Quantum cosmology for inflationary scenary

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We study quantum and super quantum cosmology for a Friedman-Robertson Walker (FRW) flat universe in the presence of an inflation exponential potential with the corresponding kinetic term for the scalar field. We exhibit exact solutions for the corresponding Wheeler-DeWitt (WDW) equation and its square root. In both cases, solutions, as is known for the Bianchi models, of the form $e^{i\Phi}$ exist, where Φ is the Hamilton-Jacobi function. For a particular factor ordering of the standard Wheeler-DeWitt equation we show a "wave packet" in which for large x (with the radius of expansion of the universe $a \sim e^{-x}$), φ the scalar field and x "compete" modulating the behaviour of the "wave packet".

Keywords: Inflation; quantum exact solutions; supersymmetric quantum cosmology; factor ordering; Einstein-Hamilton-Jacobi equation.

Se estudian soluciones cosmológicas cuánticas y supercuánticas exactas para un universo plano tipo Friedman-Robertson-Walker (FRW) en presencia de un potencial exponencial de inflación con un término cinético para el campo escalar. Se presentan soluciones exactas a la ecuación de Wheeler-DeWitt (WDW) y su correspondiente raíz cuadrada. En ambos casos, las soluciones, como es conocido de los modelos Bianchi, son de la forma $e^{i\Phi}$, donde Φ es la función de Hamilton-Jacobi. Para un valor particular del parámetro de ordenamiento de factores en la ecuación de Wheeler-DeWitt, se muestra un "paquete de ondas" en el cual para grandes valores de x (con el radio de expansión del universo definido $a \sim e^{-x}$), el campo escalar φ "compite" con x en la modulación del comportamiento del "paquete de ondas".

Descriptores: Inflación; soluciones exactas cuánticas; cosmología cuántica supersimétrica; ordenamiento de factores; ecuación de Einstein-Hamilton-Jacobi.

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1. Introduction

The best candidate for quantum gravity theory remains to be superstring matrix theory [1, 2]. If it is indeed correct, it should account for a quantum description of the universe. However, a well understood second quantization of string theory is not at hand. An attempt to search for quantum models of our universe has been proposed, namely, to solve the Wheeler-DeWitt equation for the effective action of string theory [3, 4].

Actually, the validity of this approach remains an open question. For example, within the context of Bianchi IX cosmology, in general relativity, it has been shown that imposing additional symmetry on the model alters the physical predictions [5]. It has, on the other hand, been argued that one can find conditions that must be satisfied to justify the minisuperspace approximation [6]. It has been claimed that the WDW equation corresponds to the s-wave approximation [7], in the string theory formalism. Nevertheless, the expectations seem to be that the fundamental behaviour of the wave function will be preserved [8], by considering a more general analysis.

In principle, the wavefunction of the universe yields the probability that a spatial hypersurface evolves from a given initial state. However, ambiguities arise when attempting to invoke such an interpretation due to the hyperbolic nature of the Wheeler-DeWitt equation: a conserved current with a positive-definite probability density is not possible. One possible resolution of this and related difficulties is to extend the standard quantization of the universe in a supersymmetric fashion. Supersymmetry may help in the quantization of gravity for a number of reasons.

In order to put in context the approach we present in this work, we will briefly describe the main different formalisms applied to supersymmetric quantum cosmology.

i) Those defined by means of the use of supersymmetry as a square root [9–12], in which the Grassmann variables are auxiliary variables and are not to be identified as the supersymmetric partners of the cosmological bosonic variables.

ii) The superfield formulation [13, 14], which permits local supersymmetric quantum cosmological models to be constructed in a systematic way, getting in a direct manner the corresponding fermionic partners and being able to incorporate matter [15]. These fermionic partners are not deduced directly from the gravitino.

iii) Models based on supergravity. They have been studied using the Arnowitt-Deser-Misner (ADM) canonical formulation and a four-component spinor formalism [16–18]. They have also been studied with ADM variables and a two-component spinor formalism [19, 20]. Following this scheme, matter has been also included [21, 22], taking $N = 1$ as well as $N = 2$ supergravity. Further, Ashtekar's variables have been considered [23, 24]. Some of these models have already been presented in two comprehensive and organized works, a book [25] and an extended review [26].

The realization of these models needs a homogenization ansatz, which is usually taken to be where the space coordinates dependence of the metric is eliminated. That is, the spatial derivatives are set to zero. In the case of supergravity, the homogeneity ansatz should be formulated to be consistent with supersymmetry.

It is also well known that one can transform the lagrangian in the so called string-frame to the Einstein frame [27]. In this work we will consider the Einstein frame to describe a Friedman-Robertson-Walker flat universe, the lagrangian will contain a kinetic term for the scalar field coming from the dilaton field in the string frame and a potential depending on this scalar field which we will consider an exponential function of it. It has been considered that the $B_{\mu\nu}$ field could be responsable for the presence of these kind of potentials [1]. This is one of the possible potentials which fits with the present data and can be deduced by performing a second order reconstruction of the COBE potential [27].

For this model, we will exhibit solutions to the WDW equation in Sec. 2. A "Gaussian" state is also constructed for which it is shown, for an enough large inflation scalar field, that its approach to the singularity is slowed down by the presence of the inflation field. The exponential of this scalar field "compete" with the scale factor of the universe and modifies the behaviour of the "Gaussian" state. The supersymmetric quantum wave functions are exhibited in Sec. 3, following the procedure in Refs. 9-12. We also observe a tendency for supersymmetric vacua to remain close to their semi-classical limits, because in this work and others [12], the exact solutions found are also the lowest-order WKB approximations. Section 4 is devoted to final remarks.

2. WDW equation and its solutions

We start with the following classical hamiltonian that comes from inflationary cosmological model:

$$H = e^{3x}\left(-P_x^2 + P_\varphi^2 + \beta e^{-\mu\varphi}\right), \qquad (1)$$

where x and φ are the variables in the model, with β and μ complex parameters. In the quantum scheme, this hamiltonian is up to operator and we take the following representation for the operators $P_{q^u} = -i\partial/\partial q^u$:

$$\hat{H}\Psi = e^{3x}\left(\frac{\partial^2\Psi}{\partial x^2} - \frac{\partial^2\Psi}{\partial\varphi^2} + \beta e^{-\mu\varphi}\Psi\right) = 0. \qquad (2)$$

To include the factor ordering problem, we substitute the following relation into (2):

$$e^{3x}\frac{\partial^2\Psi}{\partial x^2} \to e^{3x}\left(\frac{\partial^2\Psi}{\partial x^2} - p\frac{\partial\Psi}{\partial x}\right), \qquad (3)$$

where the real parameter p measures the ambiguity in the factor ordering. So, the Wheeler-DeWitt equation, we can read now

$$\Box\Psi + p\frac{\partial\Psi}{\partial x} - \beta e^{-\mu\varphi}\Psi = 0, \qquad (4)$$

with $\Box\Psi = -\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial\varphi^2}$ is the d'Alambertian in two dimensions with signature (-,+).

To solve (4) we use the separation of variables method, making $\Psi = X(x)Y(\varphi)$, thus we have

$$-Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{d\varphi^2} + pY\frac{dX}{dx} - \beta e^{-\mu\varphi}XY = 0. \qquad (5)$$

We obtain the following two equations:

$$-\frac{d^2X}{dx^2} + p\frac{dX}{dx} = \eta^2 X, \qquad (6)$$

$$-\frac{d^2Y}{d\varphi^2} + \beta e^{-\mu\varphi}Y = \eta^2 Y, \qquad (7)$$

where the parameter η is the separation constant.

Equation (6) is solved easily:

$$X = X_0 e^{\frac{p}{2}\left[1+\sqrt{1-(\frac{2\eta}{p})^2}\right]x} + X_1 e^{\frac{p}{2}\left[1-\sqrt{1-(\frac{2\eta}{p})^2}\right]x}. \qquad (8)$$

We can identify (7) as a Bessel differential equation, whose solution is

$$Y = Z_\nu\left(\pm i\frac{2\sqrt{\beta}}{\mu}e^{-\frac{\mu\varphi}{2}}\right), \qquad (9)$$

where Z_ν is a generic Bessel function with order $\nu = \pm i\frac{2\eta}{\mu}$.

Thus, the solution becomes

$$\Psi_\eta = XY = \left[X_0 e^{\frac{p}{2}\left[1+\sqrt{1-(\frac{2\eta}{p})^2}\right]x} + X_1 e^{\frac{p}{2}\left[1-\sqrt{1-(\frac{2\eta}{p})^2}\right]x}\right] Z_\nu\left(\pm i\frac{2\sqrt{\beta}}{\mu}e^{-\frac{\mu}{2}\varphi}\right). \qquad (10)$$

Since these solutions have the dependence in the parameter η, the general solution can be put as

$$\Psi_{gen} = \int G(\eta)\Psi_\eta d\eta, \qquad (11)$$

where $G(\eta)$ represents a weighting function.

For the particular value in the factor ordering $p = 0$, the solution is

$$\Psi(x,\varphi) = e^{\pm i\frac{2\eta}{\mu}x}K_\eta(\pm i\frac{2\sqrt{\beta}}{\mu}e^{-\frac{\mu}{2}\varphi}). \qquad (12)$$

For this solution a *wave packet* can be constructed [28]:

$$\Psi(x,\varphi) = \mathcal{N}e^{-\frac{\mu}{2}\varphi}\sinh(\frac{\mu}{2}x) \times \exp\left[-\frac{2\sqrt{\beta}}{\mu}e^{-\frac{\mu}{2}\varphi}\cosh\left(\frac{\mu}{2}x\right)\right]. \qquad (13)$$

For large x the *wave packet* reduces to

$$\Psi(x,\varphi) = \frac{\mathcal{N}}{2}e^{\frac{\mu}{2}(x-\varphi)}\exp\left[-\frac{\sqrt{\beta}}{\mu}e^{\frac{\mu}{2}(x-\varphi)}\right], \qquad (14)$$

where the exponential functions depend on $x-\varphi$. We can see that in this stadium, the scalar field φ and x *compite* modulating the behaviour of the *wave packet*. So, when the singularity is approached (large x), if φ is large enough, the *wave packet* slows its approach to zero.

On the other hand, if in (1) we substitute $P_x = \frac{\partial\Phi}{\partial x}$ and $P_\varphi = \frac{\partial\Phi}{\partial\varphi}$ (for zero factor ordering), where the Φ is known as the superpotential function, thus we obtain the Einstein-Hamilton-Jacobi equation

$$-\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial\varphi}\right)^2 + \beta e^{-\mu\varphi} = 0, \qquad (15)$$

and using the separation of variables method, we have the following solution for the superpotencial Φ:

$$\begin{aligned}\Phi &= \Phi_x(x) + \Phi_\phi(\phi)\\ &= \pm sx \pm \frac{2s}{\mu}\left[\ln\sqrt{\frac{1+\sqrt{1-\frac{\beta}{s^2}e^{-\mu\varphi}}}{1-\sqrt{1-\frac{\beta}{s^2}e^{-\mu\varphi}}}} - \sqrt{1-\frac{\beta}{s^2}e^{-\mu\varphi}}\right], \qquad (16)\end{aligned}$$

where the parameter s is the separation constant.

3. Supersymmetric quantum frame

We start giving the following super-hamiltonian:

$$H_{super} := \left(\mathcal{H}_0 + K\frac{\partial^2\Phi(x,\varphi)}{\partial q^\nu \partial q^\mu}\left[\bar{\psi}^\nu, \psi^\mu\right]\right), \qquad (17)$$

where the bosonic hamiltonian $\mathcal{H}_0$ corresponds to the one in Eq. (4), K is a complex constant. Following Ref. [12], we give the super-charges

$$Q = \psi^x\left(\partial_x + i\frac{\partial\Phi(x,\varphi)}{\partial x}\right) + \psi^\varphi\left(\partial_\varphi + i\frac{\partial\Phi(x,\varphi)}{\partial\varphi}\right), \qquad (18)$$

$$\bar{Q} = \bar{\psi}^x\left(\partial_x - i\frac{\partial\Phi(x,\varphi)}{\partial x}\right) + \bar{\psi}^\varphi\left(\partial_\varphi - i\frac{\partial\Phi(x,\varphi)}{\partial\varphi}\right). \qquad (19)$$

We suppose the following algebra for the variables ψ^μ and $\bar{\psi}^\nu$, $(\mu,\nu = x,\varphi)$ [12]:

$$\left\{\psi^\mu, \bar{\psi}^\nu\right\} = \eta^{\mu\nu}, \quad \left\{\psi^\mu, \psi^\nu\right\} = 0, \quad \left\{\bar{\psi}^\mu, \bar{\psi}^\nu\right\} = 0. \qquad (20)$$

Using the representation for these variables as $\psi^\mu = \eta^{\mu\nu}\partial/\partial\theta^\nu$ and $\bar{\psi}^\nu = \theta^\nu$, one finds the superspace hamiltonian to be written in the form

$$\begin{aligned}H_{super}\Psi &= \left\{Q, \bar{Q}\right\}\Psi = \left(Q\bar{Q} + \bar{Q}Q\right)\Psi\\ &= \left(-\partial_x^2 + \partial_\varphi^2 - \left(\frac{\partial\Phi(x,\varphi)}{\partial x}\right)^2 + \left(\frac{\partial\Phi(x,\varphi)}{\partial\varphi}\right)^2\right.\\ &\left. + i\frac{\partial^2\Phi(x,\varphi)}{\partial x^2}\left[\bar{\psi}^x, \psi^x\right] + i\frac{\partial^2\Phi(x,\varphi)}{\partial\varphi^2}\left[\bar{\psi}^\varphi, \psi^\varphi\right]\right)\Psi. \qquad (21)\end{aligned}$$

This equation is similar to the structure in (17).

So, we obtain the following relations between superpotential Φ and the potential under study, that is not other thing that the Einstein-Hamilton-Jacobi equation, whose solution is given in Eq. (16):

$$V(x,\varphi) = -\beta e^{-\mu\varphi} = -\left(\frac{\partial\Phi(x,\varphi)}{\partial x}\right)^2 + \left(\frac{\partial\Phi(x,\varphi)}{\partial\varphi}\right)^2. \qquad (22)$$

Also, in this scheme, any physical state must obey the following quantum constraints

$$\bar{Q}\Psi = 0, \qquad (23)$$

$$Q\Psi = 0. \qquad (24)$$

The wave function has the following decomposition in the Grassmann variables representation:

$$\Psi = \psi_+ + \psi_0\theta^0 + \psi_1\theta^1 + \psi_-\theta^0\theta^1, \qquad (25)$$

where the components $\psi_\pm$ are the contributions to the bosonic sector, and, ψ_0, ψ_1 are the contribution functions in the fermionic sector.

The supercharges read as

$$Q = -\left(\partial_x + iD_x\Phi_x\right)\frac{\partial}{\partial\theta^0} + \left(\partial_\varphi + iD_\varphi\Phi_\varphi\right)\frac{\partial}{\partial\theta^1}, \qquad (26)$$

$$\bar{Q} = \theta^0\left(\partial_x - iD_x\Phi_x\right) + \theta^1\left(\partial_\varphi - iD_\varphi\Phi_\varphi\right), \qquad (27)$$

where $D_x = \frac{d}{dx}$, $D_\varphi = \frac{d}{d\varphi}$.

When we use Eq. (23), we have the following set of partial differential equations

$$\left(\partial_x\psi_+ - iD_x\Phi_x\psi_+\right) = 0, \qquad (28)$$

$$\left(\partial_\varphi\psi_+ - iD_\varphi\Phi_\varphi\psi_+\right) = 0, \qquad (29)$$

$$\partial_x\psi_1 - iD_x\Phi_x\psi_1 - \partial_\varphi\psi_0 + iD_\varphi\Phi_\varphi\psi_0 = 0. \qquad (30)$$

Thus, the solutions for Eqs. (28 -29) are

$$\psi_+ = \psi_{+x}\psi_{+\varphi}, \qquad (31)$$

$$\psi_{+x} = K_{+1}e^{i\Phi_x},$$

$$\psi_{+\varphi} = K_{+2}e^{i\Phi_\varphi}, \qquad (32)$$

where K_{+1} and K_{+2} are integration constants. Finally, the structure for the function ψ_+ is

$$\psi_+ = K_+e^{i\Phi}, \qquad (33)$$

where Φ is given in Eq. (16).

Now, using Eq. (24) we have the other set equations

$$\partial_\varphi\psi_- + iD_\varphi\Phi_\varphi\psi_- = 0, \qquad (34)$$

$$\partial_x\psi_- + iD_x\Phi_x\psi_- = 0, \qquad (35)$$

$$-\partial_x\psi_0 - iD_x\Phi_x\psi_0 + \partial_\varphi\psi_1 + iD_\varphi\Phi_\varphi\psi_1 = 0, \qquad (36)$$

having the structure form for ψ_- as

$$\psi_- = K_2 e^{-i\Phi}. \qquad (37)$$

For functions ψ_0, ψ_1 we propose the following ansatz:

$$\psi_0 = \frac{\partial R}{\partial x} e^{i\Phi}, \qquad (38)$$

$$\psi_1 = \frac{\partial R}{\partial \varphi} e^{i\Phi}, \qquad (39)$$

where $R = R(x,\varphi)$ is a bosonic function.

Introducing (38) and (39) in (30), we find that the function R has the following structure:

$$R(x,\varphi) = R_x(x) + R_\varphi(\varphi), \qquad (40)$$

and re-introducing on (36), we obtain

$$D_x^2 R_x + 2iD_x R_x D_x \Phi_x = D_\varphi^2 R_\varphi + 2iD_\varphi R_\varphi D_\varphi \Phi_\varphi = a_0, \qquad (41)$$

that is easy to solve when we apply separation variables methods, for instance

$$D_x F_x + 2iF_x V_x = a_0, \qquad (42)$$

$$\text{with } F_x = D_x R_x, \quad V_x = D_x\Phi_x;$$

$$D_\varphi F_\varphi + 2iF_\varphi V_\varphi = a_0, \qquad (43)$$

$$\text{with } F_\varphi = D_\varphi R_\varphi, \quad V_\varphi = D_\varphi\Phi_\varphi.$$

The solutions for F_x and F_φ are

$$F_x = a_0 e^{-2i\Phi_x}\int e^{2i\Phi_x}dx + a_1 e^{-2i\Phi_x}, \qquad (44)$$

$$F_\varphi = a_0 e^{-2i\Phi_\varphi}\int e^{2i\Phi_\varphi}d\varphi + a_2 e^{-2i\Phi_\varphi}, \qquad (45)$$

With that, using (38) and (39), we have

$$\psi_0 = e^{-2i\Phi_x}\left(a_0\int e^{2i\Phi_x}\,dx + a_1\right)e^{i\Phi} = e^{\mp 2isx}\left(\mp\frac{ia_0}{2s}e^{\pm 2isx} + a_1\right)e^{i\Phi}, \qquad (46)$$

$$\psi_1 = e^{-2i\Phi_\varphi}\left(a_0\int e^{2i\Phi_\varphi}\,d\varphi + a_2\right)e^{i\Phi}. \qquad (47)$$

In this point, we can mention that these contributions to the wave function have a tendency to remain close to their semi-classical limits, (see Ref. 12), and the exact solutions found are also the lowest-order WKB approximations.

4. Final Remarks

We have exhibited exact solutions to the Wheeler-DeWitt equation and its square root. Some of these solutions result of the form $e^{i\Phi}$, where Φ is the Hamilton-Jacobi function. The same behaviour was found for the Bianchi models [29]. For the standard WDW equation and a particular factor ordering (3) we have found the *wave packet* (13). For large x the *wave packet* reduces to (14), where the exponential functions depend on $x - \varphi$. So, when the singularity is approached (large x), if φ is large enough, the *wave packet* slows its approach to zero.

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