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ELEMENTS OF A GRAVITATIONAL LENS BY ASSUMING AN ELLIPTICAL GALAXY MODEL

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RESUMEN

En éste trabajo se estudian algunos elementos para lentes gravitacionales por galaxias como son ecuación de la lente, ángulo de desviación, potencial de desviación y retardo temporal, modelando la distribución volumétrica de masa de la lente en forma elíptica. La función de distribución volumétrica de masa en la galaxia deflectora ρ, contiene en su centro un núcleo de radio a, una densidad volumétrica en su núcleo ρ_0 and un parámetro libre de forma b, ($b > a$). Mediante la distribución de densidad volumétrica de masa se encuentran inicialmente la densidad superficial de masa Σ, (proyectada en el plano de la lente), para después hallar los elementos de una lente gravitacional los cuales son totalmente generales y quedan en términos de los parámetros geométricos a and b, que se han relacionado mediante un factor adimensional $n = \frac{b}{a} > 1$. Los resultados encontrados se aplican a un sistema de lente por galaxia particular para hacer un análisis basado en el retardo temporal entre dos imágenes y ver las condiciones que debe cumplir el parámetro n

ABSTRACT

In this work we study elements for gravitational lenses per galaxies such as the equation of lens, deviation angle, deflection potential and time delay, modeling the distribution of the volumetric mass of the lens in elliptical form. The function of volumetric distribution of mass in the deflecting galaxy ρ has a nucleus with radius a in its core, form-free parameter b ($b > a$), and volumetric density in its nucleus ρ_0. Through the distribution of volumetric mass density ρ, we initially find surface mass density Σ (projected on plane of the lens), followed by elements of a gravitational lens which are completely general and expressed in terms of the geometric parameters a and b. These are related by the adimensional factor $n = \frac{b}{a} > 1$. Results are applied to a galaxy-specific lens system to conduct an analysis based on the temporal delay between two images and to observe the conditions with which parameter n must comply.

Key Words: Gravitational lens — the lens equation — deflection angle — deflecting potential — time delay

1. INTRODUCTION

In the study of gravitational lenses (GL) volumetric distribution of mass density of the deflector (ρ), can be projected onto a plane perpendicular to

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the line of vision between the observer and the light source. This plane is called the lens plane, according to Narayan, R. and Bartelmann, M. (1997). The lens is considered thin and the volumetric distribution of lens mass is replaced by a plane on which the surface mass density is (Σ), constituting the so-called approximation of the thin lens, in accordance with Schneider, P. et al. (1992).

In general, GL require a few basic elements such as: surface mass density (Σ), lens equation, deviation angle (α), deflection potential (Ψ) and time delay (Δt), which comprise basic tools for their application in the study of some lens systems in astrophysics. These analytical expressions are specified when applied to a model of mass volume distribution in a particular galaxy.

Some observational data shows that astronomical objects that act as lenses are modeled in diverse ways, according to Cohen, A. S. and Hewitt, J. N. (2000). In systems of gravitational lenses per galaxy, we observationally measure parameters such as: dispersion speed of the material particles that make up the lens (σ_p), angular position of the images (θ), red shifts of lens (z_L), source (z_S) and time delay (Δt); with these it becomes possible to study and assume models in said systems.

The distances from the source to the lens and from the lens to the observer are $\sim 1pc$. In this way, systems that constitute the lenses, such as deflecting galaxy, source and observer, are too far away, which causes light to travel in free space most of the time, this light only locally becoming deviated when it passes through the lens. We must therefore model the universe through which the ray of light should pass; to do this we need to set cosmological parameters, Used, for example, by the authors Adler et al. (1975), Ciufolini et al. (1995), Foster et al. (1994), Kenion (1995): vacuum density (Ω_v); matter density (Ω_m); softness parameter ($\widetilde{\alpha}$) and Hubble constant (H_0).

By applying the properties of GL to a specific system, using observational values and setting cosmological parameters, it is possible to study different galactic models.

In our case we will assume a model of a galaxy which is elliptical in its volumetric distribution of mass (ρ), with which we determine the analytical expressions of GL, used, e.g., by Brainerd et al. (1996) and Golse et al. (2002). Here we follow Hjorth et al. (1997) and Molina et al. (2006), who propose a distribution of volumetric mass density useful for models of elliptical galaxies acting as gravitational lenses.

2. LENS ELEMENTS AND ELLIPTIC MODEL IN THE GALAXY DEFLECTOR

2.1. *Lens elements*

In GL literature, the approximation of a flat lens is characterized by a surface mass density given by the projection operator, according to Miranda, C., Molina, U and Viloria, P. (2014),

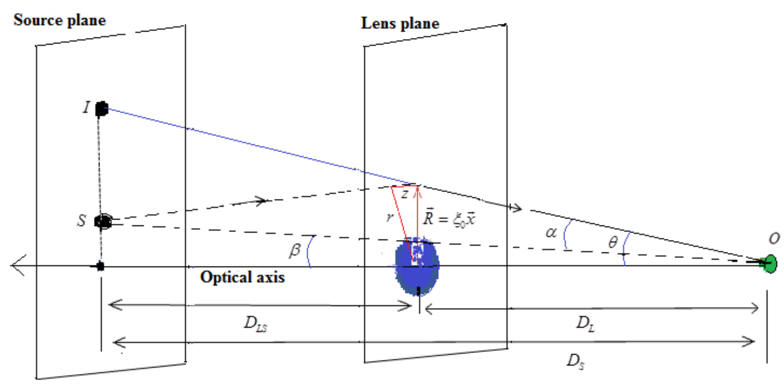

Fig. 1. Illustration of a gravitational lens system. The angular separations of the source and the image from the optic axis as seen by the observer are β and θ, respectively. The angular diameter distances between the observer and the source, the observer and the lens, and the lens and the source are D_S, D_L and D_{LS}, respectively, by Narayan et al. (1997)

$$\Sigma(\vec{R}) = \int \rho(\vec{R}, z)dz \tag{1}$$

where $\vec{R}$ is a radius vector in the lens plane as shown in Figure 1, and ρ is the volumetric distribution of the lens mass. The radius vector R, called impact parameter, can be written $R = \xi_0 x$. The quantity ξ_0 is known as scale parameter or scale factor, and is defined according to the lens model being used. In our case it is defined below in equation (10). According to Schneider et al., 1992 (p. 231), matter within the disc of radius around the mass contributes to the deflection of the ray of light, and matter outside of this disc ($x' < x$) does not contribute importantly to deflection, in such a way that the skew angle follows the expression,

$$\alpha(x) = \frac{2}{x} \int x'\kappa(x')dx' \tag{2}$$

when $x' < x$. The quantity κ defined as $\kappa(x) = \frac{\Sigma(x)}{\Sigma_{cr}}$ is the so-called convergence which indicates the existence of a minimal or critical surface density for the GL phenomenon to occur, in accordance with Narayan et al. (1997) and Schneider et al. (1992). We define surface density of critical mass as: $\Sigma_{cr} = \frac{c^2 D_S}{4.\pi.G.D_L D_{LS}}$, where D_S, D_L and D_{LS}, are the angular diameter distances between observer-source, observer-lens and lens-source, respectively, as shown in figure 1.

The deflection potential of the lens is according to Narayan et al. (1997),

$$\psi = \frac{1}{\pi} \int \kappa(x') \ln |x - x'| \, dx'^2 \tag{3}$$

The time delay Δt between two light beams detected by an observer is given by Molina et al. (2006) and Narayan et al. (1997),

$$\Delta t = \frac{(1 + z_L)}{c} \frac{\xi_0^2 D_S}{D_L D_{LS}} (\frac{1}{2}[\alpha_2^2 - \alpha_1^2] - [\psi_2 - \psi_1]) \tag{4}$$

where z_L is the redshift of the deflecting galaxy, D_L is the angular diameter distance of observer-lens, D_S is the angular diameter distance of observer-source, D_{LS} is the angular diameter distance of lens-source.

The expression (4), contains the geometric delay described in equation (2), and the gravitational potential given by equation (3).

Furthermore, the relationship between source position (β), positions of images ($\theta = \frac{\xi}{D_L}$) and deflection angle can be written in accordance with equation (2.15a) in Schneider et al., 1992 (p.31) as,

$$\beta = \frac{\xi_0}{D_L} x - \frac{D_{LS}}{D_S} \alpha(x) \tag{5}$$

which is called the lens equation. Equations (2), (3), (4) and (5) make up the group of basic elements for a GL study.

2.2. *Elliptical galaxy model deflector*

For our study, we modeled the distribution of lens mass as an elliptical galaxy, following Hjorth and Kneib (1997), who proposed a distribution of volumetric density which is useful for elliptical galaxy models acting as gravitational lenses. With these distributions we find the analytical expressions of GL. This volumetric distribution of mass is,

$$\rho(r) = \frac{\rho_0}{(1 + \frac{r^2}{a^2})(1 + \frac{r^2}{b^2})} \tag{6}$$

The model of deflecting galaxy contains a central nucleus with radius a, a free-form parameter acting as scale b ($b > a$), and a volumetric density of fixed mass in the nucleus ρ_0. Introducing the volumetric density given in equation (6), in the projection operator defined in equation (1) and making the change of variable, $z^2 = r^2 - R^2$ (see Fig. 1), we obtain the surface mass density of the lens. The distance z is typically much smaller than the distances between observer and lens and between lens and source, so after developing the integrals the surface mass density of the lens takes the following form,

$$\Sigma(n, R) = \frac{\sum_0 n^2 a}{n^2 - 1} [\frac{1}{\sqrt{a^2 + R^2}} - \frac{1}{\sqrt{n^2 a^2 + R^2}}] \tag{7}$$

where $n = \frac{b}{a} > 1$ is the adimensional parameter and $\Sigma_0 = \pi \rho_0 a$ is the surface density contained in the nucleus.

To find the analytical expressions of the deviation angle and the deflection potential, first we change $R = \xi_0 x$, in equation (7), in such a way that the surface density of the flat lens is,

$$\Sigma(n,x) = \frac{\sum_0 n^2 a}{(n^2-1)\xi_0}[\frac{1}{\sqrt{A^2+x^2}} - \frac{1}{\sqrt{n^2A^2+x^2}}] \quad (8)$$

where $A = \frac{a}{\xi_0}$ a new parameter. The convergence factor $\kappa = \frac{\Sigma}{\Sigma_{cr}}$ (defined above), now has the form,

$$\kappa(n,x) = \frac{1}{2}[\frac{1}{\sqrt{A^2+x^2}} - \frac{1}{\sqrt{n^2A^2+x^2}}] \quad (9)$$

where we choose a scale factor such as,

$$\xi_0 = \frac{8\pi G D_L D_{LS} \Sigma_0 n^2 a}{c^2 D_S (n^2-1)} \quad (10)$$

The scale factor is set by the dispersion speed of the components of the deflecting galaxy σ_p (this is explained in the following paragraphs). We observe that the scale factor is expressed in terms of the central radius a, and the adimensional parameter $n = \frac{b}{a} > 1$. We see that when the adimensional parameter approaches one, that is, $n \approx 1$, the scale factor becomes infinite, which prompts one to think that it would be preferable choose a scale that depends only on the radius of the nucleus a.

In the time delay expressed in equation (4), we see the arbitrary scale factor ξ_0, which we choose for the distribution of elliptical mass as shown in equation (10). The central volumetric density ρ_0 is not a quantity that is observationally measurable in a gravitational lens system, but it is possible to establish it by knowing the rate of dispersion σ_p of the matter components of the deflecting galaxy. To do this, through the expression proposed by Hjorth and Kneib (1997),

$$\sigma_p^2 = \frac{2G}{\Sigma(R)} \int_R^\infty \frac{M(r)}{r^2} \rho(r) \sqrt{r^2 - R^2} dr \quad (11)$$

it is possible to obtain the dispersion rate.

In this expression (11) we see that if the dispersion velocity of the matter components of the deflecting galaxy is known, the central volumetric density can be set, and with this, the scale factor (10) is also set.

2.3. *Deviation angle*

Expressed in terms of impact parameter $R = \xi_0 x$ and using the convergence factor in equation (9), the deviation angle (2) takes the form,

$$\alpha(n,R) = \frac{(n-1)a}{R} + \sqrt{1 + \frac{a^2}{R^2}} - \sqrt{1 + \frac{n^2a^2}{R^2}} \quad (12)$$

which is given in terms of central radius a and adimensional parameter $n = \frac{b}{a} > 1$. We observe that if it is the case that this parameter is approximate to one, that is, $n \approx 1$ and selecting a set scale, the deviation angle becomes $\alpha(R) \approx 0$.

2.4. *Deflection Potential*

We find the deflection potential of the lens that we are considering by replacing equation (9) in equation (3) and writing in terms of impact parameter $R = \xi_0 x$, we obtain,

$$\begin{aligned}\psi(\mathbf{n}, R) = \quad & \tfrac{a}{\xi_0}[(n-1) + \ln\tfrac{2a}{\xi_0} - n\ln\tfrac{2na}{\xi_0} + \sqrt{1+\tfrac{R^2}{a^2}} - \sqrt{n^2+\tfrac{R^2}{a^2}} \\ & - \ln\tfrac{a(1+\sqrt{1+\frac{R^2}{a^2}})}{\xi_0} + n\ln\tfrac{a(n+\sqrt{n^2+\frac{R^2}{a^2}})}{\xi_0}]\end{aligned} \tag{13}$$

given that $n > 1$. We note that if this adimensional parameter approximates one, that is, $n \approx 1$, and if we select a set scale, the deflection potential becomes zero, $\psi(R) \approx 0$.

2.5. *Time delay*

Differentiating the square of the deviation angle for two images, $\alpha_2^2 - \alpha_1^2$ and subsequently differentiating the deflection potential $\psi_2 - \psi_1$, and then replacing these two differentiations in equation (4), we get the time delay as,

$$\Delta t = \frac{(1+z_L)\xi_0^2 D_S}{cD_L D_{LS}}[\frac{h(n,a)}{2} - a\frac{g(n,a)}{\xi_0}] \tag{14}$$

Equation (13) allows us to determine the time delay between two images, where a the radius of the nucleus of lens mass distribution is and n is the adimensional parameter.

Furthermore, we have defined two new functions: $h(n,a)$ and $g(n,a)$, defined below, depend on how the nucleus radius a is set and and on how much these two new functions vary from the adimensional parameter n.

Function $h(n,a)$ is defined in the form,

$$\begin{aligned}h(n,a) = \quad & (n^2-n+1)(\tfrac{a^2}{R_2^2} - \tfrac{a^2}{R_1^2}) \\ & +(n-1)[\sqrt{\tfrac{a^2}{R_2^2}+\tfrac{a^4}{R_2^4}} - \sqrt{\tfrac{a^2}{R_2^2}+\tfrac{n^2a^4}{R_2^4}}] \\ & +(n-1)[\sqrt{\tfrac{a^2}{R_1^2}+\tfrac{n^2a^4}{R_1^4}} - \sqrt{\tfrac{a^2}{R_1^2}+\tfrac{a^4}{R_1^4}}] \\ & -\sqrt{1+\tfrac{a^2}{R_2^2}+n^2(\tfrac{a^2}{R_2^2}+\tfrac{a^4}{R_2^4})} \\ & +\sqrt{1+\tfrac{a^2}{R_1^2}+n^2(\tfrac{a^2}{R_1^2}+\tfrac{a^4}{R_1^4})}\end{aligned} \tag{15}$$

Similarly, $g(n,a)$ is defined as,

$$\begin{aligned} g(n,a) = & \sqrt{1+\frac{R_2^2}{a^2}} - \sqrt{1+\frac{R_1^2}{a^2}} \\ & + \sqrt{n^2+\frac{R_1^2}{a^2}} - \sqrt{n^2+\frac{R_2^2}{a^2}} \\ & + \ln\left|\frac{1+\sqrt{1+\frac{R_1^2}{a^2}}}{1+\sqrt{1+\frac{R_2^2}{a^2}}}\right| \\ & + n\ln\left|\frac{n+\sqrt{n^2+\frac{R_2^2}{a^2}}}{1+\sqrt{n^2+\frac{R_1^2}{a^2}}}\right| \end{aligned} \tag{16}$$

$h(n,a)$ and $g(n,a)$, are normalized so that when they are introduced into the time delay equation (14), their units are expressed in seconds.

2.6. *Lens Equation*

By substituting the deflection angle (12) in equation (5), we get,

$$\begin{aligned} \beta = & \frac{R}{D_L} - \frac{D_{LS}}{D_S}\frac{(n-1)a}{R} \\ & -\frac{D_{LS}}{D_S}[\sqrt{1+\frac{a^2}{R^2}} - \sqrt{1+\frac{n^2a^2}{R^2}}] \end{aligned} \tag{17}$$

This formula represents the equation of the lens in the elliptical lens model we propose, which is a function of the impact parameter and adimensional parameter n.

3. APPLICATION OF THE PROPOSED GRAVITATIONAL LENS MODEL TO GALACTIC LENS B0218 + 357

The expressions articulated in the previous section are completely general and can be applied to any gravitational lens system. In our case, by way of example, we choose to apply them to the B0218 + 357 lens system, to analyze the effectiveness of the results we obtain.

Some researchers, such as, notably, Wucknitz et al. (2004) who, in their work Models for the Lens and Source of B0218 + 357 determine the Hubble constant H_0, and discuss different models for the B0218 + 357 galactic lens. In addition to this, in order to obtain an estimate of the Hubble constant, A.D. Biggs et al. (1999) model the B0218 + 357 system using the lens model described by Kormann et al. (1994) as a Singular Isothermal Ellipsoid (SIE) mass. More information on the morphology of the received images can be found in the work of C. Spingola et al. (2015). Observational data for the

B0218 + 357 lens system deposited in the CASTLES Survey, according to Cohen, A. S., and Hewitt, J. N., (2000) is summarized in table 1 below: the dispersion speed σ_p, the angular positions of two images θ_1 and θ_2, the redshifts for lens z_L and the source z_S; and the difference in time delay between the two images Δt, as follows:

Table 1. Data observed in B0218 +357 lens system

σ_p	θ_1	θ_2	z_L	z_S	Δt
150**km**/s	40mas	290mas	0.68	0.94	10.5±0.2$days$

For our case, we choose cosmological parameters within the range of values most accepted in literature; we thus rely on the work of several authors, and among these point especially to: Kessler et al. (2009), Boughn and Crittenden (2001), Bartelmann et al. (1997), Weinberg (1972), Grogin et al. (1996) and others. The parameters we choose for our elliptical lens model are: Hubble Constant, $H_0 = 76\frac{km/s}{pc}$, Vacuum density $\Omega_v = 0.7$, matter density $\Omega_d = 0.3$, and softness parameter $\widetilde{\alpha} = 0.5$. This softness parameter of matter in the universe is smoothly distributed (that is, it is not bound up in galaxies), note Dyer and Roeder (1973), see also P. Schneider et al. (1992) (p.138) and Xi Yang et al. (2013).

According to Dyer, C. C. (1973) and Wucknitz et al. (2004), by using, for the B0218 + 357 galactic lens system, the previous cosmological parameters and the following values: cross section $\frac{D_L D_{LS}}{D_S}$, observer-lens distance D_L, impact parameters of the two images R_1 and R_2, we obtain: $\frac{D_L D_{LS}}{D_S} = 2.25 \times 10^7 pc$, $D_L = 1.364 \times 10^9 pc$, $R_1 = 264.58pc$ and $R_2 = 1918.21pc$.

With these quantities, we can find the elements of the proposed lens model.

3.1. *Scale factor, surface mass density of the lens, deviation angle and deflection potential.*

Because there are two images in the B0218 + 357 lens system, the previous calculations on the two impact parameters $R_1 = 264.58pc$ and $R_2 = 1918.21pc$ allow us to set the radius of the nucleus in the range of values of these two parameters (R_1 and R_2). In this work, we give the radius of the nucleus approximately the value of the minor impact parameter, expressing the radius of the nucleus approximately as $a = 264pc$, which corresponds to the impact parameter with least deviation. To facilitate analysis we define the adimensional quantity λ as $\lambda = \frac{R}{a} > 1$; by setting the nucleus radius and varying the impact parameter according to this nucleus radius, we can also set the surface density in the nucleus as the approximate value of $\Sigma_0 = 58.62kg/m^2$.

Furthermore, the scale factor in equation (10), is in terms of the adimensional parameter n, that is,

$$\xi_0 = (200.16\mathbf{pc})\frac{n^2}{(n^2-1)} \tag{18}$$

being $pc = 3.086 \times 10^{16} m$. The surface mass density of the lens, expressed in equation (8) is then found to be,

$$\Sigma(n,\lambda) = \frac{58.62n^2}{n^2-1}[\frac{1}{\sqrt{1+\lambda^2}} - \frac{1}{\sqrt{n^2+\lambda^2}}]kg/m^2 \tag{19}$$

where for this particular lens, the impact parameter satisfies the condition $1 \leq \lambda \leq 7.25$ and $n > 1$. When the value of n is fixed, expression (19) allows us to estimate the surface mass density of the lens as a function of λ, in the given interval.

After making the replacements required by our proposed system, the deviation angle represented in equation (12), takes the form,

$$\alpha(n,\lambda) = \frac{n-1}{\lambda} + \sqrt{1+\frac{1}{\lambda^2}} - \sqrt{1+\frac{n^2}{\lambda^2}} \tag{20}$$

in which we know that $1 \leq \lambda \leq 7.25$ and $n > 1$.

At the same time, the deflection potential expressed in equation (13), takes the new form,

$$\begin{aligned}\psi(\mathbf{n},\lambda) = \;& \tfrac{a}{\xi_0}[(n-1) + \ln\tfrac{2a}{\xi_0} - n\ln\tfrac{2na}{\xi_0} \\ & +\sqrt{1+\lambda^2} - \sqrt{n^2+\lambda^2} \\ & - \ln\tfrac{a(1+\sqrt{1+\lambda^2})}{\xi_0} \\ & +n\ln\tfrac{a(n+\sqrt{n^2+\lambda^2})}{\xi_0}]\end{aligned} \tag{21}$$

which depends upon the scale factor represented in equation (18) and the following conditions: $1 \leq \lambda \leq 7.25$ and $n > 1$.

3.2. *Time Delay Model*

By using the values obtained from the B0218 + 357 lens system, we reduce the time delay stated in equation (14) is reduced to,

$$\begin{aligned}\Delta t = \;& (5.68days)\tfrac{n^2}{(n^2-1)^2}[\tfrac{h(n)}{2} \\ & -\tfrac{1.32(n^2-1)g(n)}{n^2}]\end{aligned} \tag{22}$$

Quantities h and g, and equations (15) and (16), depend only upon the adimensional parameter n. This allows us to establish values for the time delay. We take advantage of the fact that the time delay between the two images is measured observationally, as shown in table 1 (10.5 days), and a series of different values of n are explored in equation (22), until observed value is reached. The time delay of equation (22), for some values of n, is as follows:

Table 2. Time delay values, for some values of n, in the B0218 +357 lens system, according to equation (22).

n	2	2.1	2.2
$\Delta t(days)$	9.67	10.3	10.9

When we compare the observational time delay $\Delta t = 10.5 \pm 0.2 days$, as shown in table 1, with equation (22), we see that for the B0218 + 357 lens system, the adimensional parameter has a range of certainty of $2 \leq n \leq 2.2$. We can appreciate the most approximate value to $n = 2.1$ in table 2.

Establishing the value of parameter n allowed us to estimate the geometric parameters of the elliptical lens, a and b, and afterwards find the numerical values for the other basic elements such as the surface density of the lens, the deviation angle, the scale factor and the deflection potential, as indicated in the following section.

3.3. *Estimation of deviation angle, deflection potential and lens equation.*

Given that for the B0218 + 357 lens system, the approximate radius of the nucleus is, the adimensional parameter is $a = 264pc$, the cross section is $\frac{D_L D_{LS}}{D_S}$ and the observer-lens distance is D_L, we can estimate the deviation angle, deflection potential and the lens equation in this model:

1. Using as our premise equation (19), taking into account that the value of the adimensional parameter is $n = 2.1$, the surface density diminishes as the impact parameter increases, and because this is in the interval $1 \leq \lambda \leq 7.25$, the surface mass density of the lens for the proposed system is estimated in the range of $0.32kg/m^2 \leq \Sigma \leq 32kg/m^2$.

2. In accordance with the equation (20), and taking into account that the parameter λ is in the range $1 \leq \lambda \leq 7.25$, we deduce that deviation angle oscillates between 120mas and 220mas.

3. In accordance with equation (18), and with the values of adimensional parameter n, represented in section 3.1, we can establish an approximate value of the scale factor, that is, $\xi_0 = 260pc$. Furthermore, as the values of parameter λ oscillate between $1 \leq \lambda \leq 7.25$, then the values of the deflection potential, according to equation (21), oscillate between 150mas and 930mas.

4. CONCLUSIONS

This work is based on a volumetric mass distribution which describes a fast relaxation scenario, similar to a model of an SIS isothermal sphere, according to Kenion, I. R. (1995), where the mass of the lens is considered spherically symmetric. The model of elliptical density contains a central nucleus with radius a, mass density ρ_0 in the central nucleus and free-form parameter a, a mass density in the central core ρ_0 and also a free shape parameter b $(b > a)$. From this volumetric mass distribution we find new analytical expressions of the lens elements. These new elements are equations for: lens surface density, deviation angle, deflection potential and time delay. These expressions depend upon the impact parameter of the images and on geometric lens elements a and b, related by the adimensional parameter $n = \frac{b}{a} > 1$.

The analytical expressions of the surface mass density of the lens, deviation angle, deflection potential and time delay used in this work to describe our proposed model can be used to analyze any other system of galaxy lens. Our equations are quite general and for their application to study a specific lens system only require the observational measurements indicated in Table 1.

To implement the results of section (2), one basically needs a system of lenses per galaxy whose mass density distribution fits the elliptical model we describe in this paper, the analytical expressions found here being a good point of departure for further research. In this work the proposed gravitational lens model is applied specifically to the B0218 + 357 lens system, using observational values shown in Table 1. To do this, we have used the cosmological parameters most widely accepted in literature, used, e.g., by authors Bartelmann, M., at al. (1997), Boughn et al. (2001), Foster et al. (1994), Kessler et al. (2009), Schneider et al. (1992),such as the Hubble constant, vacuum density, matter density and softness parameter contained in the angular diametric distances of observer-lens, observer-source and lens-source. This allows us to find the approximate radius of the nucleus of the deflecting galaxy, the angular diametrical distances, the scale factor synthesized in equation (18), and afterwards, adjust the adimensional parameter n.

The adimensional parameter n is adjusted through the time delay represented in equation (22), and compared to the observed time delay $\Delta t_{obs} = 10.5 days$, shown in Table 1. To do this, it was necessary to write the impact parameter R, in terms of the nucleus radius ; in the form of $\lambda = \frac{R}{a} \geq 1$.

Because the impact parameter is expressed in terms of the radius of the nucleus, we were able, once we have set cosmological parameters, determine the theoretical time delay given in equation (22), depending only on the adimensional parameter n. Thus, when we compare the theoretical time delay given in equation (22), with the observationally measured time delay in table 1, we find that the range of values of the adimensional parameter n is $2 \leq n \leq 2.2$.

Finally, as evidenced in section (3.3), by adopting the approximate value $n = 2.1$ for the adimensional parameter and the interval for the impact parameter $1 \leq \lambda \leq 7.25$, we were able to estimate numerical values for the surface density of the lens, deviation angle, scale factor and deflection potential in our proposed system.

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