Trace anomaly for 4D higher derivative scalar-dilaton theory

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Trace anomaly for conformally invariant higher derivative 4D scalar-dilaton theory is obtained by means of calculating divergent part of one-loop effective action for such system. Its applications are briefly mentioned.

Keywords: Quantum field theory; quantum gravity; conformal anomaly

Se obtiene la anomalía de traza para el sistema escalar-dilaton en cuatro dimensiones, considerando una teoría con derivadas altas e invariancia conforme. Se mencionan algunas aplicaciones.

Descriptores: Tería cuántica de campo; gravitación cuántica; anomalía conforme

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The inflationary brane-world scenario at the early universe based on a trace anomaly induced effective action in the frame of Randall-Sundrum compactification [1] has been suggested in Ref. 2. However, before developing this scenario is necesary to calculate conformal anomaly of brane matter.

Conformally invariant theories are very interesting objects. On the classical level, conformal symmetry is manifested in the fact that the stress-energy tensor for the system is traceless. But at the quantum level, after applying the regularization and (or) renormalization, such symmetry is broken. This fact leads to so-called Weyl, conformal or trace anomaly. Once the symmetry is broken, the next step is to integrate this anomaly for finding a finite (non-local) effective action and then add it to classical gravity for describing the effects emerging from quantum theory.

So, it is well-known that conformally invariant field theories give rise to trace anomaly at quantum level [3]. For the 4D higher derivative scalar field theory it was found that trace anomaly can be obtained from the following action:

$$S = \int d^4x \, \sqrt{-g} \, \varphi \nabla^4 \varphi,$$

where

$$\nabla^4 \equiv \Box^2 - 2R^{\mu\nu}\nabla_\mu\nabla_\nu + \frac{2}{3}R\,\Box - \frac{1}{3}(\nabla_\mu R)\nabla^\mu,$$

$\Box \equiv \nabla_\mu \nabla^\mu$ and[a] φ with dimension l^0; ∇^4 has the unique conformally covariant structure for a fourth-order differential operator, and it is self-adjoint [4]. It is interesting that conformal anomaly can be found also in the case of external gravity-dilaton background: Nojiri and Odintsov found a similar situation for 2D or 4D conformally invariant scalar-dilaton system [5].

In this work we search for the trace anomaly for 4D higher derivative scalar-dilaton theory by calculating the divergent part of the effective action for such theory.

Thus, first we must calculate trace anomaly. For this, it is helpful to remember that effective action is constructed in such a way that its variational derivative with respect to metric tensor give us the mean value of stress-energy tensor for the conformally invariant system which is under consideration. Thus, the form of trace anomaly corresponds with the divergent part of such effective action [3]. Then, we calculate one-loop effective action for theory (1) and substract from it its divergent part.

The action for higher derivative dilaton coupled scalar is

$$S = \int d^4x\sqrt{-g}\; f(\sigma)\varphi\nabla^4\varphi, \tag{1}$$

where φ is the scalar field, σ is the dilaton field and there is non-minimal coupling between scalar and dilaton fields. The self-interaction of dilaton is not considered here.

We shall calculate the one-loop efective action using the background fields (*i.e.*, classical fields). For this, we must split all fields in the action (1) into their classical and quantum parts, according to the rule $\sigma \rightarrow \tilde{\sigma}$, $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu}$ and $\varphi \rightarrow \varphi + \tilde{\varphi}$, where the fields with tilde are taken as being classical.

One-loop effective action is given by the expression [6]

$$\Gamma^{(1)} = \frac{i}{2}\mathrm{Tr}\ln\frac{\delta^2 S[\varphi]}{\delta\varphi(x)\delta\varphi(y)}.$$

Applying this to (1) we get

$$\Gamma^{(1)} = \frac{i}{2}\mathrm{Tr}\ln H,$$

where

$$H \equiv \Box^2 + L^\mu\nabla_\mu\Box - V^{\mu\nu}\nabla_\mu\nabla_\nu - N^\mu\nabla_\mu + U, \tag{2}$$

and we have adopted the notations

$$L^{\mu}=2\frac{(\nabla^{\mu}f)}{f},\qquad V^{\mu\nu}=\left[\frac{(\Box f)}{f}-\frac{2}{3}R\right]g^{\mu\nu}+2\left[R^{\mu\nu}+\frac{(\nabla^{\mu}\nabla^{\nu}f)}{f}\right],$$

$$N^{\mu}=2\frac{(\nabla^{\mu}\Box f)}{f}+\frac{1}{3}(\nabla^{\mu}R)-\frac{2}{3}\frac{(\nabla^{\mu}f)}{f}R,\qquad U=\frac{(\Box^{2}f)}{f}+\frac{1}{6}\frac{(\nabla^{\mu}f)}{f}(\nabla_{\mu}R)+\frac{(\nabla^{\mu}\nabla^{\nu}f)}{f}R_{\mu\nu}-\frac{1}{3}\frac{(\Box f)}{f}R. \quad (3)$$

The method for calculating the divergences of one-loop effective action is based on the universal-trace method [7]. The divergent part of effective action given by the trace-log of operator (2) has been calculated in Refs. 6 and 8 and it is

$$\mathrm{Tr}\ln H=\frac{2i}{\varepsilon}\mathrm{Tr}\Big\{-U+\frac{1}{4}L^{\mu}N_{\mu}+\frac{1}{6}L^{\mu}\nabla_{\mu}V-\frac{1}{6}L^{\mu}\nabla^{\nu}V_{\mu\nu}-\frac{1}{24}VL^{\mu}L_{\mu}-\frac{1}{12}V^{\mu\nu}L_{\mu}L_{\nu}+\frac{1}{2}P^{2}$$
$$+\frac{1}{12}S_{\mu\nu}S^{\mu\nu}+\frac{1}{6}R_{\mu\nu}R^{\mu\nu}-\frac{1}{24}R^{2}+\frac{1}{12}RV-\frac{1}{6}R^{\mu\nu}V_{\mu\nu}+\frac{1}{48}V^{2}+\frac{1}{24}V^{\mu\nu}V_{\mu\nu}+\frac{1}{60}F-\frac{1}{180}G\Big\}, \quad (4)$$

where $F\equiv C^{2}_{\mu\nu\rho\tau}$ is the square of the conformal Weyl tensor, G is the Gauss-Bonnet topological invariant and

$$P\equiv\frac{1}{6}R-\frac{1}{2}\nabla^{\mu}L_{\mu}-\frac{1}{4}L_{\mu}L^{\mu},\qquad V\equiv V^{\mu}_{\mu},$$

$$S_{\mu\nu}\equiv\nabla_{[\nu}L_{\mu]}+\frac{1}{2}L_{[\nu}L_{\mu]},\qquad A_{[\mu}B_{\nu]}\equiv\frac{1}{2}(A_{\mu}B_{\nu}-A_{\nu}B_{\mu}).$$

Substituting relation (3) into (4) and carrying out elementary transformations we obtain

$$\mathrm{Tr}\ln H=\frac{2i}{\varepsilon}\mathrm{Tr}\Big\{-\frac{(\Box^{2}f)}{f}-\frac{1}{3}\frac{(\nabla_{\alpha}f)(\nabla^{\alpha}R)}{f}-R_{\mu\nu}\frac{(\nabla^{\mu}\nabla^{\nu}f)}{f}+\frac{1}{6}R\frac{(\Box f)}{f}+4\frac{(\nabla_{\alpha}f)(\nabla^{\alpha}\Box f)}{f^{2}}$$
$$-\frac{11}{3}\left(\frac{\nabla_{\alpha}f}{f}\right)^{2}\frac{(\Box f)}{f}-\frac{4}{3}R_{\mu\nu}\frac{(\nabla^{\mu}f)(\nabla^{\nu}f)}{f^{2}}-\frac{4}{3}\frac{(\nabla^{\mu}f)(\nabla^{\nu}f)(\nabla_{\mu}\nabla_{\nu}f)}{f^{3}}+\frac{19}{12}\left(\frac{\Box f}{f}\right)^{2}+$$
$$\frac{1}{6}\left(\frac{\nabla^{\mu}\nabla^{\nu}f}{f}\right)^{2}+\frac{1}{60}F-\frac{1}{180}G\Big\}. \quad (5)$$

In the limit $f(\sigma)=1$ (5) is reduced to

$$\mathrm{Tr}\ln H=\frac{2i}{60\varepsilon}\mathrm{Tr}\left(F-\frac{1}{3}G\right)$$

which (excluding surface terms) is the same result as the one obtained in Ref. 4.

The effective action is related with the trace of the energy momentum tensor $T_{\mu\nu}$ (see *e.g.*, Refs. 3, 4 and 6). According to these results, trace anomaly for theory (1) has the same structure as (5), *i.e.*

$$T\equiv\langle T^{\mu}{}_{\mu}\rangle=\mathrm{Tr}\Big\{-\frac{(\Box^{2}f)}{f}-\frac{1}{3}\frac{(\nabla_{\alpha}f)(\nabla^{\alpha}R)}{f}-R_{\mu\nu}\frac{(\nabla^{\mu}\nabla^{\nu}f)}{f}+\frac{1}{6}R\frac{(\Box f)}{f}+4\frac{(\nabla_{\alpha}f)(\nabla^{\alpha}\Box f)}{f^{2}}-\frac{11}{3}\left(\frac{\nabla_{\alpha}f}{f}\right)^{2}\frac{(\Box f)}{f}$$
$$-\frac{4}{3}R_{\mu\nu}\frac{(\nabla^{\mu}f)(\nabla^{\nu}f)}{f^{2}}-\frac{4}{3}\frac{(\nabla^{\mu}f)(\nabla^{\nu}f)(\nabla_{\mu}\nabla_{\nu}f)}{f^{3}}+\frac{19}{12}\left(\frac{\Box f}{f}\right)^{2}+\frac{1}{6}\left(\frac{\nabla^{\mu}\nabla^{\nu}f}{f}\right)^{2}+\frac{1}{60}F-\frac{1}{180}G\Big\}. \quad (6)$$

As usual in field theory, the energy-momentum tensor give us information about the kind of matter considered in the model [3, 6]. Thus, conformal anomaly as quantum effect is related with the quantum matter present in the theory. Furthermore, one can integrate (6) to find the anomaly induced effective action and take it as a quantum correction for a specific model (*e.g.*, FRW). It is also interesting that for some value of dilaton (defined to satisfy $T=0$) the conformal anomaly is absent. This indicates that such value of dilaton is kind of fixed point where conformal symmetry may be restored and then in that point the theory (1) becomes finite. Let us remark that without dilaton, *i.e.*, $f(\sigma)=1$, there is no way to avoid divergences in theory (1) [4].

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[a]. In this work we shall set $c, \hbar = 1$.

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