*Revista Mexicana de Astronomía y Astrofísica*, **41**, 477–482 (2005)

# APPLICATION OF NON-LINEAR ANALYSIS TO VARIABILITY OF 3C390.3

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*Received 2005 April 27; accepted 2005 June 28*

RESUMEN

En este artículo, aplicamos métodos no lineales en el análisis de la curva de luz de 3C390.3 con el fin de investigar su comportamiento temporal complejo y poder distinguirlo de los modelos lineales. Aplicamos pruebas estadísticas avanzadas a la serie temporal, asi como a modelos lineales y no lineales de referencia. Las diferentes pruebas aplicadas sistemáticamente rechanzan la hipótesis de un mecanismo generador lineal, independientemente de si se utiliza parcialmente la curva de luz, lo que implica una naturaleza no lineal.

ABSTRACT

In this paper, we apply non-linear data analysis methods to the light curve of 3C390.3 in order to investigate its complex time-evolution behavior and distinguish it from simple linear models. We applied advanced statistical tests to the time series and to linear and non-linear mathematical models, in order to have a comparison reference. The different tests applied systematically reject the hypothesis of a linear generating mechanism for any part of the time series, implying that the light curve of 3C390.3 has indeed a non-linear nature.

*Key Words:* **METHODS: DATA ANALYSIS — METHODS: STATISTICAL — QUASARS: INDIVIDUAL (3C390.3)**



## 1. INTRODUCTION

The study of variability in astronomical objects such as AGNs provides insight to the understanding of the dynamics that govern the explosive events that take place on them. Some features of these objects can be described by means of stochastic models. The unpredictability of such events leads us to believe in the inadequacy of applying periodic analysis to their light curves. Even more, in these systems, some non-linear dependencies are observed, which are necessary to explain their luminosity bursts. This non-linear behavior is expected if one sees the central luminosity source in AGNs as a complex system, with many parameters non-linearly related. Examples of these may well be luminosity dependent accretion and the appearance or disappearance of hot spots in an accretion disk.

## 2. THE LIGHT CURVE

As part of the photometric and spectral monitoring campaigns of AGNs, in 1998 a group of astronomers from the former Soviet Union, Europe, and México observed the central regions of 11 AGNs with different telescopes in a common optical monitoring program. In particular, the object 3C390.3 was included in the sample (Bochkarev & Shapovalova 1999).

Spectral observations were taken with the 6 m and 1 m telescopes of SAO RAS (Russia) and the 2.1 m of Cananea (México), in the wavelength range 4000–8000 Å, with resolutions from 3–15 Å, and a S/N = 50 for the continuum in H$\alpha$ and H$\beta$.

Photometric observations in the $BVR$ bands were conducted in 3 different observatories, i.e., SAO RAS (Special Astrophysical Observatory), CL SAI (Crimea Laboratory Sternberg Astronomical Institute) and AAO (Abastumani Astrophyscial Observatory). Yet, the observed number of points for the light curves was insufficient to carry out a study of the long-term dynamics of this object. So the historic light curve in the $B$ band, which dates back to 1966, was taken from the literature. Included are the photoelectric photometry obtained by Sandage from

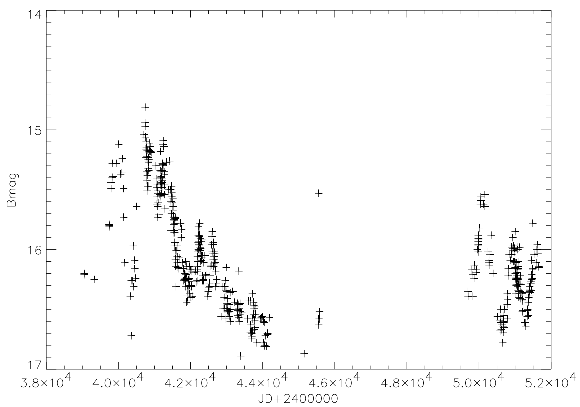


Fig. 1. Historic light curve of 3C390.3, *B* band.

1965–1967, 1971–1972 (Sandage 1973), Nevizvestny (1986); spectrophotometric observations from 1970–1979 made by Yee & Oke (1981), from which *B* magnitudes were derived. Also are included photographic observations available from 1967–1980 from Cannon, Penston, & Brett (1971), Babadzhanyants et al. (1973;1974;1975;1976;1984), Selmes, Tritton, & Wordsworth (1975), Scott et al. (1976), Pica et al. (1980). Photographic observations were averaged in 5 day intervals. We also included observations from Perez et al. (1988), Lawrence et al. (1996) and the data from the International Monitoring AGN Watch Consortium from 1995-1995 (Dietrich et al. 1998). The resulting light curve is shown in Figure 1, where one can notice an increment in the luminosity with significant fluctuations between 1965 and 1977. The variability analysis of the emission in H$\beta$ was discussed by Shapovalova et al. (2001).

## 3. NON-LINEARITY

In time-series analysis of chaotic systems, there are certain physical parameters of special interest (e.g., dimension) that determine some useful invariant concepts. The particular problem of the detection of non-linear behavior is normally complicated by the presence of long time coherence, besides the usual problems related to finite precision, measurement errors and signal noise. In such cases, the use of surrogate data sets has proven to be an excellent technique to compare with. In surrogates one seeks, generally, for a statistical property that differs at a significant level from the original data. If the generating mechanism of the synthetic data is linear, it is relatively easy to test whether or not the signal is linear.

From all possible non-linearity tests, we have chosen a assortment, such as the McLeod-Li test (McLeod & Li 1983), Engle's test (Engle 1982), BDS test (Brock, Dechert, & Scheinkman 1996), Tsay's test (Tsay 1986), Hinich's Bicovariance test (Hinich 1996) and the Hinich's Bispectral test (Hinich 1982).

### 3.1. *Engle's Test*

In most time series analysis, emphasis is given to the first moments (i.e., the linear part), neglecting serial dependencies in any higher order moments, bypassing them as an unimportant by-product. But, in the case of complex series, the search for non-stationarity has led people to seek for more plausible models that can deal with non-constant higher order moments. Engle introduced the concept of conditional heteroskedastic time series. This approach makes a distinction between the conditional and the unconditional second order moments. Autoregressive univariate models frequently produce errors with non constant variance once fitted to the original data. The standard supposition of constant variance of most models results then unlikely. The key of the Engle's ARCH model lies in the fact that while the unconditional covariance of the variable of interest may be time invariant, the conditional ones (covariance and variance) show strong dependencies on the past values of the system.

The variance of the error regressors of Engle's test takes the form of: $\sigma^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + ... + \alpha_p\epsilon_{t-p}^2$, where $\alpha_0, \alpha_1, ..., \alpha_p$ and $\epsilon_{t-i}$ is the regression error in the *i-th* delay.

Since the model seeks explicitly for second order moment non-linearities, in its simplest form, the ARCH(p) process can be written as:

$$y_t = \beta_1 + \beta_2 x_{2t} + ... + \beta_k x_{kt} + \epsilon_t \,,$$

$$\epsilon_t \sim N(0, \sigma_t) \,,$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2 + ... + \alpha_p\epsilon_{t-p}^2 \,.$$

If the null hypothesis $H_0 : \alpha_0 ... \alpha_p = 0$ is accepted, any ARCH effect is neglected. Instead the alternative hypothesis $H_1 : \alpha_i \neq 0$ accepts them. Engle proposes a test by means of Lagrange Multipliers, due to the constant conditional variance, whose statistic can be calculated as $nR^2$, where $n$ is the number of observations and $R^2$ is the correlation coefficient in the regression of $\epsilon_t^2$. This statistic has an asymptotic distribution of the form $\chi^2$.

### 3.2. *McLeod-Li Test*

McLeod-Li's test is based fact that if $x_t$ is a Gaussian stationary process, then the autocorrelation:

$$Corr(x_t^2, x_{t-i}^2) = Corr(x_t, x_{t-i})^2, \ \forall i \,, \qquad (1)$$

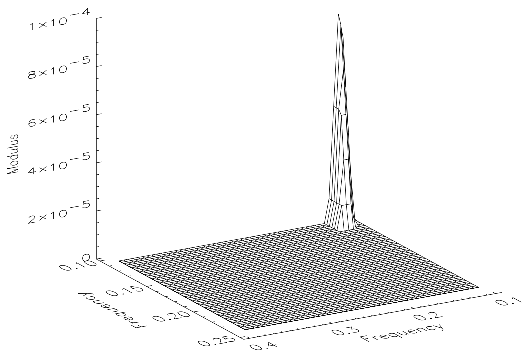


Fig. 2. Linear Model.

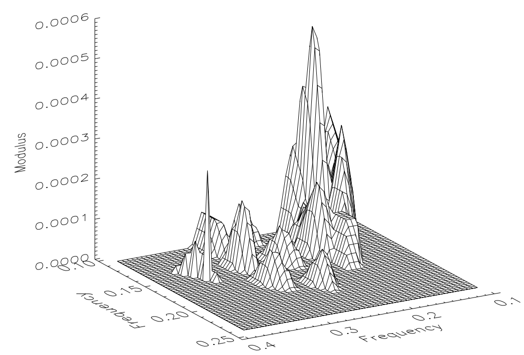


Fig. 3. Non-Linear Model.

where $Corr(.,.)$ is the correlation coefficient given by:

$$\hat{r}(i) = \frac{\sum_{i=1}^{N}(x_t^2 - \sigma^2)(x_{t-i}^2 - \sigma^2)}{\sum_{t=1}^{N}(x_t^2 - \sigma^2)}, \quad (2)$$

$$\sigma^2 = \sum_{t=1}^{N} \frac{x_t^2}{N}. \quad (3)$$

Any deviation from the former behavior points to non-linearity. McLeod-Li's test requires the existence of eighth order moments for the $\chi^2$ distribution convergence. The Box-Ljung statistical estimator $Q$, asymptotically converges to a $\chi^2$ distribution if the generating process is of the iid kind, and is given by:

$$Q = N(N+2)\sum_{i=1}^{L} \frac{\hat{r}^2(i)}{N-i}. \quad (4)$$

### 3.3. *BDS Test*

This is a non-parametric test that examines serial independence based on the correlation integral of $x_t$. For a given number of embedding dimensions $m$, the generated sequence of histories is:

$$x_t^m = (x_t, ..., x_{t+m-1})', \quad (5)$$

where $'$ denotes transposed.

For a $T$ number of observations, $T+m-1$ histories can be constructed. The integral correlation $C_{m,T}(\epsilon)$ is given by:

$$C_{m,T}(\epsilon) = \sum_{t=1s}^{T-m+1} \sum_{s=1}^{t-1} I_\epsilon(x_t^m, x_s^m) / \frac{T_m(T_m-1)}{2}, \quad (6)$$

$$I_\epsilon(x_t^m, x_s^m) = \begin{cases} 1 & \text{if } \|x_t^m - x_s^m\| < \epsilon \\ 0 & \text{otherwise}. \end{cases}$$

Basically, $C_{m,T}$ counts the number of $m$-histories that lay in an hypercube of size $\epsilon$. Under the null of an iid process:

$$H_0 : C_m(\epsilon) - C(\epsilon)^m = 0, \quad (7)$$

or expressed also as:

$$H_0 : C_{m,T}(\epsilon) - C_T(\epsilon)^m, \quad (8)$$

the BDS estimator

$$w_{m,T} = \sqrt{T}\frac{C_{m,T}(\epsilon) - C_T(\epsilon)^m}{\sigma_{m,T}(\epsilon)}, \quad (9)$$

converges asymptotically to zero with probability 1.

### 3.4. *Tsay Test*

A non-linear stationary series can be expressed as a Volterra expansion of the form:

$$x_t = \mu + \sum_{i=-\infty}^{\infty} a_i\epsilon_{t-i} + \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} a_{ij}\epsilon_{t-i}\epsilon_{t-j} + \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} a_{ijk}\epsilon_{t-i}\epsilon_{t-j}\epsilon_{t-k} + ... \quad (10)$$

where $\epsilon_t$ is an iid process. The null of stationarity is obtained fixing $a_{ij} = a_{ijk} = ... = 0$. If at least one of these coefficients is different from zero, then non-linearity is assumed.

The regression equation in this case takes the form:

$$x_t = \gamma_0 + \sum_{i=1}^{K} \gamma_i \hat{v}_{t,i} + \eta_t, \quad (11)$$

where $\hat{v}_{t,i}$ is the projection of all possible cross-products of the form $x_{t-i}x_{t-j}$. The Tsay estimator is the usual F-test.

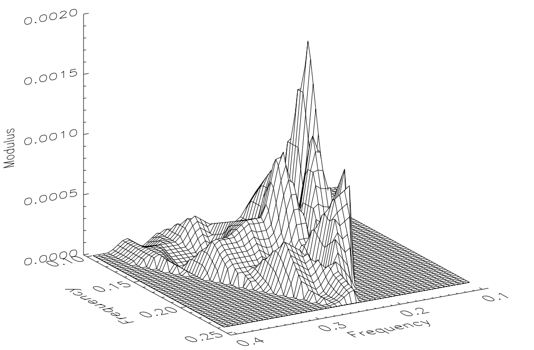


Fig. 4. 3C390.3.

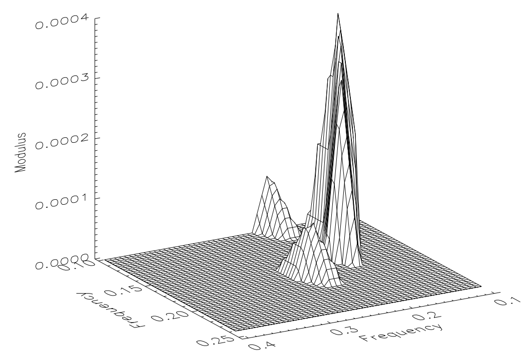


Fig. 5. 3C390.3 without gap, simple interpolation.

This particular test is a generalization of Keenan's test that explicitly seeks for quadratic dependence in the data. Though Keenan's test has been used in the astronomical community in the analysis of variability (Vio et al. 1992), Tsay's test is a better choice because it has proved to be a more robust one.

### 3.5. *Hinich Bicovariance Test*

This test assumes that $x_t$ is a realization of a stationary stochastic process of third order. The sample's bicovariance is defined as:

$$C_3(r,s) = (N-s)^{-1} \sum_{t=1}^{N-s} x_t x_{t+r} x_{t+s}\,,\ 0 \leq r \leq s\,, \tag{12}$$

which is a generalization of the *skewness* parameter. The $C_3(r,s)$ are all zero for an iid process. Values different from zero are expected for data in which $x_t$ depends on delay cross-products $x_{t-i}x_{t-j}$ and others of higher order.

### 3.6. *Hinich Bispectral Test*

This is a non-parametric test that examines third order moments in the frequency domain. The test obtains a direct estimator of *any* non-linear mechanism independently of any linear dependence present in the data. As a result, the validity of data filtering, through variants of AR modeling (i.e., prewhitening of data) is irrelevant.

The third order correlation function is:

$$Corr(r,s) = E[x_t, x_{t+r}, x_{t+s}]\,, \tag{13}$$

where the bispectrum of $x_t$ for a pair of frequencies $(f_1, f_2)$ is the double Fourier transform:

$$F(f_1,f_2) = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} Corr(r,s)$$

$$\exp[-i2\pi(f_1(r) + f_2(s))]\,. \tag{14}$$

The series $x_t$ will be linear only if can be expressed as:

$$x_t = \sum_{n=0}^{\infty} a(n)u(t-n)\,, \tag{15}$$

where $u_t$ iid (i.e., $\bar{x} = 0, \sigma^2 = 0$), and the coefficients $a(n)$ are fixed. The bispectrum of $x_t$ takes a simple form when the mechanism is linear:

$$F(f_1,f_2) = \mu_3 A(f_1)A(f_2)A^*(f_1+f_2)\,, \tag{16}$$

$$A(f) = \sum_{s=0}^{\infty} a(s) exp[-i2\pi fs]\,, \tag{17}$$

where $A^*(f)$ is the complex conjugate and $\mu_3$ is $E\{u(t)^3\}$.

## 4. RESULTS

In Fig. 1 the light curve of 3C390.3 is shown. It has been binned in regular intervals of 10 days, as the tests we are applying require regular sampling of the signal. The compromise when choosing a binning interval is that secondary peaks of the original signal do not disappear. In the reconstruction, linear interpolation was adopted, because without *a priori* knowledge of the system's dynamics there is no reason for introducing a more sophisticated interpolation. In addition there is the fact that for some parts of the light curve there are lapses that cannot be reconstructed by any algorithm. In other words,

TABLE 1

CONFIDENCE LEVELS FOR NON-LINEARITY TESTS

| Signal | Bispectral | McLeod-Li | BDS | Bicovariance | Engle | Tsay |
|---|---|---|---|---|---|---|
| Linear Model | 0.988 | 0.017 | 0.700 | 0.898 | 0.173 | 0.941 |
| Nonlinear Model | 0.072 | 0.118 | 0.542 | 0.090 | 0.120 | 0.177 |
| 3C390 | 0.045 | 0.003 | 0.000 | 0.002 | 0.004 | 0.012 |
| 3C390 simple interp. | 0.098 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3C390 interp gauss. | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.013 |
| 3C390 no gap | 0.023 | 0.005 | 0.000 | 0.006 | 0.009 | 0.070 |
| 3C390 no gap simp interp | 0.058 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3C390 no gap interp. gauss. | 0.010 | 0.000 | 0.000 | 0.000 | 0.000 | 0.032 |



we cannot reproduce what was not measured. However, numerical simulations can help us to estimate the interpolation effects in our analysis.

The tests were also applied to the signal contaminated by Gaussian noise and for partial lapses of the signal, avoiding empty spaces of the order of 500 days. We must say, though, that there is no reason to consider the noise to be Gaussian. By virtue of the central limit theorem, under weak conditions, the sum of noise sources is *crudely approximated* (i.e. limited by *the form* of the sum) by a Gaussian distribution. In fact, most signals in physics are non-Gaussian in nature. We just have to add noise in order to make numerical simulation with contaminated data.

Table 1 shows the results of the tests with the confidence intervals for each one, under the null hypothesis of a linear generating mechanism. In general, a high consistency is found between the different tests, though each one shows variations due to the specific power each one has against certain non-linear mechanism detection. Nevertheless, none of the tests has the ability to isolate the origin of the non-linearity or the presence of chaos in the series. Because each test has different hypothesis, there is no compatibility between them. As a result, it is hard to use the statistical estimators jointly, though some are complementary.

In the case of the bispectral test, the statistical estimator has little power against some forms of chaos that produce irregular peaks widely spaced in the bispectrum, even though these are visually evident. That is why, in this test, the non-linear behavior can be graphically evident. In Figures 2 to 5 are shown the bispectra for the different signals.

For the bispectral test, also used were a couple of "toy signals", in order to compare them visually, namely a linear and a non-linear model of the form:

$$\dot{x} = \Theta x(t) + \sigma w(t)\,, \qquad (18)$$

$$\dot{x} = (\alpha - 0.5)\beta - x(t) + \sqrt{2\beta x(t)}w(t)\,. \qquad (19)$$

There is a clear difference between linear and non-linear models, where the bispectrum in the non-linear case shows higher order dependencies in the form of peaks away from the origin. This same behavior is observed in the bispectra of 3C390 for any form of reconstruction and for the complete series and the partial ones (i.e., without empty spaces). This shows that the non-linear nature of the light curve of 3C390 is not affected by any of the criteria taken in the application of this analysis.

## 5. CONCLUSIONS

Different alternatives have been presented in order to explain AGN variability. Most are linear in nature, such those that employ explosive time-independent events (i.e., shot noise) combined with fixed or variable decay time. Other exploit the stochasticity in the form of spatially independent active regions (i.e.. blobs). But it is because of this independence that the resulting approach is linear.

Other approaches, like local instabilities in the accretion rate with spatial independence, exist, but the light curves reproduced by these models are very uniform, and such regularity is not observed in real AGN curves.

Although others have made non-linear analyses, the use of only one kind of test, usually a less robust one than the ones presented here, is strange, and so the use of an assortment of tests becomes relevant. We have presented an exhaustive testing of the light curve of 3C390.3 with powerful statistical tools, which show with high consistency that 3C390.3 has a non-linear nature. This leads us to think that the modeling of the nuclear events of AGNs must take into account these non-linearities, i.e., the existing *feedback* between different mechanisms in the core of AGN and its environment, and not just select any single physical parameter and expect it to explain the central energetic processes of these objects.

We thank M. Hinich for useful suggestions on issues related to Gaussian noise source restrictions, which improved the paper. We acknowledge CONACYT for supporting partially this work through a grant fellowship.

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