; aceptado el 25 de febrero de 2003

We calculate the photonic band structure of a two dimensional (2D) photonic crystal (PC) formed by a hexagonal array of circular rings embedded in air. This crystal has been used as the cladding of a photonic crystal fibre whose core is composed of air. We solve the Maxwell equations using the plane wave method to expand the fields and the dielectric constant. Our calculations reveal that for propagation in the plane of periodicity, the PC possesses no complete photonic band gaps. However we demonstrate the existence of complete photonic band gaps for propagation out of the plane of periodicity.

*Keywords:* Band structure, photonic crystals, fibre optics.

Calculamos la estructura de bandas fotónica para un cristal fotónico bidimensional formado por un arreglo hexagonal de aros circulares en una matriz de aire. Este cristal se ha usado como la envolvente de una fibra óptica de cristal fotónico, cuyo núcleo está compuesto de aire. Resolvemos las ecuaciones de Maxwell y usamos el método de ondas planas para expandir los campos y la constante dieléctrica. Nuestros cálculos revelan que, para la propagación en el plano de periodicidad, el cristal fotónico no posee brechas fotónicas completas. Sin embargo, demostramos la existencia de brechas fotónicas completas para propagación fuera del plano de la periodicidad.

*Descriptores:* Estructura de bandas, cristales fotonicos, fibras opticas.

PACS: 71.20.-b, 42.70.Qs, 42.81.-i

## 1. Introduction

Photonic crystal fibres (PCF's) are optic fibres which guide light through a defect surrounded by a periodic array of air holes running along the entire length of the fibre. These fibres have been shown to possess numerous unusual properties, including highly tuneable dispersion, nonlinearity, and single mode operation at all wavelengths [1-4]. Such properties are of fundamental interest in several areas, including soliton propagation and supercontinuum generation [5-7]. Two different guiding mechanisms for PCF's have been identified. The first mechanism is conventional guiding, which relies on total internal reflection and requires that the core have a higher refractive index than the cladding. The second employs a defect mode in a two-dimensional photonic band gap. This mechanism uses the structure which stops the propagation in any transverse direction, and can, in principle, allow light to propagate in an air core. Recently a fibre has been fabricated in which light can be guided in a hollow core [8]. In this type of fibre, light is trapped in the air by a full two-dimensional photonic band gap of the cladding instead of by total internal reflection.

In this work, we calculate the cladding's photonic band structure for the hollow core photonic crystal fibre reported in Ref. 8. The cladding of this fibre can be considered as a 2D hexagonal array of infinitely long rings of silica embedded in air. We consider both propagation in the plane of periodicity and propagation out of the plane of periodicity. Our results show that for propagation out of the plane of periodicity the structure possesses a complete photonic band gap. We solve Maxwell's equations using the plane wave expansion.

## 2. Theoretical model

The photonic band structure is obtained solving Maxwell's equations, with no sources and currents. Taking the time dependence as  $\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r})e^{i\omega t}$ , and eliminating the electric field, we obtain

$$\nabla \times \left[ \frac{1}{\epsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right] = \left( \frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r}). \quad (1)$$

Because we have a periodic system, we expand the inverse of the dielectric constant  $\eta(\mathbf{r}) = 1/\epsilon(\mathbf{r})$  and the magnetic field  $\mathbf{H}(\mathbf{r})$  as

$$\eta(\mathbf{r}) = \sum_{\mathbf{G}} \eta(\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}}, \quad (2)$$

$$\mathbf{H}(\mathbf{r}) = \sum_{\mathbf{G}} \mathbf{H}(\mathbf{G}) e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}}, \quad (3)$$

where  $\mathbf{G} = l\mathbf{a}^* + m\mathbf{b}^* + n\mathbf{c}^*$  is a vector of the reciprocal lattice. If we substitute these expansions in Eq. (1) we obtain

$$\begin{aligned} & - \sum_{\mathbf{G}'} \eta(\mathbf{G} - \mathbf{G}') [(\mathbf{k} + \mathbf{G}) \times [(\mathbf{k} + \mathbf{G}') \times \mathbf{H}(\mathbf{G}')] ] \\ & = \left( \frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{G}), \quad (4) \end{aligned}$$

which is an algebraic eigenvalue problem. In the previous equation,  $\eta(\mathbf{G} - \mathbf{G}')$  are the Fourier coefficients of the function  $\eta(\mathbf{r}) = 1/\epsilon(\mathbf{r})$ . In general, they are given by

$$\eta(\mathbf{G}) = \frac{1}{V_c} \int_{V_c} \eta(\mathbf{r}) e^{-i\mathbf{G}\cdot\mathbf{r}} d\mathbf{r}, \quad (5)$$

where the integration is over the volume unit cell. In this work we are interested in a two-dimensional (2D) photonic crystal formed by a hexagonal array of circular infinitely long cylinders of silica with dielectric constant  $\epsilon_a = 2.1025$ , embedded in air ( $\epsilon_b = 1$ ) [8]. The cylinders have internal radius  $r_1$  and external radius  $r_2 = a_0/2$ , the lattice constant being  $a_0$ . The inset of Fig. 1 shows schematically the distributions of rings where for clarity, the repeated unit cell of this periodic system appears in the centre. For 2D photonic crystals, the Fourier coefficients of the function  $\eta(\mathbf{G})$  are

$$\eta(\mathbf{G}) = \begin{cases} f\eta_a + (1-f)\eta_b, & \mathbf{G} = 0 \\ (\eta_a - \eta_b) \frac{1}{A_a} \int_{A_a} e^{-i\mathbf{G}\cdot\mathbf{r}} d\mathbf{r}, & \mathbf{G} \neq 0 \end{cases}$$

where  $f = A_A/A_C$  is the fraction of the unit cell occupied by the "atoms" forming the crystal. For the case considered here  $f = \pi(r_2^2 - r_1^2)/A_C$ , and the integration is over the area occupied by the rings. For the special configuration of rings shown in Fig. 1, the Fourier coefficients for  $\mathbf{G} \neq 0$  are

$$\eta(\mathbf{G} \neq 0) = 2(\eta_a - 1) \left[ (f - f_1) \frac{J_1(|G|r_2)}{|G|r_2} - f_1 \frac{J_1(|G|r_1)}{|G|r_1} \right], \quad (6)$$

where  $f_1 = \pi r_1^2/A_C$ ,  $J_1$  is the Bessel function of order 1, and we have substituted the value of the dielectric constant of the background,  $\epsilon_b = 1.0$ . If we consider propagation in the plane of periodicity, then  $\mathbf{k}$  and  $\mathbf{G}$  are 2D vectors, and we can uncouple Eq. (4) in two sets of equations,

$$\sum_{\mathbf{G}'} \eta(\mathbf{G} - \mathbf{G}') (\mathbf{k} + \mathbf{G}) \cdot (\mathbf{k} + \mathbf{G}') H_z(\mathbf{G}') = \left(\frac{\omega}{c}\right)^2 H_z(\mathbf{G}), \quad (7)$$

$$\sum_{\mathbf{G}'} \eta(\mathbf{G} - \mathbf{G}') [(\mathbf{k} + \mathbf{G}) \cdot (\mathbf{k} + \mathbf{G}')] - (\mathbf{k} + \mathbf{G}')(\mathbf{k} + \mathbf{G}) \cdot \mathbf{H}_T(\mathbf{G}') = \left(\frac{\omega}{c}\right)^2 \mathbf{H}_T(\mathbf{G}). \quad (8)$$

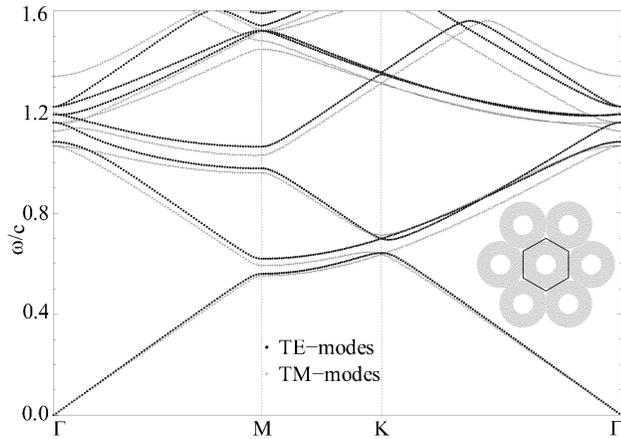


FIGURE 1. Photonic band structure for the TE and TM modes of a hexagonal array of circular rings embedded in air. The propagation is in the plane of periodicity along the principal directions of the Brillouin zone. The inset shows the distributions of rings, and the unitary cell.

Equation (7) has the magnetic field parallel to the cylinders and it describes the propagation of electromagnetic waves with the electric field transversal to the axis of the cylinders. This is referred to as transversal electric (TE) modes. Equation (8) is referred to as transversal magnetic (TM) modes, since the magnetic field is in the plane of periodicity. We have used Eqs. (6)-(8) and the parameters reported in Ref. [8]. These parameters are: lattice constant  $4.9\mu$ , and  $r_1 = 1.45\mu$ . Figure 1 shows the photonic band structure along the principal directions of the Brillouin zone for both the TE and the TM modes. As we can see from the figure, there is no complete photonic band gap for both polarisations. We have done calculations for different values of the inner radius  $r_1$ , but close to the reported value. In all the cases considered here there is not a complete photonic band gap for propagation in the plane of periodicity. However, if we consider propagation of electromagnetic waves out of the plane of periodicity, complete photonic band gaps start to open up. In this case the propagation vector  $\mathbf{k}$  is no longer two dimensional, and we cannot obtain equations for the TE and TM modes. In this case we must solve directly Eq. (4). Figure 2 shows the photonic band structure for propagation out of the plane of periodicity for  $k_z = 5.0$  ( $k_z$  is in units of  $2\pi/a_0$ ). As we can see from the figure there are two complete photonic band gaps which are indicated by the shadowed area. Figure 3 shows the frequencies of the band edges against the component of the propagation vector out of the plane of periodicity. As we can see from this figure, complete photonic band gaps exist if the  $k_z$  component is sufficiently large. If we introduce a defect breaking the periodicity of the structure to form the core of an optical fibre, any guided mode with propagation constant ( $\beta = k_z$ ) within a complete photonic band gap cannot penetrate the cladding and must be totally localised in the core. This is in fact the result reported in Ref. 8. In Fig. 4 of Ref. 8, the authors observed a peak in the intensity spectrum of the transmitted light through the air core for a wavelength around 810 nm. This means that the photonic crystal sur-

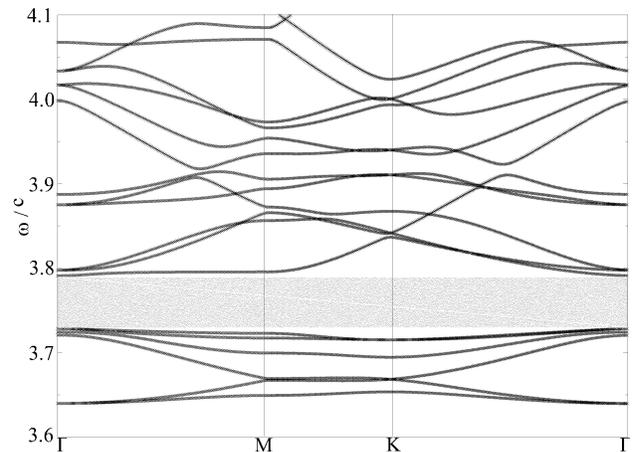


FIGURE 2. Photonic band structure for propagation out of the plane of periodicity, along the principal directions of the Brillouin zone for  $k_z = 5.0$ ,  $k_z$  is in units of  $2\pi/a_0$ .

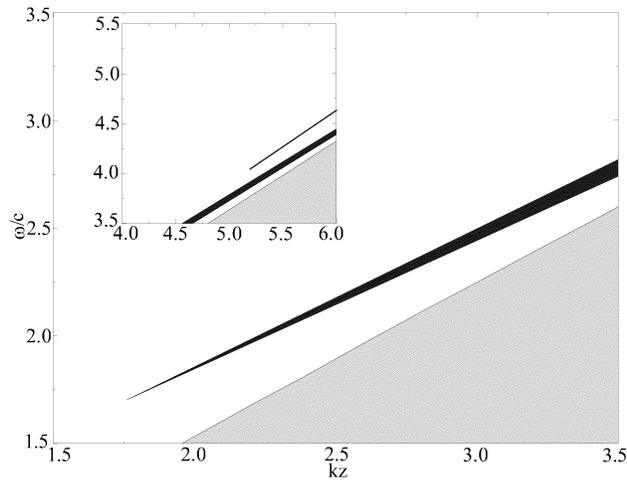


FIGURE 3. Full 2D photonic band gaps for propagation out of the plane of periodicity.

rounding the core must have a complete photonic band gap at around  $(\omega/c) = 7.75\mu\text{m}^{-1}$ . We obtain a complete photonic band gap at that value of  $(\omega/c)$  for  $k_z = 10.93$ .

### 3. Conclusions

We have calculated the photonic band structure for the cladding of a hollow core crystal fibre. We show that for propagation out the plane of periodicity the cladding possesses complete photonic band gaps, allowing light to propagate along the hollow core.

### 4. Acknowledgements

This work has been partially supported by Conacyt, México, under grant No. 33808-E.

1. J.C. Knight, T.A. Birks, P. St. J. Russell, and D.M. Atkin, *Opt. Lett.* **21** (1996) 1547.
2. J.C. Knight, J. Broeng, T.A. Birks, and P. St. J. Russell, *Science* **282** (1998) 1476.
3. J.K. Ranka and R.S. Windeler, *Optics and Photonics News* **11** (2000) 20.
4. J.C. Knight, T.A. Birks, B. Mangan, and P. St. J. Russell, *Optics and Photonics News* **13** (2002) 26.
5. W.J. Wadsworth, J.C. Knight, A. Ortigosa-Blanch, J. Arriaga, E. Silvestre, and P. St. J. Russell, *Electronics Lett.* **36** (2000) 53.
6. J.K. Ranka, R.S. Windeler, and A.J. Stentz, *Opt. Lett.* **25** (2000) 25.
7. S. Coen *et al.*, *Opt. Lett.* **26** (2001) 1356.
8. R.F. Cregan, B.J. Mangan, J.C. Knight, T.A. Birks, P. St. J. Russell, P.J. Roberts, D.C. Allan, *Science* **285** (1999) 1537.