Earthquake-Induced Helmholz Resonance in Manzanillo Lagoon, Mexico

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On 9th October, 1995, at 9:36 local time, an earthquake of magnitude 8.0 occurred on the west coast of Mexico. The epicenter was located at 18° 51.5' N and 104° 8.4' W, at approximately 25 km of Manzanillo. This important commercial harbor is connected by a narrow channel to a lagoon. A tidal gauge recorded the forced oscillations occurred in the lagoon. A dominant period of 36 minutes was observed. Analytical solutions for ideal rectangular flat basins and numerical modeling for realistic geometries show that the lagoon and the channel produce a Helmholz resonance with a period similar to that observed in the earthquake.

Keywords: Tsunamis; resonance; numerical modeling; coastal oceanography

El 9 de octubre de 1995, a las 9:36 hora local, un terremoto de magnitud 8.0 ocurrió en la costa occidental de México. El epicentro estuvo localizado en 18° 51.5' N y 104° 8.4' O, a aproximadamente 25 km de Manzanillo. Este importante puerto comercial está conectado por un estrecho canal a una laguna. Un mareógrafo registró las oscilaciones que fueron forzadas en la laguna. Se observa una señal dominante con un periodo de 36 minutos. Las soluciones analíticas para cuencas rectangulares planas ideales y la modelación numérica para geometrías más reales muestran que la laguna y el canal producen un modo de resonancia de Helmholz con un periodo similar al observado en el terremoto.

Descriptores: Tsunamis; resonancia; modelación numérica; oceanografía costera

PACS: 92.10.-c; 92.10.Hm; 92.10.Sx

1. Introduction

An earthquake of magnitude Mx = 8.0 occurred on October 9, 1995, at 9:30 local time, at 18° 51.5' N and 104° 8.4' W, off the west coast of Mexico (Fig. 1). Effects and damages of the tsunami generated by the earthquake were documented by Borrero [1]. A device to measure conductivity, temperature and depth (CTD), installed 50 m below the ocean surface and 2240 m offshore, recorded the tsunami at Barra de Navidad, about 70 km Northwest of the epicenter [2]. The tsunami traveled 10-12 minutes to reach this site. In this paper, we discuss the oscillations in the Manzanillo bay-lagoon generated by the tsunami. The epicenter of the earthquake was located about 25 km southeast of the port of Manzanillo.

The CTD spectra at Barra de Navidad contain peaks of 25, 13, 8 and 6 minutes [2]. However, the tidal gauge record at Manzanillo yields a spectral peak with a period of about 36 minutes. The shape and dimensions of the basins do not suggest a mode of resonance with this period. We show that analytical solutions for ideal rectangular basins and numerical modeling of the water circulation in the bay-lagoon system of Manzanillo suggest a Helmholz resonance with a period of around 36 minutes.

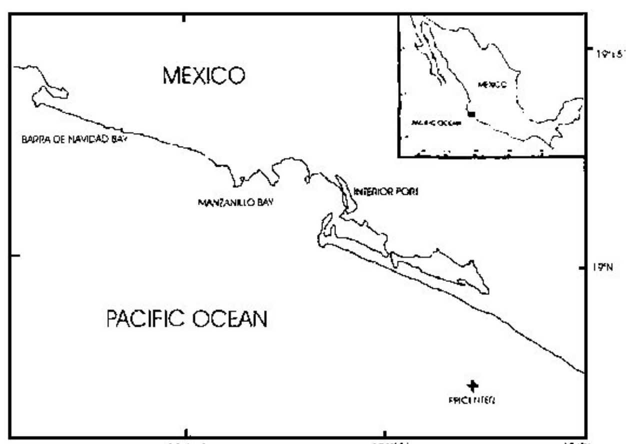

FIGURE 1. Manzanillo Bay and Manzanillo Lagoon. The epicenter of the earthquake is shown.

2. Data

Manzanillo Bay covers about 90 km^2. It is about 12 km wide and 7.5 km long (Fig. 2a). A maximum depth of 86 m is found at the open boundary with the Pacific Ocean. The lagoon, or Inner Port, is connected to the outer port by a narrow channel. The lagoon has a length of approximately 3200 m and an average width of 1000 m. The channel is 650 m long and its width is about 200 m. The lagoon has a maximum depth of 16 m (Fig. 2b). A tidal gauge is near the entrance of the channel [3].

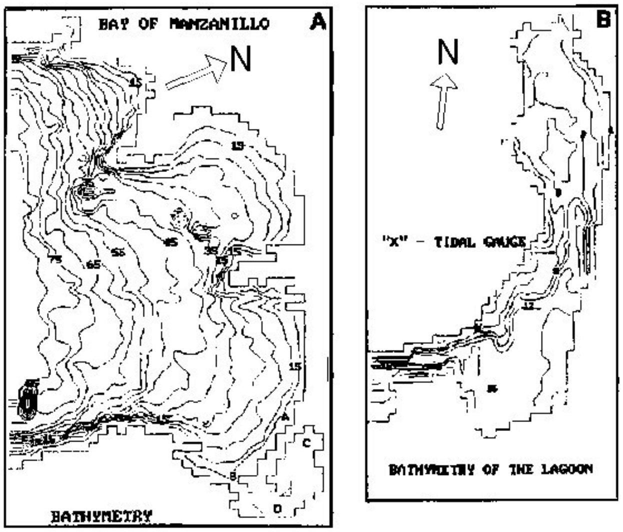

FIGURE 2. Bathymetry of the whole of Manzanillo Bay (A), and of the Inner Port or lagoon (B). The position of the tidal gauge is indicated.

The earthquake occurred during spring tides (Fig. 3). The tidal range was about 0.7 m. The depth of the mean sea level increased immediately after the earthquake and it returned slowly almost to its original level. The first shock wave produced abrupt changes in the sea surface elevation of about 2.0 m. Amplitudes of this order lasted for 7 hours (Fig. 3b). The signal decreased in approximately 24 hours to amplitudes it had before the earthquake occurred. This oscillation of around 36 minutes seems to be easily excited, since it is detected in all data recorded at this station. The permanent sea level change was about 2.5 cm, corresponding to coastal subsidence in the area between Manzanillo and Barra de Navidad generated by the earthquake.

A 6 minute window was used to estimate the periods of the tsunami oscillations. In Fig. 4, the power spectrum, estimated applying FFT, is displayed using 512 data points. The peak oscillation has a period of 36 minutes. Other important periods of about 27, 22, and 14 minutes can be observed.

3. Modes of resonance

Merian's formula [4]

$$T = \frac{2L}{n\sqrt{g\overline{H}}} \qquad \text{for } n = 1, 2, 3 \ldots, \qquad (1)$$

is valid for rectangular closed basins, where T is the period, L is the length of the basin, g is the acceleration of gravity and $\overline{H}$is the mean depth. We model the Inner Port as a rectangular closed basin. Assuming $L \approx$3200 m, g=9.8, $\overline{H}$= 8 m and n=1 we find a period of T = 12 minutes. From Merian's formula for rectangular semi-enclosed basins,

$$T = \frac{2L}{(n+\frac{1}{2})\sqrt{g\overline{H}}} \qquad \text{for } n = 0, 1, 2, 3 \ldots, \qquad (2)$$

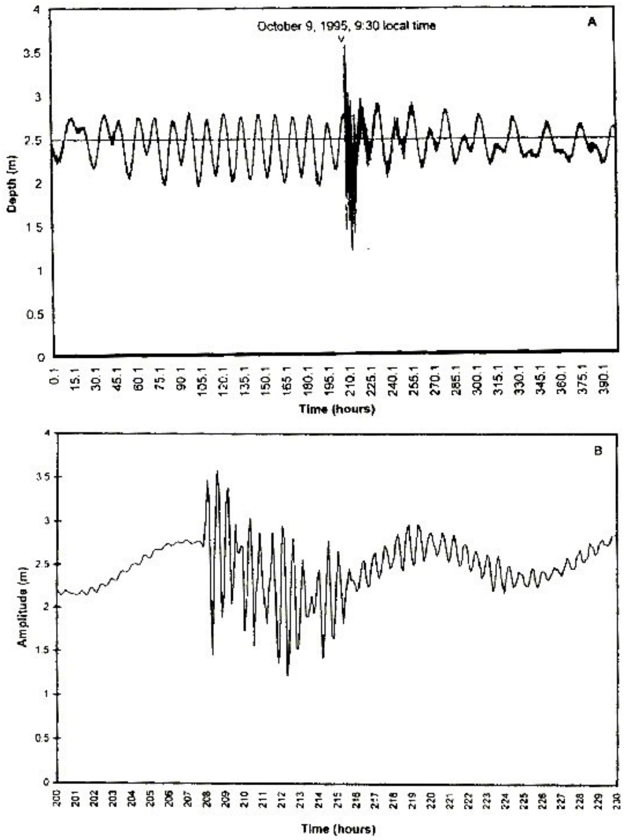

FIGURE 3. Time series of sea surface elevation observed at Manzanillo (A) for a period of 16 days and (B) for a period of 9.6 days.

we find for Manzanillo Bay, with $L \approx$7500 m, $\overline{H}$=40 m and $n = 0$ a period of approximately 25 minutes in agreement with the peak of 22 minutes appearing in Fig. 4. For transverse oscillations of Manzanillo Bay, we use Eq. (1), with $L \approx$12000 m; then T = 20 minutes. These results suggest that the dimensions of the bay and of the lagoon are inadequate to explain the dominant period of 36 minutes.

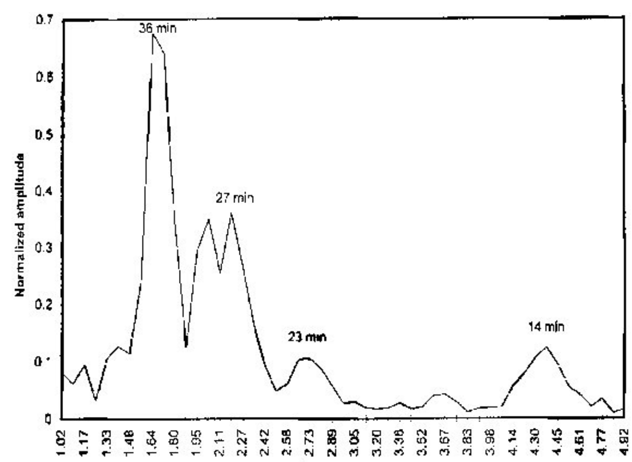

FIGURE 4. Power spectrum of the observed sea surface oscillation at the tidal gauge using Fast Fourier Transform. The amplitudes are normalized.

The position of the tidal gauge at the entrance to the channel suggests that the lagoon as well as the channel should both influence the period of oscillation. Each of these water bodies has a fundamental period of resonance on the order of 12 and 3.5 minutes respectively. Let us consider a combined system such as that described in Fig. 5. Such connected systems have additional free oscillations called "pumping" or Helmholz modes, due to periodic exchange of mass through the channel [5]. Using the formula

$$T_H = \frac{2\pi}{\sqrt{\frac{b}{B}\frac{g\overline{H}}{L_1 L_2}}}, \tag{3}$$

for connected systems [4,6], a rough estimate of Helmholz may be obtained. Let L_1= 650 m, L_2= 3200 m, b= 150 m, B= 800 m, $c = \sqrt{g\overline{H}}$ and $\overline{H}$= 8 m, we find a period of 34 to 38 minutes. This preliminary result agrees with the period of the oscillation observed during the earthquake.

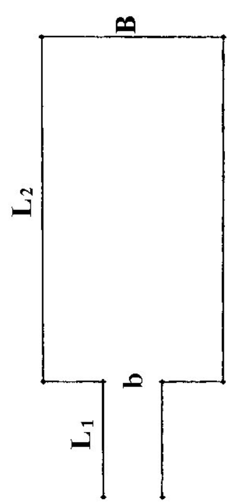

FIGURE 5. Idealized rectangular lagoon and channel systemused to estimate the Helmholz mode of resonance.

4. Numerical simulation

We estimate the modes of resonance of Manzanillo Bay from a bidimensional, non-linear, semi-implicit numerical shelf model [7]. This model has been used for the North Sea [8] and the Gulf of California [9]. The equations of motion are

$$\frac{\partial U}{\partial t} + \frac{U}{(H+\zeta)}\frac{\partial U}{\partial x} + \frac{V}{(H+\zeta)}\frac{\partial U}{\partial y} - fV = -g(H+\zeta)\frac{\partial \zeta}{\partial x} + \frac{\partial}{\partial x}\left(A_H\frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_H\frac{\partial U}{\partial y}\right) + \tau_S^{(x)} - \tau_B^{(x)}, \tag{4}$$

$$\frac{\partial V}{\partial t} + \frac{U}{(H+\zeta)}\frac{\partial V}{\partial x} + \frac{V}{(H+\zeta)}\frac{\partial V}{\partial y} + fU = -g(H+\zeta)\frac{\partial \zeta}{\partial y} + \frac{\partial}{\partial x}\left(A_H\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_H\frac{\partial V}{\partial y}\right) + \tau_S^{(y)} - \tau_B^{(y)}, \tag{5}$$

and the equation of continuity is

$$\frac{\partial \zeta}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \tag{6}$$

where $U$and V are the horizontal components of transport in the x and y directions, $f = 2\Omega sin\phi$ is the Coriolis parameter, Ω= 7.29×10^{-5} is the angular velocity of the Earth, ϕ is the latitude,ζ is the sea surface elevation, t is the time, $H$is the water depth and A_H is the coefficient of eddy viscosity. The parameters $\tau_S^{(x)}$, $\tau_B^{(x)}$, $\tau_S^{(y)}$ and $\tau_B^{(y)}$ are as follows:

$$\tau_S^{(x,y)} = \lambda_W(W^{(x)}, W^{(y)})\sqrt{W^{(x)2} + W^{(y)2}}, \tag{7}$$

$$\tau_B^{(x,y)} = \frac{r}{(H+\zeta)^2}(U,V)\sqrt{U^2+V^2}, \tag{8}$$

where $\lambda_W = 3.2\times 10^{-6}$ is a constant, $W^{(x)}$ and $W^{(y)}$ are the components of the wind speed in the x and y directions and $r = 0.003$ is the coefficient of friction. In the present study no wind forcing is considered; therefore $W^{(x)} = W^{(y)} = 0$. For closed boundaries the condition $V_n = 0$ obtains. V_n is the component of the velocity normal to the boundary. For open boundaries we have

$$\frac{\partial V_n}{\partial x_n} = 0, \tag{9}$$

where x_n is a coordinate normal to the open side of the bay. At closed boundaries a semi-slip condition is applied. The semi-enclosed region is forced at the open boundary by a wave

$$\zeta(t) = A\cos(\omega t - \Phi), \tag{10}$$

where A is the amplitude, ω is the frequency and Φ is the phase of the forcing signal.

The velocities may be vertically integrated in the form

$$\overline{u} = \frac{1}{(H+\zeta)}\int_{-H}^{\zeta} udz, \tag{11}$$

where u is the velocity in the x direction and $\overline{u}$ the corresponding vertical average. The derivation of the energy equation is given by Trepka [10] and Zahel [11]. The energy equation is

$$\frac{d(H(\overline{u}^2+\overline{v}^2)/2)}{dt} + \frac{d(g\zeta^2/2)}{dt} + r(\overline{u}^2+\overline{v}^2)^{\frac{3}{2}} - A_H\left(\overline{u}\nabla^2\overline{u} + \overline{v}\nabla^2\overline{v}\right) + \left(\frac{\partial(H\overline{u}\zeta)}{\partial x} + \frac{\partial(H\overline{v}\zeta)}{\partial y}\right) = 0, \tag{12}$$

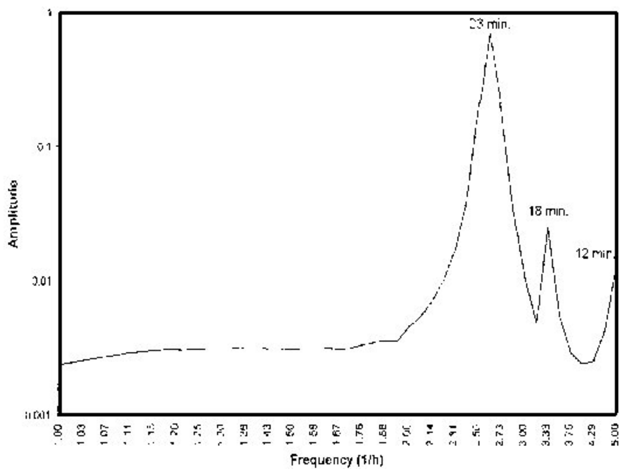

FIGURE 6. Spectrum of the Bay of Manzanillo from numerical modeling.

FIGURE 7. Spectrum of the lagoon + channel configuration predicted from numerical modeling.

where the first term represents the time change of kinetic energy, the second term is the potential energy, the third term is the rate of dissipation of energy by bottom friction, the fourth term is the rate of dissipation of energy by eddy viscosity and the fifth term is the energy flux across the open boundary. The integration of all energy terms over the area has a periodic variation. If the integrated terms are averaged over the period of the forcing wave, the resulting mean values must satisfy the relations

$$\frac{d(\overline{E}_k + \overline{E}_p)}{dt} = \overline{E}_d + \overline{E}_v + \overline{E}_f = 0, \qquad (13)$$

where $\overline{E}_k$ is the average kinetic energy in the basin, $\overline{E}_p$ is the average potential energy, $\overline{E}_d$ is the mean rate of dissipation of energy by bottom friction, $\overline{E}_v$ is the mean rate of dissipation of energy by eddy viscosity and $\overline{E}_f$ is the mean flux of energy across the open boundary. Obviously the total mean dissipation rate $\overline{E}_D = \overline{E}_d + \overline{E}_v$ must be balanced by the mean flux of energy $\overline{E}_f$; thus $\overline{E}_f = \overline{E}_D$.

For each frequency, we calculate the average flux of energy $\overline{E}_f$ into the system. The amplitude of the forcing waves is taken to be constant. The spectrum is constructed by plotting frequency against $\overline{E}_D$ or $\overline{E}_f$. In the first simulation, we compute the spectrum of Manzanillo Bay, for waves with periods between 12 and 60 minutes. All waves had an amplitude of 0.2 m. In Fig. 6 the resulting normalized spectrum is displayed. We find a large energy in waves with periods of about 23 minutes, which agrees with the preliminary estimates from Eq. (2). We suggest that this oscillation is the fundamental mode of resonance of Manzanillo Bay. It differs markedly from T = 36 minutes as observed in Fig. 3. Now we carry out a simulation for the composite basin formed by the lagoon and the channel. The result is shown in Fig. 7. This figure resembles the spectrum obtained from the observations (see Fig. 4). A peak with a period of 36 minutes appears in Fig. 7. Thus, the lagoon and the channel constitute the appropriate geometry to produce a Helmholz mode of resonance with a period similar to that produced by the earthquake. There is also a good agreement with other observed modes of resonance or seiches, specially with the frequencies of 1.17 and 2.73 cycles per hour.

In Fig. 8, we show the predicted time series of the rates of dissipation by bottom friction and viscosity of tidal energy, and flow of energy across the open boundary. The energy flux across the open boundary of Manzanillo Bay with the Pacific Ocean is much larger than the dissipation rates, but the total rate of dissipation of energy $\overline{E}_D$ is balanced with the mean energy flux $\overline{E}_f$. We estimate that about 80% of $\overline{E}_D$ was dissipated inside the lagoon. Fig. 9 provides a qualitative description of the time series of sea surface elevation and currents. For this calculation, Manzanillo Bay was forced at the open boundary by a wave with an amplitude of 1 meter. The positions of points A, B, C and D are shown in Fig. 2a. Note the large phase lag in sea surface elevation between the Bay and the Lagoon. In Helmholz resonance, the phase lag is due to the time it takes to fill and empty the lagoon. In Manzanillo Bay, changes of sea surface elevation are roughly sinusoidal, but inside the lagoon the friction effects are reflected by the distortion of the curves. These double peaks arise due to higher harmonics M_4 and M_6 generated by nonlinear friction terms in the equations of motion. Within the lagoon, the velocity at different points is mostly in phase (Fig. 10). The lag between the southern and northern side is 3.5 minutes or 35°. Asymmetry of the time series of velocities in the channel is due to nonlinear effects from bottom friction. The velocity at shallow point A is periodic but not sinusoidal. The predicted velocities in the channel are very high, in agreement with reported observations of strong currents.

5. Discussion and conclusions

Tsunamis in Manzanillo Bay are well observed. Farreras [12] discusses tsunamis in Manzanillo Port in 1957, 1964, 1965, 1968, 1975, 1976. The 1995 earthquake generated oscillations with periods of approximately 14, 23, 27 and 36 minutes in the lagoon. Filonov [13] calculated the principal oscillations before, during and after the passage of the tsunami waves at Barra de Navidad, 70 kilometers NW of Manzanillo.

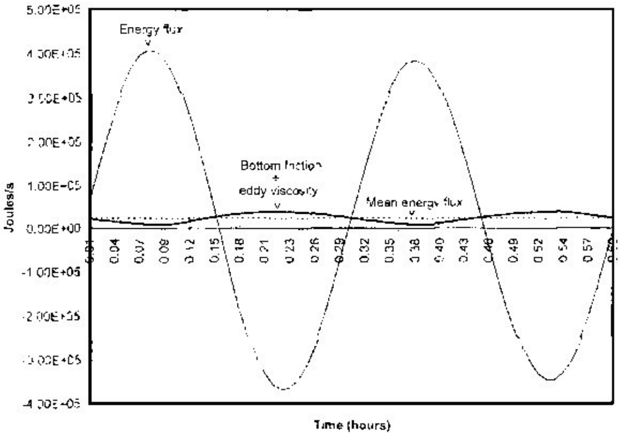

FIGURE 8. Predicted time series of the dissipation rates of energy, of the energy flux at the open boundary of the Bay of Manzanillo and of the mean energy flux.

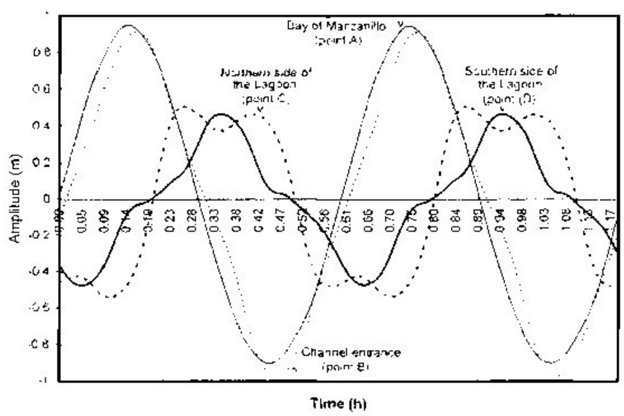

FIGURE 9. Predicted time series of sea surface elevation at different points of the system.

Before the earthquake periods of 40 and 25 minutes were dominant. The spectrum at Barra de Navidad showed periods of 25, 13, 8 and 6 minutes. The 36 minute oscillation does not appear in Filonov's results. It must be a local response of the lagoon-channel configuration in Manzanillo. In Helmholz modes most of the kinetic energy is concentrated in the connecting channel [14]. This is due to the fact that the Helmholz resonance is associated with the time to fill up or empty the lagoon [15]. In Fig. 10, the velocity is higher in the channel than in the lagoon, which suggests dominance of Helmholz resonance. Our numerical experiment also suggests that the channel-lagoon configuration shows a maximum of energy flow at a period of 36 minutes. This result agrees well with the period of the signal observed in the lagoon.

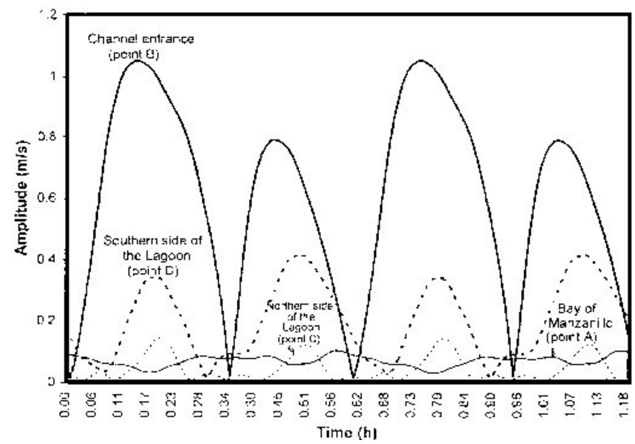

FIGURE 10. Predicted time series of the absolute value of the velocity at different points.

In the Port of Manzanillo, the ships at anchor oscillate and tend to collide. This tendency might be controlled by modifying the channel, thus increasing the period of the Helmholz resonance. The type of study presented in this work may be useful in understanding the physics of Manzanillo Bay and to correct the present design of the port.

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