

Fashion with Snobs and Bandwagoners in a Three-Type Households and Three-Sector Neoclassical Growth Model

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Abstract

This study deals with interactions between fashion, economic growth and income and wealth distribution in a growth model of heterogeneous households with economic structure. The modelling of fashion and preference change is inspired by the economic model of fashion recently published by Giovino and Naimzada (2015). We introduce fashion consumption, fashion industry and preference change with regards to fashion goods into a general equilibrium dynamic growth model proposed by Zhang (2012, 2013). The economy consists of three groups of people (the snob, the bandwagoner, and the consumer) and three production (capital goods, consumer goods, and fashion) sectors. The motion of the economic system is described by a set of differential equations. For illustration, we simulate the motion of the economic system. We identify the existence of a unique stable equilibrium point. We also carry out comparative dynamic analysis. The comparative analyses provide some insights into the complexity of economic growth with fashion.

JEL Classification: C23, G21, L2.

Keywords: Fashion, Snob, Bandwagoner, Fashion Industry, Neoclassical Growth Theory, Income and Wealth Distribution.

Representación del consumo: Modelo de Crecimiento Neoclásico con Tres Factores

Resumen

El trabajo muestra las interacciones entre la moda, el crecimiento económico y la distribución del ingreso y la riqueza en un modelo de crecimiento de los hogares heterogéneos con la estructura económica. La modelación de la moda y la preferencia de cambio se basan en el modelo económico de la moda recientemente publicado por Giovino y Naimzada (2015). Introducimos el consumo de moda, industria de la moda y el cambio de preferencias con respecto a los artículos de moda en un modelo de equilibrio general dinámico crecimiento propuesto por Zhang (2012, 2013). La economía se compone de tres grupos de personas (el snob, el bandwagoner, y el consumidor) y tres sectores de producción (bienes de capital, bienes de consumo y moda). El movimiento del sistema económico es descrito por un conjunto de ecuaciones diferenciales. Se simula el movimiento del sistema económico. Identificamos la existencia de un punto de equilibrio estable único. Finalmente llevamos a cabo análisis de dinámica comparativa el cual proporcionan algunas ideas sobre la complejidad de crecimiento económico considerando la moda.

Clasificación JEL: C23, G21, L2.

Palabras Clave : Moda, Esnob, Bandwagoner, Industria de la Moda, Modelo Neoclásico de Crecimiento, Distribución del Ingreso y la Riqueza.

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1. Introduction

Daniels (1951) describes: “Fashion is important because it is in almost everything.” In modern times a fashion started in a metropolitan area in one country tends to rapidly globally spread over the world due to modern communications. Fashion has become a very important industry. According to Hemphill and Suk (2009: 1148), “It is the major output of a global business with annual U.S. sales of more than \$200 billion - larger than those of books, movies, and music combined. (...). It has provided economic thought with a canonical example in theorizing about consumption and conformity. Social thinkers have long treated fashion as a window upon social class and social change. Cultural theorists have focused on fashion to reflect on symbolic meaning and social ideals.” There are many studies about fashion and its interactions with other aspects of societies, even though the literature on formally modelling economics of fashion are not so many (*e.g.*, Pollak, 1976; Stigler and Becker, 1977; Abel, 1990; Karni and Schmeidler, 1990; Bianchi, 2002; Nakayama and Nakamura, 2004; Caulkins, *et al.*, 2007; Zhou, *et al.*, 2015). Simmel (1904) points out that fashion dynamics is enforced by common people’s imitation of elite. Giovinazzo and Naimzada (2015) build a model to investigate the dynamics of the fashion cycle described by Simmel (1904). They argue that the originality of Simmel’s thought has been lost in the modern literature on the subject. A main reason is that the... “models generally assume that preferences are exogenous and overlook the fact that individual tastes change in time, partly in line with choices previously made by the social group.” Nevertheless, there are only a few formal economic models which explore dynamic complexity of fashion within context of economic growth and development theory. An analytical reason for this slow development is that how to deal with heterogeneity of goods and households with microeconomic foundation are still challenging issues in economics. The lacking of a proper analytical framework means that it is difficult for economists to deal with fashion and wealth accumulation in an integrated theory.

As far as fashion and classification of the population are concerned, this paper is inspired by the economic model of fashion by Giovinazzo and Naimzada (2015). Their model is based on Benhabib and Day (1981). The population in their model is classed into the snob and the bandwagoner (Leibenstein, 1950). In their model there are one conspicuous good and one normal good. The bandwagoner’s preference is characterized by that a particular kind of consumption increases in association with the rise in the social group’s average consumption of the previous period. The snob’s preference for a certain purchase falls with the rise of the average collective consumption of the preceding period. We use this idea to take account of fashionable behavior and their impact on macroeconomic as well as microeconomic variable in a general equilibrium dynamic model proposed by Zhang (2012, 2013). It should be remarked that Shi (1999) builds a wealth accumulation model with fashion and two groups of agents. Fashion is treated as an externality due to dependence of individual agents’ time preference on the two groups’ per-capita consumption habit. In Shi’s approach stronger and more persistent fashion tends to generate oscillations in wealth. This study differs from Shi’s approach in that Shi employs the traditional Ramsey approach to household behavior and this study uses Zhang’s approach (Zhang, 1993, 2012). The paper is organized as follows.

Section 2 develops the growth model of wealth and income distribution among heterogeneous households with fashion. Section 3 examines dynamic properties of the model and simulates the model with three groups. Section 4 carries out comparative dynamic analysis. Section 5 concludes the study.

2. The Basic Model

We consider an economic system consisting of fashion, capital goods, and consumer goods sectors. The capital goods and consumer goods sectors are the same as in the Uzawa two sector model (Uzawa, 1961; Burmeister and Dobell 1970; and Barro and Sala-i-Martin, 1995). Services are classified as consumer goods. The fashion sector produces fashion goods. It should be noted that it is straightforward to further classify fashion goods as durable and instantly consumed goods. For simplicity of analysis, we consider fashion goods as instantly consumed goods. Capital goods are used as inputs in the three sectors. Capital depreciates at a constant exponential rate $\delta_k (0 < \delta_k < 1)$, which is independent of the manner of use. Households own assets of the economy and distribute their incomes to consume and save. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. Rather than classifying the population into the snob, the bandwagoner as in Giovinazzo and Naimzada (2015), we classify the population into the snob, the bandwagoner, and common consumer. The population consists of three groups, called consumers (who have no interest in fashion), the snob, and bandwagoner. Each group has a fixed population \bar{N}_j , indexed by $j = C, S, B$. Let prices be measured in terms of capital goods and the price of the commodity be unity. We denote the wage rates and rate of interest by $w_j(t)$ and $r(t)$ respectively. Let $p_C(t)$ and $p_F(t)$ denote prices of consumer and fashion goods, respectively. The total capital stock $K(t)$ is allocated between the three sectors. We use subscript index, i, s and f to stand for capital, consumer, and fashion goods sectors, respectively. We use $N_m(t)$ and $K_m(t)$ to stand for the labor force and capital stocks employed by sector m . The total population \bar{N} and total qualified labor supply N are

$$\bar{N} = \bar{N}_C + \bar{N}_S + \bar{N}_B, \quad N = h_C \bar{N}_C + h_S \bar{N}_S + h_B \bar{N}_B \quad (1)$$

in which h_j is the human capital of group j . The assumption of labor force being fully employed implies

$$N_i(t) + N_s(t) + N_f(t) = N. \quad (2)$$

2.1 The Capital Goods Sector

Let $F_m(t)$ stand for the production function of sector $m, m = i, s, f$. The production function of capital goods sector is specified as follows

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \quad (3)$$

where A_i, α_i and β_i are positive parameters. The marginal conditions for the capital goods sector are given by

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \quad (4)$$

where w_t is the wage rate of per qualified labor input. It should be noted that the wage rate of group j is

$$w_j(t) = h_j w(t) \quad (5)$$

2.2 Consumer Goods and Fashion Goods Sectors

The production functions of the two sectors are

$$F_m(t) = A_m K_m^{\alpha_m}(t) N_m^{\beta_m}(t) \alpha_m + \beta_m = 1 \alpha_m \beta_m > 0, \quad m = s, f, \quad (6)$$

where A_m, α_m and β_m are the technological parameters of the two sectors. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_m p_m(t) F_m(t)}{K_m(t)}, \quad w(t) = \frac{\beta_m p_m(t) F_m(t)}{N_m(t)} \quad (7)$$

2.3 Current Income and Disposable Income

In this study, we use the approach to modeling behavior of households by Zhang (1993). Let $\bar{k}_j(t)$ stand for per capita wealth of group j . We have $\bar{k}_j(t) = \bar{K}_j(t) / \bar{N}_j$, where $\bar{K}_j(t)$ is the total wealth held by group j . Per capita current income from the interest payment $r(t) \bar{k}_j(t)$ and the wage income is

$$y_j(t) = r(t) \bar{k}_j(t) + w_j(t).$$

The per capita disposable income is the sum of the current disposable income and the value of wealth. That is

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \quad (8)$$

2.4 Budget Constraints with Fashion Goods

The disposable income is used for saving and consumption. The representative household from group j would distribute the total available budget between savings $s_j(t)$, consumer goods $c_j(t)$, and fashion goods $f_j(t)$. It should be noted $f_C(t) = 0$. As far as no confusion occurs, we similarly use indexes without explicitly distinguishing groups. The budget constraints are

$$p_C(t) c_j(t) + s_j(t) + p_F(t) \bar{f}_j(t) = \hat{y}_j(t). \quad (9)$$

2.5 Utility Functions with Fashion

We assume that utility level $U_j(t)$ is dependent on $c_j(t)$, $s_j(t)$ and $f_j(t)$ as follows

$$U_j(t) = c_j^{\xi_{j0}(t)}(t) s_j^{\lambda_{j0}(t)}(t) f_j^{\theta_{j0}(t)}(t), \quad \xi_{j0}(t), \lambda_{j0}(t) > 0, \theta_{j0}(t) \geq 0 \quad (10)$$

where $\xi_{j0}(t)$, is the propensity to consume consumer goods, $\lambda_{j0}(t)$ the propensity to save, and $\theta_{j0}(t)$ is group j 's propensity to consume fashion goods (remark, $\theta_{C0}(t) = 0$). Following Giovinnazzo and Naimzada (2015), we consider the propensities of the snobs and the bandwagoners for consuming fashions endogenous. We will specify preference changes later on. It should be noted that although there are many heterogeneous-households growth models with endogenous wealth accumulation, the heterogeneity in most of those studies is by the differences in the initial endowments of wealth among different types of households rather than in preferences (see, for instance, Chatterjee, 1994; Caselli and Ventura, 2000; Maliar and Maliar, 2001; Penalosa and Turnovsky, 2006; and Turnovsky and Penalosa, 2006). Different households are essentially homogeneous in the sense that all the households have the same preference utility function in the approach.

2.6 Optimal Household Behavior

Maximizing the utility function subject to (9) yields

$$p_C(t) c_j(t) = \xi_j(t) \hat{y}_j(t) s_j(t) = \lambda_j(t) \hat{y}_j(t) p_F(t) f_j(t) = \theta_j(t) \hat{y}_j(t), \quad (11)$$

where

$$\begin{aligned} \xi_j(t) &\equiv \rho_j(t) \xi_{j0}(t) \lambda_j(t) \equiv \rho_j(t) \lambda_{j0}(t) \theta_j(t) \\ &\equiv \rho_j(t) \theta_{j0}(t) \rho_j(t) \equiv \frac{1}{\xi_{j0}(t) + \lambda_{j0}(t) + \theta_{j0}(t)}. \end{aligned}$$

2.7 A Simplified Literature Review on Fashion Dynamics and Habit Formation

Before we specify how $\theta_{S0}(t)$ and $\theta_{B0}(t)$ change over time, we review the approach to fashion in discrete time accepted by Giovinnazzo and Naimzada (2015). Their model is based on Benhabib and Day (1981). The population is classed into the snob and the bandwagoner, subscripted by $i = SB$. There are one conspicuous good and one normal good. The bandwagoner's preference is characterized by that a particular kind of consumption increases in association with the rise in the social group's average consumption of the previous period. The snob's preference for a certain purchase falls with the rise of the average collective consumption of the preceding period. The utility function is specified as

$$U(t) = x_i^{\alpha_i(t)}(t) y_i^{1-\alpha_i(t)}(t) \quad (12)$$

where x_i and y_i are respectively the consumption levels of conspicuous good and normal good by i . The budgets with constant prices are

$$p x_i(t) + q y_i(t) = m_i(t). \quad (13)$$

It should be noted that price is fixed in the approach. As the model is partial (and for short-term problems), the fixed prices assumption is acceptable. The

population is normalized unity. The population shares of the bandwagoners and the snobs are respectively ω and $1 - \omega$. The average conspicuous consumption of period t is

$$\bar{x}(t) = \omega x_B(t) + (1 - \omega) x_S(t) \quad (14)$$

The snob's preference change is given by $\alpha_S(t) = f_S(\bar{x}(t - 1))$, where $f_S(\bar{x}(t - 1))$ is decreasing. The bandwagoner's preference change is given by $\alpha_B(t) = f_B(\bar{x}(t - 1))$, where $f_B(\bar{x}(t - 1))$ is increasing. It is straightforward to conduct dynamic analysis of the model with some specified preference change functions.

As in Giovanazzo and Naimzada (2015), we define the average conspicuous consumption in time t as

$$\bar{f}(t) = n f_B(t) + (1 - n) f_S(t), \quad (15)$$

where

$$n = \frac{\bar{N}_B}{\bar{N}_B + \bar{N}_S}.$$

Each individual takes the average behavior of large populations $\bar{f}(t)$ as given when he makes decision. Following Giovanazzo and Naimzada, we assume that the snobs' and the bandwagoners' propensities to consume fashions as follows

$$\theta_j(t) = \Lambda_j(\bar{f}(t)), \quad j = B, S. \quad (16)$$

We also generally assume that $\theta_S(t)$ ($\theta_B(t)$) is decreasing (increasing) in $\bar{f}(t)$. As shown in the appendix, we can describe behavior of the model with any type of preference changes. It should be noted that fashion dynamics is a far more complicated phenomenon than we can accurately describe in one model. Hemphill and Suk (2009: 1148) describe this widely spread complex processes as follows, "People flock to ideas, styles, methods, and practices that seem new and exciting, and then eventually the intensity of that collective fascination subsides, when the newer and hence more exciting emerge on the scene. Participants of social practices that value innovation are driven to partake of what is "original," "cutting edge," "fresh," "leading," or "hot." But with time, those qualities are attributed to others, and another trend takes shape." In fact, there are different theories of fashion with regards to types of fashions. For instance, Miller *et al.* (1993) summarize types of models in theory of fashion developed more than two decades ago as follows: (i) external-individual models - aesthetic perceptions and learning model; (ii) external-social models - social conflict model, art movement model, ideas of beauty model, mass market model, market infrastructure model; (iii) internal-individual models - demand model, scarcity-rarity model, conspicuous consumption model, individualism-centered model, conformity-centered model, uniqueness motivation model; (iv) internal-social models - trickle-down theory, collective behavior theory, adoption and diffusion model, symbolic communication model, subcultural leadership, model, spatial diffusion model, historical resurrection model, and historical continuity model. In our approach we use a highly

aggregated variable called fashion good to describe fashion dynamics. This obviously can provide only limited insights into complexity of fashion dynamics.

2.8 Wealth Accumulation

According to the definition of $s_j(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t) \quad (17)$$

This equation states that the change in wealth is equal to saving minus dissaving.

2.9 Demand and Supply of the Three Sectors

The demand and supply equilibrium for the consumer goods sector is

$$c_C(t) \bar{N}_C + c_S(t) \bar{N}_S + c_B(t) \bar{N}_B = F_s(t) . \quad (18)$$

The demand and supply equilibrium for the fashion sector is

$$f_S(t) \bar{N}_S + f_B(t) \bar{N}_B = F_f(t) \quad (19)$$

As output of the capital goods sector is equal to the depreciation of capital stock and the net savings, we have

$$S(t) - K(t) + \delta_k K(t) = F_i(t) \quad (20)$$

where

$$\begin{aligned} S(t) &\equiv s_C(t) \bar{N}_C + s_S(t) \bar{N}_S + s_B(t) \bar{N}_B, \\ K(t) &= \bar{k}_C(t) \bar{N}_C + \bar{k}_S(t) \bar{N}_S + \bar{k}_B(t) \bar{N}_B \end{aligned}$$

2.10 Capital being Fully Utilized

Total capital stock $K(t)$ is allocated to the three sectors

$$K_i(t) + K_s(t) + K_f(t) = K(t) \quad (21)$$

We completed the model. The model is structurally general in the sense that some well-known models in economics can be considered as its special cases. For instance, if the population is homogeneous and there is no fashion industry, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). It is structurally similar to the Walrasian general equilibrium model (note that in our approach the population can be classified into any number of groups) if the wealth is fixed and depreciation is neglected. The new aspect of this paper is the introduction of fashion into the general equilibrium growth model proposed by Zhang.

3. The Dynamics and its Properties

As the dynamic system is nonlinear and highly dimensional with wealth accumulation, it is difficult to get explicitly analytical properties. We can conduct simulation to follow the motion of the dynamic system. The following lemma shows that the dimension of the dynamical system is the same as the

number of groups. We also provide a computational procedure for calculating all the variables at any point in time. First, we introduce a new variable $z(t)$.

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}. \quad (22)$$

3.1 Lemma

The motion of the economic system is determined by differential equations with $z(t)$, $\bar{k}_B(t)$ and $\bar{k}_S(t)$ as the variables

$$\begin{aligned} \dot{z}(t) &= \tilde{\Lambda}_z(z(t), \bar{k}_B(t), \bar{k}_S(t)), \\ \dot{\bar{k}}_B(t) &= \tilde{\Lambda}_z(z(t), \bar{k}_B(t), \bar{k}_S(t)), \\ \dot{\bar{k}}_S(t) &= \tilde{\Lambda}_z(z(t), \bar{k}_B(t), \bar{k}_S(t)), \end{aligned} \quad (23)$$

in which $\tilde{\Lambda}_j(t)$ are unique functions of $z(t)$, $\bar{k}_B(t)$ and $\bar{k}_S(t)$ defined in the appendix. At any point in time the other variables are unique functions of $z(t)$, $\bar{k}_B(t)$ and $\bar{k}_S(t)$ determined by the following procedure: $r(t)$ and $w_j(t)$ by (A2) $\rightarrow p_C(t)$ and $p_F(t)$ by (A3) $\rightarrow \bar{f}(t)$ by (A12) $\rightarrow \theta_j(t)$, $\rho_j(t)$, $\xi_j(t)$ and $\lambda_j(t)$ by their definitions $\rightarrow \bar{k}_C(t)$ by (A14) $\rightarrow \hat{y}_t(t)$ by (A9) $\rightarrow K(t) = \sum_j \bar{k}_j(t) \bar{N}_j \rightarrow c_j(t)$, $s_j(t)$ and $f_t(t)$ by (10) $\rightarrow w_t(t) = w(t)/h_1 \rightarrow K_f(t)$ by (A5) $\rightarrow N_s(t)$ by (A7) $\rightarrow N_i(t)$ by (A6) $\rightarrow K_j(t)$ by (A1) $\rightarrow F_j(t)$ by the definitions.

The lemma gives a computational procedure for following the motion of the economic system. We specify the preference changes as follows

$$\theta_{B0}(t) = \bar{\theta}_B + \tilde{\theta}_B \bar{f}(t), \quad \theta_{S0}(t) = \bar{\theta}_S - \tilde{\theta}_S \bar{f}(t), \quad (24)$$

where $\tilde{\theta}_j$ and $\bar{\theta}_j$ are non-negative parameters. To satisfy this requirement, we have to specify how $\xi_{j0}(t)$ and $\lambda_{j0}(t)$ $j = B, S$ are changed. We specify

$$\begin{aligned} \xi_{B0}(t) &= \bar{\xi}_B - \omega_B \tilde{\theta}_B \bar{f}(t) \quad \lambda_{B0}(t) = \bar{\lambda}_B - (1 - \omega_B) \tilde{\theta}_B \bar{f}(t), \quad 0 \leq \omega_B \leq 1, \\ \xi_{S0}(t) &= \bar{\xi}_S + \omega_S \tilde{\theta}_S \bar{f}(t) \quad \lambda_{S0}(t) = \bar{\lambda}_S + (1 - \omega_S) \tilde{\theta}_S \bar{f}(t), \quad 0 \leq \omega_S \leq 1, \end{aligned} \quad (25)$$

where ω_B and ω_S are “weight” parameters. For instance, if ω_B is small, it simply implies that a change in the propensity to consume fashion is associated with a relatively small change in the propensity to consume consumer goods and a relatively large change in the propensity to save. Under (24) and (25) we have

$$\xi_j(t) \equiv \rho_j \xi_{j0}(t) \lambda_j(t) \equiv \rho_j \lambda_{j0}(t) \theta_j(t) = \rho_j \theta_{j0}(t) \rho_j \equiv \frac{1}{\xi_j + \bar{\lambda}_j + \bar{\theta}_j} j = B, S. \quad (26)$$

We have that $\rho_j(t)$ are constant. From (11) and (10) we have

$$\bar{f}(t) = n \frac{\rho_B \theta_{B0}(t) \hat{y}_B(t)}{p_F(t)} + (1 - n) \frac{\rho_S \theta_{S0}(t) \hat{y}_S(t)}{p_F(t)}, \quad (27)$$

where we use (26). Inserting (24) in (27) yields.

$$\bar{f}(t) = \frac{n \bar{\theta}_B \rho_B \hat{y}_B(t) + (1 - n) \bar{\theta}_S \rho_S \hat{y}_S(t)}{p_F(t) + (1 - n) \bar{\theta}_S \rho_S \hat{y}_S(t) - n \bar{\theta}_B \rho_B \hat{y}_B(t)} \quad (28)$$

We specify parameter values as follows

$$A_i = 1 \quad A_s = 1, \quad A_f = 1.3, \quad \alpha_i = 0.31, \quad \alpha_s = 0.3, \quad \alpha_f = 0.36, \quad \delta_k = 0.07,$$

$$\begin{aligned} \begin{pmatrix} N_C \\ N_B \\ N_S \end{pmatrix} &= \begin{pmatrix} 100 \\ 10 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} h_C \\ h_B \\ h_S \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}, \quad \begin{pmatrix} \lambda_{C0} \\ \lambda_{B0} \\ \lambda_{S0} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.9 \\ 0.95 \end{pmatrix}, \\ \begin{pmatrix} \xi_{C0} \\ \xi_{B0} \\ \xi_{S0} \end{pmatrix} &= \begin{pmatrix} 0.15 \\ 0.11 \\ 0.1 \end{pmatrix}, \quad \begin{pmatrix} \bar{\lambda}_{10} \\ \bar{\lambda}_{20} \\ \bar{\lambda}_{30} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.75 \\ 0.7 \end{pmatrix}, \quad (29) \\ \begin{pmatrix} \bar{\theta}_B \\ \bar{\theta}_S \end{pmatrix} &= \begin{pmatrix} 0.01 \\ 0.01 \end{pmatrix}, \quad \begin{pmatrix} \tilde{\theta}_B \\ \tilde{\theta}_S \end{pmatrix} = \begin{pmatrix} 0.001 \\ 0.001 \end{pmatrix}, \quad \begin{pmatrix} \omega_B \\ \omega_S \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}. \end{aligned}$$

The population of the consumer is largest, while the population of the bandwagoner is the next. The capital goods and consumer goods sector's total productivities are respectively 1.3 and 1. We specify the values of the parameters, α_j in the Cobb-Douglas productions for the capital goods and consumer goods sectors approximately equal to 0.3 (for instance, Miles and Scott, 2005; Abel *et al.*, 2007). The depreciation rate of physical capital is specified at 0.05. A change in the propensity to consume fashion is associated with the equal changes (in the opposite direction) in the propensities to consume and to save. We specify the initial conditions as follows

$$z(0) = 0.06, \quad \bar{k}_2(0) = 10, \quad \bar{k}_3(0) = 4.$$

The motion of the variables is plotted in Figure 1. In Figure 1, the national income is

$$Y(t) = F_i(t) + p_C(t) F_s(t) + p_F(t) F_f(t). \quad (30)$$

Due to the fixed positions of the initial state, the national output and wealth/capital fall over time. The output level of the capital goods sector and fashion sector rise and the output level of the consumer goods sector fall overtime. The price of consumer goods falls. The rate of interest rises in association with falling wage rates. The prices of consumer goods and fashion goods and the propensities to consume fashion goods change slightly. Labor force is shifted from the consumer goods sector to the other two sectors. The capital input of the consumer goods sector is reduced and the capital inputs of the two other sectors are increased. The snob consume slightly less fashion goods and the bandwagoner more fashion goods. The average consumption level of fashion goods is increased over time. The snob consumes less consumer goods and has less wealth. The bandwagoner consumes more consumer goods and has more wealth. The consumer consumes less consumer goods and has less wealth.

Figure 1. The Motion of the of the Economic System

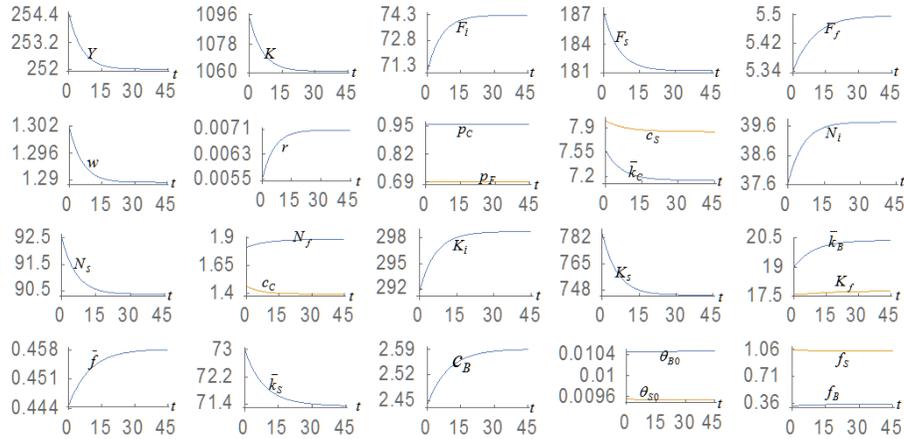


Figure 1 shows that that all the variables become stationary in the long term. This implies the existence of an equilibrium point. The simulation confirms that the system has a unique equilibrium. We list the equilibrium values in (25)

$$\begin{aligned}
 Y &= 252, K = 1061.7, F_i = 74.28, F_s = 181.24, F_f = 5.5, \bar{f} = 0.46, w_C = 1.29, \\
 w_B &= 2.58, w_S = 7.74, r = 0.007, p_C = 0.96, p_F = 0.69, N_i = 39.75, \\
 N_s &= 90.37, N_f = 1.89, K_i = 298.8, K_s = 744.64, K_f = 17.76, \bar{k}_C = 7.15, \\
 \bar{k}_B &= 20.38, \bar{k}_S = 71.34, c_C = 1.4, c_B = 2.59, c_S = 7.84, \theta_{B0} = 0.011, \\
 \theta_{S0} &= 0.01, f_B = 0.343, f_S = 1.306
 \end{aligned}$$

It is straightforward to calculate the three eigenvalues as follows

$$\{-0.15, -0.12, -0.10\}.$$

The eigenvalues are real and negative. The unique equilibrium is locally stable. This guarantees the validity of exercising comparative dynamic analysis.

4. Comparative Dynamic Analysis

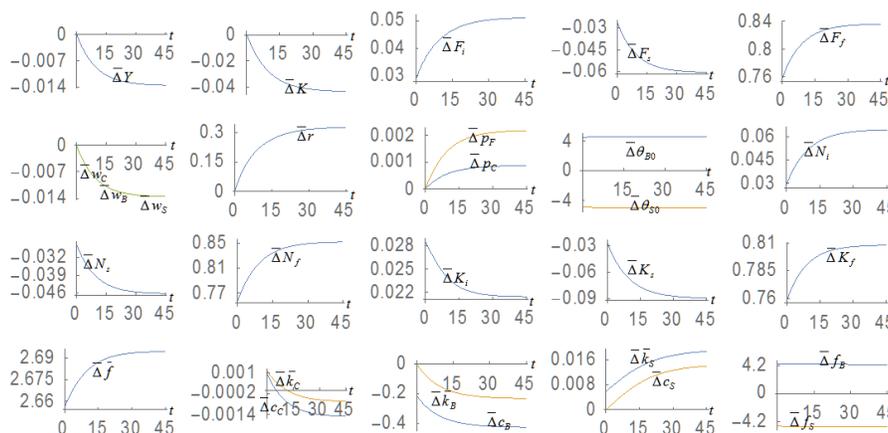
We simulated the motion of the national economy under (29). It is significant to examine how the economic system reacts to exogenous changes. As the lemma provides the computational procedure to calibrate the motion of all the variables, it is straightforward to examine effects of change in any parameter on transitory processes as well stationary states of all the variables. We introduce a variable $x_j(t)$, which stands for the change rate of the variable, $\Delta x_j(t)$ in percentage due to changes in the parameter value.

4.1 The Bandwagoner's Propensity to Consume Fashion Goods Being More strongly affected by the average fashion consumption

We now examine the effects that the bandwagoner's propensity to consume fashion goods is more strongly affected by the average fashion consumption as

follows: $\tilde{\theta}_B : 0.001 \Rightarrow 0.002$ The bandwagoner's propensity to consume fashion goods is more sensitive to change in the average consumption. This implies that, for instance, if the snob consumes more fashion goods, the bandwagoner tend to consume more fashion goods with the same disposable income. The preference change brings about lower national income and wealth. The output levels of the capital goods and fashion goods sectors are augmented and the level of the consumer goods sector is reduced. The bandwagoner's propensity to consume fashion goods is increased and the snob's propensity to consume fashion goods is reduced. The prices of the consumer and fashions are slightly increased. The rate of interest is rising in association with falling in wage rates. The average consumption of fashion goods is expanded. The bandwagoner consume more fashion goods and less consumer goods and have less wealth. The snob consume less fashion goods and more consumer goods and have more wealth. The consumer's behavior is slightly affected.

Figuer 2. The Bandwagoner's Propensity to Consume Fashion Goods Being more Strongly Affected

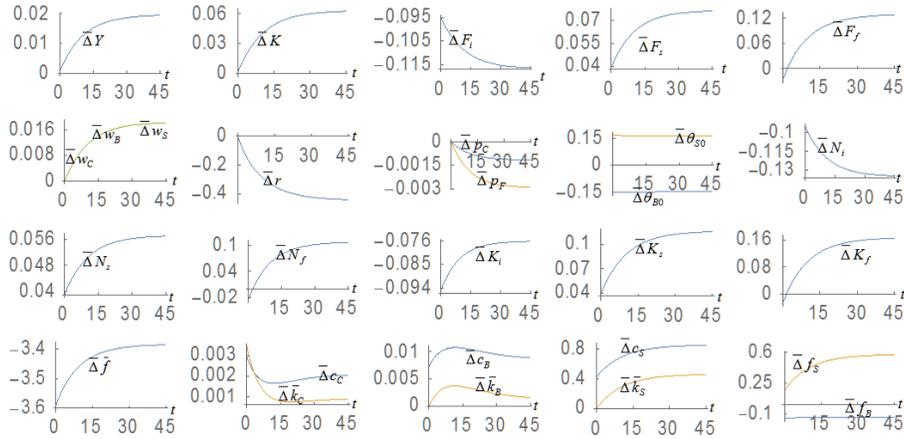


4.2 The Snob's Propensity to Consume Fashion Goods being More Negatively affected by the Average Fashion Consumption

We now examine the effects that the snob's propensity to consume fashion goods is more negatively affected by the average fashion consumption as follows: $\tilde{\theta}_S : 0.001 \Rightarrow 0.003$. The change in the parameter implies that the snob's propensity to consume fashion goods is more sensitive to change in the average consumption. The average consumption level of fashion goods is reduced. This enhances the snob's propensity to consume fashion goods. The bandwagoner's propensity to consume fashion goods is lowered. The bandwagoner consume less fashion goods and more consumer goods and have more wealth. The snob consume more fashion goods and more consumer goods and have more wealth. The consumer consume more consumer goods and have more wealth. The preference change enhances national income and wealth. The output levels of the consumer goods and fashion goods sectors are augmented and the level of the capital goods sector is reduced. The prices of the consumer and fashions

are slightly reduced. The rate of interest is reduced in association with raising in wage rates.

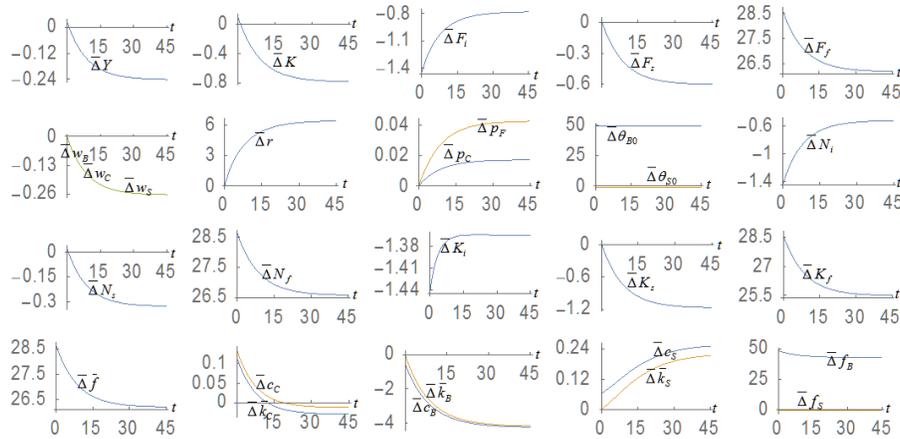
Figuer 3. The Snob's Propensity to Consume Fashion Goods Being more Negatively Affected



4.3 The Bandwagoner's Propensity to Consume Fashion Goods Being Increased

We now examine the effects that the intercept in the function of the bandwagoner's propensity to consume fashion goods is increased as follows: $\bar{\theta}_B : 0.01 \Rightarrow 0.015$. If the average consumption level of fashion goods is kept constant, the change in the parameter implies a rise in the propensity to consume fashion goods. The snob's preference is slightly affected. The national output and wealth are reduced as the bandwagoner has stronger preference for fashion goods. The output of the fashion sector is increased and the output levels of the other two sectors are reduced. Labor force is shifted to the fashion sector from the other two sectors. The capital input of the fashion sector is augmented and the capital inputs of the other two sectors are reduced. The rate of interest is increased in tandem with falling in the wage rates. The prices of the consumer and fashions are slightly increased. The average consumption level of fashion goods is increased. The bandwagoner consume more fashion goods and less consumer goods and have less wealth. The snob consume less fashion goods and more consumer goods and have more wealth.

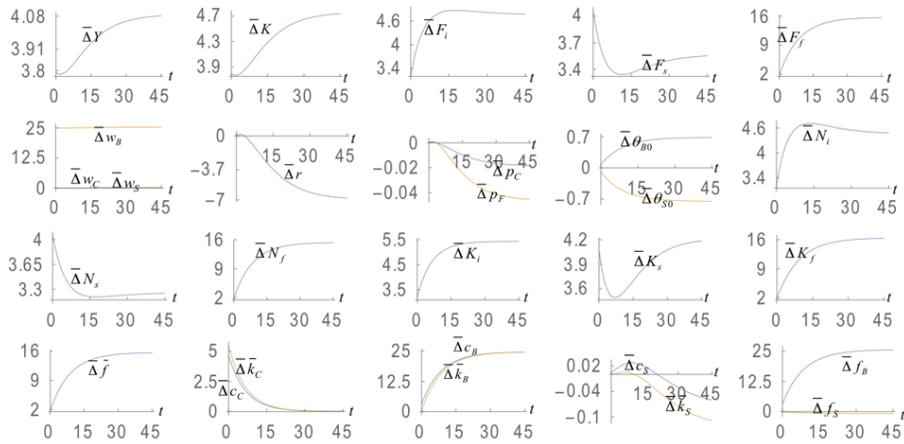
Figure 4. The Bandwagoner's Propensity to Consume Fashion Goods being Increased



4.4 The Bandwagoner's Human Capital Being Improved

We now examine the effects that the bandwagoner's human capital is improved as follows: $h_B : 2 \Rightarrow 2.5$. The total labor is augmented. Each sector employs more labor force. The national wealth and output are increased. All the three sectors' output levels and capital inputs are augmented. The bandwagoner's wage rate is increased and the other two groups' wage rates are slightly affected. The rate of interest and prices of the consumer and fashion goods are reduced. The bandwagoner's preference to consume fashion goods is increased and the snob's preference to consume fashion goods is reduced. The average consumption of fashion goods is expanded. The bandwagoner consume more fashion and consumer goods and have more wealth. The snob consume less fashion goods. The snob consume more consumer goods and have more wealth initially and consume less consumer goods and have less wealth in the long term. The consumer's behavior is slightly affected in the long term.

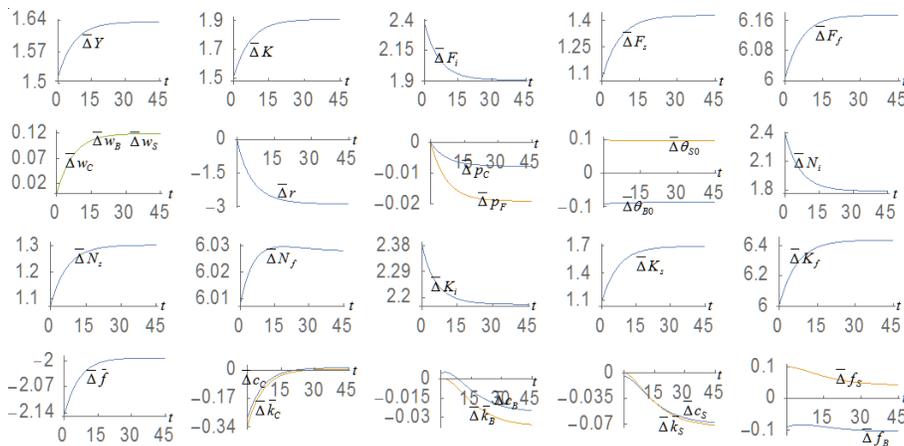
Figure 5. The Bandwagoner's Human Capital being Improved



4.5 A Rise in the Bandwagoner's Population

We now examine the effects that the bandwagoner's population is expanded as follows: $N_B : 10 \Rightarrow 11$. The total labor is augmented. Each sector employs more labor force. The national wealth and output are increased. All the three sectors' output levels and capital inputs are augmented. All the wage rates are enhanced. The rate of interest and prices of the consumer and fashion goods are reduced. The average consumption of fashion goods is expanded. The bandwagoner's preference to consume fashion goods is lowered and the snob's preference to consume fashion goods is enhanced. The bandwagoner consume more fashion and consumer goods and have more wealth. The snob consume less fashion goods. The snob consume more consumer goods and have more wealth initially and consume less consumer goods and have less wealth in the long term. The consumer's behavior is slightly affected in the long term.

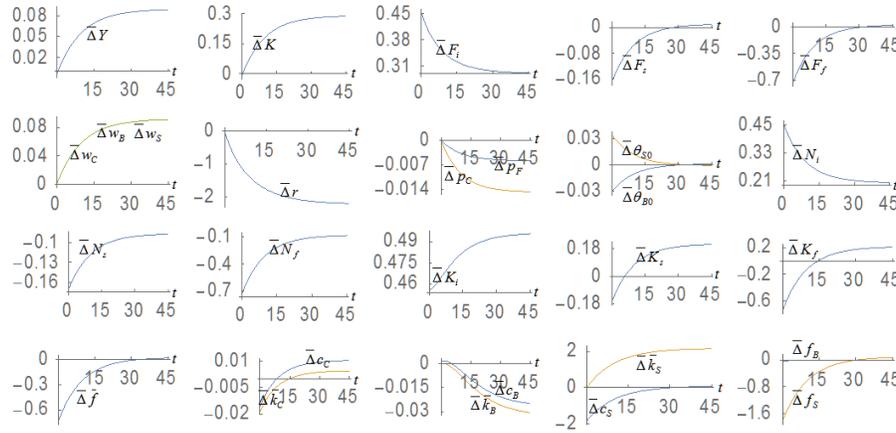
Figure 6. A Rise in the Bandwagoner's Population



4.6 A Rise in the Snob's Propensity to Save

We now examine the effects that the intercept of the function of the snob's propensity to save as follows: $\bar{\lambda}_S : 0.95 \Rightarrow 0.97$. The national wealth and output are increased. The capital goods sector's output level and labor and capital inputs are augmented. All the wage rates are enhanced. The rate of interest and prices of the consumer and fashion goods are reduced. The average consumption of fashion goods is reduced initially and is slightly changed in the long term. The bandwagoner's preference to consume fashion goods is lowered initially and is almost not changed in the long term. The snob's preference to consume fashion goods is enhanced initially and is almost not changed in the long term. The bandwagoner's consumption behavior and wealth are almost not affected. In the long term the snob have more wealth and consume almost the same as before.

Figure 7. A Rise the Snob's Propensity to Save



5. Concluding Remarks

This study dealt with interactions between fashion, economic growth and income and wealth distribution in an economic growth model of heterogeneous households with economic structure. The model emphasized the role of fashion on economic structural change and wealth and income distribution. It was inspired by some ideas in the literature of economics of fashion. The model was developed within the framework of the integrated Walrasian general equilibrium and neoclassical growth theory by Zhang (2012, 2013). For illustration, we simulated the motion of the economic system. We identified the existence of a unique stable equilibrium point. We also carry out comparative dynamic analysis with regard to the snob's propensity to consume fashion goods, the bandwagoner's propensity to consume fashion goods, the bandwagoner's human capital, the bandwagoner's population, and the snob's propensity to save. We examined how changes in these parameters affect fashion development, wealth accumulation, and inequalities in income and wealth. From our simulation, we see that relations between fashion dynamics, inequality and economic growth are complicated in the sense that these relations are determined by interrelated in a very complicated manner. As

our analytical framework is quite general, it is possible to generalize and extend the model in different aspects. One important direction is to introduce taxation into the general dynamic equilibrium model with fashion. We dealt with a simplified fashion dynamics. Fashion appears in arts, sciences, politics, academics, business, food, clothing, morality and entertainment in different forms and changes in different speeds. To analyze detailed structural changes within fashion industry in a macroeconomic framework with endogenous knowledge and wealth is a challenging issue in theoretical economics.

Appendix: Proving the lemma

By (4) and (7), we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{N_i}{\beta_i K_i} = \frac{N_s}{\bar{\beta}_s K_s} = \frac{N_f}{\bar{\beta}_f K_f} \quad (\text{A1})$$

where $\bar{\beta}_j \equiv \beta_m / \alpha_m$ Insert (A1) in (4) and (5)

$$r = \alpha_r z^{\beta_i} - \delta_k, \quad w_j = \alpha_w h_j z^{-\alpha_i} \quad (\text{A2})$$

where

$$\alpha_r = \alpha_i A_i \bar{\beta}_i^{\beta_i}, \quad \alpha_w = \beta_i A_i \bar{\beta}_i^{-\alpha_i}.$$

Hence, we determine the rate of interest and the wage rates as functions of z . From (6) and (7), we have

$$p_m = \frac{\bar{\beta}_m^{\alpha_m} z^{\alpha_m} w}{\beta_m A_m}. \quad (\text{A3})$$

Insert $p_C c_j = \xi_j \hat{y}_j$ in (23)

$$\xi_C \bar{N}_C \hat{y}_C + \xi_B \bar{N}_B \hat{y}_B + \xi_S \bar{N}_S \hat{y}_S = \frac{w N_s}{\beta_s} \quad (\text{A4})$$

where we use $p_C F_s = w N_s / \beta_s$. Similarly for fashion goods we have

$$\theta_B \bar{N}_B \hat{y}_B + \theta_S \bar{N}_S \hat{y}_S = \frac{w N_f}{\beta_f}. \quad (\text{A5})$$

Insert (A4) and (A5) in (2)

$$N_i = \hat{N} - \hat{n} \hat{y}_C, \quad (\text{A6})$$

$$\hat{N} \equiv N - \frac{(\theta_B \beta_f + \xi_B \beta_s) \bar{N}_B \hat{y}_B + (\theta_S \beta_f + \xi_S \beta_s) \bar{N}_S \hat{y}_S}{w}, \quad \hat{n} \equiv \frac{\xi_C \beta_s \bar{N}_C}{w}.$$

Insert (A1) in (21)

$$zK = \frac{N_i}{\beta_i} + \frac{N_s}{\beta_s} + \frac{N_f}{\beta_f}. \quad (A7)$$

Substituting (A4)-(A6) into (A7) yields

$$zK = \hat{N}_1 + \hat{n}_1 \hat{y}_C, \quad (A8)$$

where

$$\hat{N}_1 \equiv \frac{\hat{N}}{\beta_i} + \frac{\xi_B \beta_s \bar{N}_B \hat{y}_B + \xi_S \beta_s \bar{N}_S \hat{y}_S}{\beta_s w} + \frac{\theta_B \beta_f \bar{N}_B \hat{y}_B + \theta_S \beta_f \bar{N}_S \hat{y}_S}{\beta_f w}$$

$$\hat{n}_1 \equiv \frac{\xi_C \beta_s \bar{N}_C}{\beta_s w} - \frac{\hat{n}}{\beta_i}.$$

From the definitions of \hat{y}_j we have

$$\hat{y}_j = (1 + r) \bar{k}_j + w_j. \quad (A9)$$

Insert (A9) in (A8)

$$zK = \hat{N}_1 + \hat{n}_1 w_C + (1 + r) \hat{n}_1 \bar{k}_C. \quad (A10)$$

From (10) and (11), we have

$$\bar{f} = \frac{n \theta_B (\bar{f}) \hat{y}_B}{p_F} + \frac{(1 - n) \theta_S (\bar{f}) \hat{y}_S}{p_F}. \quad (A11)$$

We assume that we can solve (A11) with \bar{f} as the variable as follows

$$\bar{f} = f_0(z, \bar{k}_B, \bar{k}_S). \quad (A12)$$

This implies that θ_j , ρ_j , ξ_j and λ_j can be treated as functions of $z\bar{k}_B$, and

$$\bar{k}_S.$$

From (16) and (A10) we have

$$\hat{N}_1 + \hat{n}_1 w_C + (1 + r) \hat{n}_1 \bar{k}_C = z \bar{k}_C \bar{N}_C + z \bar{k}_S \bar{N}_S + z \bar{k}_B \bar{N}_B. \quad (A13)$$

Solve (A13) with \bar{k}_C as the variable

$$\bar{k}_C = \Lambda(z, \bar{k}_B, \bar{k}_S) \equiv \frac{z \bar{k}_S \bar{N}_S + z \bar{k}_B \bar{N}_B - \hat{N}_1 - \hat{n}_1 w_C}{(1 + r) \hat{n}_1 - z \bar{N}_C}. \quad (A14)$$

It is straightforward to confirm that all the variables can be expressed as functions of z, \bar{k}_B , and \bar{k}_S , by the following procedure: r and w_j by (A2) $\rightarrow p_C$ and p_F by (A3) $\rightarrow \bar{f}$ by (A12) $\rightarrow \theta_j, \rho_j, \xi_j$ and λ_j by their definitions $\rightarrow \bar{k}_C$ by (A14) $\rightarrow \hat{y}_j$ by (A9) $\rightarrow K = \sum_j \bar{k}_j N_j \rightarrow c_j, s_j$ and f_j by (10) $\rightarrow w = w_1/h_1 \rightarrow K_f$ by (A5) $\rightarrow N_s$ by (A7) $\rightarrow N_i$ by (A6) $\rightarrow K_j$ by (A1) $\rightarrow F_j$ by the definitions. From this procedure, (A12) and (17), we have

$$\dot{\bar{k}}_C = \tilde{\Lambda}_C(z, \bar{k}_B, \bar{k}_S) \equiv \lambda_C \hat{y}_C - \bar{k}_C, \quad (A15)$$

$$\dot{\bar{k}}_j = \tilde{\Lambda}_j(z, \bar{k}_B, \bar{k}_S) \equiv \lambda_j \hat{y}_j - \bar{k}_j, \quad j = B, S. \quad (A16)$$

Taking derivatives of equation (A14) with respect to t implies

$$\dot{\bar{k}}_C = \frac{\partial \Lambda}{\partial z} \dot{z} + \tilde{\Lambda}_B \frac{\partial \Lambda}{\partial \bar{k}_B} + \tilde{\Lambda}_C \frac{\partial \Lambda}{\partial \bar{k}_C}. \quad (A17)$$

Here we don't give explicit expressions as they are tedious. Equal the two right-hand sides in (A15) and (A17)

$$\dot{z} = \tilde{\Lambda}_z(z, \bar{k}_B, \bar{k}_S) \equiv \left(\tilde{\Lambda}_C - \tilde{\Lambda}_B \frac{\partial \Lambda}{\partial \bar{k}_B} - \tilde{\Lambda}_C \frac{\partial \Lambda}{\partial \bar{k}_C} \right) \left(\frac{\partial \Lambda}{\partial z} \right)^{-1}. \quad (A18)$$

The differential system (A16) and (A18) has variables, z, \bar{k}_B and \bar{k}_S and consists of differential equations. In summary, we proved the lemma.

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