SUMMARY: It is acknowledged by proponents of the Artifactual Theory of Fiction that literary works sometimes involve real or immigrant characters. However, their conception of cross-fictional identity faces serious difficulties. In this paper, we set the problem in the context of a modal framework, in relation to quantification across a plurality of possible worlds. Quantification is explained in terms of Hintikka’s notion of world lines; i.e. the possible values of bound variables are individuals that are not reduced to their manifestations. We finally offer another understanding of identity across weakly impermeable domains, in a way compatible with the Artifactual Theory of Fiction.

KEY WORDS: fictionality, identity, world lines, intentionality, creation

1. Introduction

Fictional works sometimes involve reference to real or historical characters (e.g. Napoleon, Stalin), to real places (e.g. London, Paris, Seville), and even to real events (e.g. Battle of Borodino, D-Day). They can also involve reference to fictional characters native to other fictions, as is the case in parodies (e.g. Maurice Leblanc’s Herlock Sholmès), in series (e.g. Alice, Holmes), and literary traditions and cycles (e.g. Pinocchio, Faust). Following Parsons (1980, pp. 51ff.)
we can call these characters from other fictions immigrants, by contrast with native fictional characters. Fictional discourse in which reference is made to entities that lie beyond the fiction itself is precisely what we call cross-fictional discourse. Cross-fictional discourse assumes identity preservation of characters across reality and fictional works; this is what we mean by cross-fictional identity.

In this paper, we call into question this assumption from the perspective of the Artifactual Theory of Fiction (ATF); mainly the theory initiated by Thomasson (1999) who considers that fictional entities are inhabitants of the actual world just as non-fictional ones are. They are abstract artifacts, created by an author and maintained in existence thanks to the existence of copies of the original work (or the memory of the members of a cultural community). Creation is accounted for by Thomasson in relation to the phenomenological notion of ontological dependency, which forms part of the identity conditions of fictional characters. Interestingly, this explains their contribution to the meaning of sentences (apparently) referring to them. Thomasson (2003) then holds the thesis that the ATF allows a uniform account for fictional discourse, by combining de re pretended assertions and reference to abstract entities. Cross-fictional identity is thus easily explained, since it depends on the author’s intention to refer to real or immigrant characters within his own fiction. Things are even easier if we think about the use of proper names from a Kripkean setting, where they refer to the same object in every context.1 Finally, an author can refer to an abstract artifact he has actually created and pretend to assert things about it in fiction; for example, that his character is a detective living at 221b Baker Street. Since it involves an abstract artifact in the actual world and in fictional discourse, cross-fictional reference is fundamental in Thomasson’s account.

However, Thomasson’s proposal faces serious difficulties. First, her account is incompatible with cases in which an author has the intention to merge or split different characters, regardless of whether these are real or fictional. We think that a modal semantics for fictionality helps in clarifying the problem, in relation to quantification across a plurality of worlds. In this context, we see the limits of a Kripkean setting, in which we directly quantify over objects that form part of the domain of different worlds. We propose a solution in the context of Hintikka’s world line semantics (WLS), in which

1 When speaking of Kripkean setting, we refer to the theses he held in Naming and Necessity (Kripke 1980).
individuals are world lines that connect different objects in different worlds. Cross-world identity is not unproblematic: World lines can merge or split, or even fail to indicate an individual in one world or another, depending on how they have been drawn. In intensional contexts, we must assume that world lines have been drawn and these world lines are the individuals we quantify over. Second, quantification over fictional characters presupposes that fictionally individuated world lines have been drawn. A criterion of individuation is needed to admit fictional characters in our ontology; i.e. in the domain of discourse. Thomasson’s criterion of identity does not provide a sufficient condition for the individuation of fictional characters. As a solution, we propose a syncretistic criterion according to which a character must have been created; and a world line must have been drawn between the manifestations of one and the same individual under different perspectives. In accordance with the ATF, fictional discourse can thus be explained in terms of reference to and quantification over created fictional entities. Third, we still have to explain how cross-fictional identity is possible. We argue that domains of fictions are weakly impermeable. That is, although a fiction cannot prescribe reference beyond itself, cross-fictional identity can result from interpretive efforts of the readers. More formally, this is explained by appealing to a distinction between extensional and intentional modes of predication.

We will restrict ourselves to literary fictional characters, although the point might be extended to other kinds of fictions. It is worth noting that we are concerned with a specific question concerning identity within and across fictions. We will not put forward new ontological or metaphysical arguments in favor of the ATF. We will not discuss the definition of ontological dependence in a modal framework either; this would require technicalities that go beyond the scope of the present paper. Actually, we can assume the semantics of Fontaine and Rahman (2014), which can be applied to WLs mutatis mutandis. Thus, we begin by discussing how cross-fictional identity is understood in the ATF, by Thomasson in particular (section 2). Then, we address the problem of merging and splitting characters in the context of a general modal framework for fictionality (section 3). A suitable notion of individual for fictional characters in

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a modal framework compatible with the ATF can be put forward in Hintikka’s WLs (section 4). Afterwards, we discuss the failure of Existential Generalization in WLs and argue that quantification in fictional contexts presupposes that world lines have been drawn, which in turn presupposes a more fine-grained criterion of individuation (section 5). Finally, we advocate an understanding of cross-fictional identity in a WLs for the ATF with weakly impermeable domains (section 6).

2. Artifactual Theory of Fiction

According to artifactualists, Thomasson (1999) in particular, fictional objects are inhabitants of domains of worlds just like non-fictional ones. On the one hand, they are creations or, more precisely, artifacts like chairs and buildings. On the other hand, they are abstract creations such as marriages, universities and theories. Fictional objects are bound to the everyday world by dependences on books, readers and authors. Fictional entities can therefore pertain to the domain of discourse and we can explain their contribution to the meaning of sentences like “Sherlock Holmes was created by Conan Doyle” or “there is a fictional character that is a detective in the fictions written by Conan Doyle”. Fictional objects can be the referents of both proper names and (definite) descriptions, and can serve as values of bound variables.

Thomasson (1999, pp. 35 ff.) identifies several types of ontological dependences. To define identity conditions for fictional characters, she combines the notions of historical and constant dependences with those of rigid and generic dependences. The fictional character Holmes has its origin in a particular creative intentional act of Conan Doyle, and is thus historically dependent on Conan Doyle. The ontological dependence of Holmes on Conan Doyle is of a rigid kind: Holmes depends historically on one fixed individual, namely Conan Doyle. That Holmes is rigidly historically dependent on Conan Doyle means that Holmes could not have existed if Conan Doyle (and nobody else) had not created it. After Conan Doyle’s death, Holmes survives as an abstract artifact because Holmes is ontologically sustained by copies of Conan Doyle’s work and a competent readership. Holmes’s dependence on copies is of a generic kind since there is no fixed copy on which Holmes depends. That Holmes generically constantly depends on the existence of copies means that Holmes would cease to exist if no copy existed (or if nobody remembered it). Ontological dependences form part of the identity of fictional characters.
Holmes could not be Holmes if he had not been created by Conan Doyle. In the famous short story written by Borges, *Pierre Menard, Author of the Quixote* (Borges 1944), Pierre Menard completely re-creates a story exactly identical with Cervantes’s *Don Quixote*. Since there are two disconnected creative acts, there are two corresponding created fictional characters. Although the two Don Quixote are internally indistinguishable, one is rigidly historically dependent on Cervantes and the other on Pierre Menard.

According to Thomasson (1999, pp. 52ff.), fictional characters are not odd entities. The Kripkean scheme of use of proper names can be applied in their case as well as for concrete entities. When Conan Doyle introduces the name “Sherlock Holmes”, he intentionally performs a baptism by giving a name to a fictional character he has created in the story. Afterwards, the referent of the name is maintained along a chain of dependences and transmitted in the literary community. Finally, when a competent user of the language correctly uses the name “Holmes”, he (rigidly) refers to that character created by Conan Doyle. However, how can we understand statements like “Holmes is a detective” given that no abstract artifact can have the property of being a detective? In order to understand the point, we must distinguish between the external and the internal viewpoints on fiction. Let the actual world be the world in which fictions are created: “Holmes was created by Conan Doyle” is true from an external perspective, in the actual world, but not in the fiction. By contrast, “Holmes is a detective” is true within fiction, from an internal viewpoint, but not in the actual world. Following Woods (1974), both viewpoints can be articulated and explicitly distinguished by means of a fictionality operator, to be read “according to the fiction”, in the scope of which the internal viewpoint is grasped. Whereas “according to the fiction, Frankenstein is a creation of Dr. Frankenstein” is true, “according to the fiction, Frankenstein is a creation of Mary Shelly” is not.

In a similar way, Thomasson (1999, pp. 105ff.) distinguishes between “real contexts” and “fictional contexts” by means of a “story operator”. Later, she develops her account and explains the semantics of the story operator in terms of *pretense* (Thomasson 2003, p. 205). The pretense account was put forward by Searle (1975). In a nutshell, the main idea is that there is no means to differentiate statements of fictional discourse from genuine (non-fictional) assertions. The difference is pragmatic and concerns the intentions of the author: although an author does not commit himself to tell the truth, he does not have the intention to lie either. He only has the intention to tell a story.
According to Searle, the author plays a game of pretense; that is, he only performs pretended assertions.\(^3\) When Lewis Carroll says in the fiction “Alice had not the slightest idea what Latitude was, or Longitude either, but she thought they were nice grand words to say”, he only pretends to assert something. Moreover, although Searle is an artifactualist, the pretended assertion is \textit{de dicto}, since by pretending to assert the author also pretends to refer. The author can have the intention to pretend to assert something about a concretely existent entity, in which case the pretense can be \textit{de re}. For example, when it is said “Napoleon gave commands concerning an invasion of England” in \textit{War and Peace}, the real Napoleon is the reference of the pretended assertion.

Nonetheless, why couldn’t existent abstract artifacts be the reference of \textit{de re} pretended assertions? Thomasson (2003) suggests that this would provide a more uniform account of fictional discourse: no matter whether their reference is a concrete entity or an abstract artifact, pretended assertions of fictional discourse can be \textit{de re}. When the story says “Frankenstein is a creation of Dr. Frankenstein”, the author pretends to assert of a monster that it is a creation of Dr. Frankenstein. When it is said “Frankenstein is a creation of Mary Shelly”, no pretense is involved. Cross-fictional reference is easily explained as well: When Lewis Carroll refers to Alice in \textit{Alice in Wonderland}, he may refer to a character previously created in \textit{Alice’s Adventures Underground} and pretend to assert other things about her. A necessary condition for the identity of characters \(x\) and \(y\) appearing in two different literary works \(K\) and \(L\) respectively is the following: The author of \(L\) is competently acquainted with \(x\) of \(K\) and intends to import \(x\) as \(y\) in \(L\) (Thomasson 1999, p. 67). Combined with other pragmatic criteria, this condition could provide a sufficient ground to decide issues of cross-fictional identity.

3. Merging and Splitting Characters

The \textit{de re} pretense account of internal discourse finds itself in trouble when an author has the intention to import different characters \(x\) and \(y\) as a unique \(z\) into the fiction \(L\); that is, to merge two characters in one. The same if he has the intention to split one character in two.

\(^3\) Another approach that focuses on the reader’s perspective is the \textit{make-believe} put forward by Currie (1990) and Walton (1990). For example, when a reader is sad because of a fiction, what actually happens is that the book serves as a prop for a game in which the reader makes believe of himself that he reacts to the story without really believing it.
different ones. For example, in *Animal Farm*, Orwell may have had the intention to use the name “Napoleon” to refer to a fusion of the historical Napoleon and Stalin (the real ones). When it is said in the story “Napoleon was a large, rather fierce-looking Berkshire boar, the only Berkshire on the farm, not much of a talker, but with a reputation for getting his own way”, what is the *de re* pretense about? If that Berkshire is both Napoleon and Stalin, we should draw the unwanted conclusion that Napoleon is Stalin by transitivity of the identity relation (for every $x$, $y$, $z$, if $x = y$ and $y = z$, then $x = z$). The same would follow if we considered that Orwell created a fictional character, since it would be identical with both. Is *Animal Farm* constituted of *de dicto* pretense, in which the reference to Napoleon and Stalin is only pretended? What would motivate this explanation if we agree with Thomasson that fictional discourse is made of *de re* pretense? We cannot escape the difficulty: we must explain how we can refer to or quantify over fictional entities that can merge or split.

We tackle the difficulty in a modal framework. We can thus ask the question in terms of cross-fictional reference and quantification. We begin by defining the semantics of fictional discourse with respect to a plurality of worlds. We will also consider worlds compatible with creation, and we will sketch the articulation with external considerations relative to ontological dependencies, as in the modal-temporal framework of Fontaine and Rahman (2014). The set of worlds of our modal framework will contain an actual world, in which the fictions are created. It allows for a (partial) representation of the external viewpoint. The internal viewpoint is interpreted in terms of a plurality of worlds compatible with fiction. We define the *content* of a fiction as the set of all the logical consequences of what is explicitly said in the fiction. A world compatible with the fiction is a world in which its content is true. Such worlds are accessible as long as there exist copies (or memories) of the original work. Given that fictions are usually incomplete descriptions, a plurality of worlds allows for the representation of different possible interpretations of the fiction. That is, what is left undetermined by the content can nevertheless be true or false in some (but not all) worlds compatible with the fiction, possibly on the basis of reading hypotheses. Although we need not

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4 Whether or not he really had this intention does not matter here. Such cases surely exist in the literature. See e.g. Voltolini (2012, p. 565) for other examples.

5 According to Wildman and Folde (2017), inconsistency and explosion (if it holds in fiction) may yield what they call “universal fiction”; i.e. a complete fiction in which everything is true.
commit ourselves with modal realism,\textsuperscript{6} we can follow Lewis (1978) who provides the semantics of the fictionality operator in a modal framework. Given that we focus on the behavior of characters across the worlds compatible with the fiction, a full and precise semantics of the fictionality operator is not needed.\textsuperscript{7}

4. Fictionality in WLs

We introduce in the language\textsuperscript{8} a fictionality operator $[F]_\alpha$, and its dual $<F>\alpha$, where the subscript restricts the scope of the operator to a determinate fiction $\alpha$. The former, read as “according to the fiction $\alpha$...”, is a content-relative operator. The latter is an interpretation-relative operator, to be read as “it is compatible with the fiction $\alpha$ that...”. For example, “Holmes is a detective” is true according to the content of \textit{A Study in Scarlet}. However, if the story is silent about Holmes wearing underpants, “Holmes wears underpants” is only compatible with the content. It needs not be true in every world compatible with the fiction.

The language is interpreted with respect to a modal framework $(W, R, D)$. $W$ is a set of worlds $w$. $R$ is an accessibility relation between those worlds. We can consider different relations $R_{F\alpha}$, $R_{F\beta}$, ..., respectively. Let $R_{F\alpha}(w)$ be the set of worlds compatible with the fiction $\alpha$, accessible from $w$. In the ATF we can add the restriction that $R_{F\alpha}(w)$ is not empty only if there exists at least a copy of $\alpha$ in $w$ (the accessibility is thus preserved by a constant ontological dependency). $D$ is the domain of individuals $d$.

What are the individuals of the domain $D$? How is it possible to refer to and to quantify over them in a modal framework? Can they form part of the domains of different worlds? Inspired by the work

\textsuperscript{6}That is, we consider possible worlds or alternatives as conceptual tools useful for semantic purpose, possibly human constructions rather than realities distinct from ours.

\textsuperscript{7}A full semantics of the fictionality operators should involve a study of inferences in its scope; e.g. to determine truth inferred from the content. For example, Friend’s “Reality Assumption” claims that reality is defeasibly imported in fiction (Friend 2017). Similar principles were already put forward by Woods; e.g. the “Antic-Closed World Assumption” or the “No Spin No Reader Thesis” (Woods 1974, 2010, 2018). Sophistication might be required to handle inconsistent fictions; e.g. by introducing impossible worlds as in Hintikka (1975), Rantala (1982a, 1982b), and later Priest (2016). By contrast, as stressed by Heyd (2006), if the narrator is not reliable, then it is the explicit content that could become untrue.

\textsuperscript{8}See Appendix [D2].
of MacDonald (1954) and Searle (1975), Genette (1991, pp. 49ff.) considers that fiction is impermeable to elements of reality. Indeed, there is no means to characterize fictional statements syntactically and we are left with the intention of the authors, which is to write a fiction. Therefore, even though sentences apparently describing reality might be used to build a fiction, the whole fiction is more fictional than its parts. As a consequence, fiction cannot refer beyond itself. Every attempt to express cross-fictional reference would be immediately fictionalized. Genette illustrates his proposal by a metaphor attributed to the French philosopher Paul Valéry: As a lion amounts to little more than digested mutton, fiction is little more than fictionalized reality. If each world has its own domain and if these domains are exclusive (i.e. no entity appears in more than one domain), then cross-fictional discourse is not possible. Even referring in fiction to actually existent fictional characters would not be possible. So, impermeability does not seem to be a suitable conception of the domains of fictions. From the perspective of the ATF, we wish to preserve the intuition that an abstract artifact created by Holmes is a detective in *A Study in Scarlet*. We may also wish to claim that London appears in *A Study in Scarlet* or that Faust is a character appearing in different literary works.

Permeability seems more appropriate to the ATF. It can be handled by means of a unique domain $D$, of which the domain $D_w$ of each world $w$ is a subset, and such that for two different worlds $w$ and $w'$ their respective domains $D_w$ and $D_{w'}$ may overlap.\(^9\) We can therefore quantify over individuals that appear in different worlds. In addition, and as suggested by Thomasson (1999, pp. 46ff.), given that fictional characters are existent abstract artifacts, fictional names can also be interpreted rigidly in accordance with Kripke’s theses (1980). Once the referent of a fictional name has been fixed in the course of an intentional baptism, the name refers to the same entity in every world. Just as “Napoleon” rigidly refers to Napoleon in the worlds compatible with *War and Peace*, “Holmes” rigidly refers to the character created by Conan Doyle in the worlds compatible with *A Study in Scarlet*, and this independently of their properties in these worlds. A well-known consequence of rigidity is the necessity of true identities between proper names. For example, if “Hesperus” and “Phosphorus” both refer to Venus in every world (in which it

\[^9\] That is, if domains are impermeable, no entity appears in more than one world. Then for every $w$ and $w'$ such that: $w \neq w' : D_w \cap D_{w'} = \emptyset$. But, if domains are permeable, then it can be the case that $D_w \cap D_{w'} \neq \emptyset$.  

*doi:10.22801/ifs.18704905c.2020.1174*  

*Crítica*, vol. 52, no. 154 (abril 2020)
exists), and if their semantic content is restricted to their reference, then they are necessarily identical. And if they refer to different objects, their reference is necessarily different. What is expressed by identity statements is nothing more than the relation that every object bears to itself. If rigidity is applied in fictional contexts, then \((k_1 = k_2) \rightarrow [F](k_1 = k_2)\) and \((k_1 \neq k_2) \rightarrow [F](k_1 \neq k_2)\) are valid principles. Therefore, if Napoleon and Stalin are different, “Napoleon” cannot refer to both of them in Animal Farm. We might think that giving up rigidity would be sufficient to avoid the difficulty. However, given that fiction should be about characters created in the actual world, we would be left with the validity of \((\forall x)(\forall y)((x = y) \rightarrow [F](x = y))\) which would be problematic as well. Therefore, the individuals of our modal framework must be conceived otherwise.

Individuals are possible values of bound variables. If modal language makes sense, it presupposes a notion of individual which explains how we can speak of the same entity across several possible worlds. Quantifying over objects that appear as such in different worlds is problematic. So, what does it mean to say that an object \(X\) in a world \(w_1\) is the same as an object \(Y\) in a world \(w_2\)? According to Tulenheimo (2017, pp. 11–12), this question is meaningless if we understand identity in quantitative terms. Indeed, two objects in two different worlds are inevitably different. Therefore, when we quantify in intensional contexts, we presuppose a notion of individual that is not reducible to its manifestations. In other words, possible values of bound variables are not objects pertaining to the domain of particular worlds. This notion of individual is the notion of world line, as it has been introduced by Hintikka in several of his works. Given a modal framework, each possible world has its proper domain of (world-bounded) objects. In addition, a set of individuals, which manifest themselves in different possible worlds by taking the appearance of world-bounded objects, is presupposed. Individuals are not part of any world in particular. There is no mark of them in the modal language either. They are nothing more but a precondition of first-order modal languages. They are supposed to be those entities that have manifestations in various possible worlds. To put it in

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10 It is worth noting that Kripke’s thesis was initially stated for metaphysical modality, and not for other kinds of intentional modalities like fictionality.

11 See Hintikka and Sandu 1995, and Tulenheimo’s 2017 (p. 20) ‘transcendental interpretation’ of the world lines, on that point. For temporal identity in world line semantics, see also Tulenheimo 2015.
Hintikka’s terms: “[I]n a context involving modal notions individuals have to be considered as members of several different possible worlds. An individual virtually becomes, for logical purposes, tantamount to the ‘world line’ [. . . ] connecting its manifestations in these possible worlds (1970a, p. 870).”

When we say that an object \(X\) in a world \(w_1\) is the same as an object \(Y\) in a world \(w_2\), we mean that \(X\) and \(Y\) are linked by a world line. Stated otherwise, \(X\) and \(Y\) are the manifestations or the embodiments of the same individual in different possible worlds. As suggested by Hintikka (1970b, p. 412), world lines can be mathematically represented by individual functions, whose argument is a possible world \(w\) and whose value is an object of the domain of that world \(w\).

The quantified modal language is now interpreted with respect to a model, defined as a tuple \(M = < W, R, J, \text{Int} >\).\(^{12}\) \(W\) is a non-empty set of worlds \(w\), each \(w\) having its own non-empty set \((\text{dom}(w))\) of local objects. \(R\) is a set of accessibilities \(R_{F\alpha}, R_{F\beta}, \ldots\), which relate to the worlds compatible with the fiction \(\alpha, \beta, \ldots\), respectively. \(\text{Int}\) is a function assigning to every \(n\)-ary predicate \(Q\) and world \(w\) a subset of \(\text{dom}(w)^n\), and to every individual constant \(c\) and world \(w\) an element of the set \(\text{dom}(w)\) when \(c\) has a referent in \(w\). \(J\) is a collection of world lines, and each element \(I \in J\) is a non-empty partial function on \(W\), assigning an element of \(\text{dom}(w)\) to every \(w\) on which this partial function is defined. When \(I(w) \in \text{dom}(w)\), we say that \(I\) is realized in \(w\) and we call the element \(d \in \text{dom}(w)\) such that \(I(w) = d\) the manifestation or realization of \(I\) in \(w\). Although world lines are the individuals available for quantification, they are not part of any world in particular.\(^{13}\)

It is worth noting that the value of a bound variable is a world line; i.e., if \(g\) is an assignment defined on \(x\), then \(g(x)\) is a world line. If this world line is realized in \(w\), the result \(g(x)(w)\) of applying the function \(g(x)\) to the world \(w\) is a local object belonging to \(\text{dom}(w)\).\(^{14}\) By contrast, the value of an individual constant \(c\) in a world \(w\) is an object pertaining to the domain \(\text{dom}(w)\) of that world, in accordance with its interpretation. The interpretation of a constant \(c\) in a world \(w\) can be empty. It is also non-rigid; i.e., its value need not be the same for every possible world.\(^{15}\)

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\(^{12}\) See Appendix [D3].

\(^{13}\) Our WLs is an adaptation of Tulenheimo’s semantics (2017, pp. 30–32).

\(^{14}\) See Appendix [D4].

\(^{15}\) See Appendix [D5].
Truth is defined with respect to a model $M$, a world $w$ and an assignment $g$ in a standard way. Quantifiers range over locally manifested individuals. Thus, when evaluating $(\exists x)[F]P(x)$ in $M$ in $w$, we must assign to the bound variable an individual realized in $w$. That is, $M, w, g \models (\exists x)[F]P(x)$ iff there is an individual $I \in J$ such that $I(w) \in \text{dom}(w)$ and $M, w, g[x: I] \models [F]P(x)$, iff for every $w' \in R_F(w)$: $M, w', g[x: I] \models P(x)$, iff $I(w') \in \text{Int}(P, w')$. This means that in de re readings like $(\exists x)[F]P(x)$, we quantify over individuals that manifest themselves in $w$ and every $w' \in R_F(w)$. By contrast, $M, w, g \models [F](\exists x)P(x)$ only assumes that the individual at stake appears in the worlds $w' \in R_F(w)$.

The interpretation of predicates and individual constants in a world $w$ is defined over the local objects pertaining to $\text{dom}(w)$. Individual constants are interpreted non-rigidly; that is, their reference is world-relative and need not be the same in every world. Identity is also world relative: it is the relation that every local object entertaines with itself. As a consequence, identity between individual constants (proper names) is contingent: it can hold in one world and not in another; $(k_1 = k_2) \rightarrow [F](k_1 = k_2)$ is not valid anymore. It is worth noting that rigidity would not be sufficient to rehabilitate the validity of this principle. In their virulent charge against Kripke, and more generally what they call “the New Theory of Reference”, Hintikka and Sandu (1995, p. 270) argued that we should assume in addition that modal individuals cannot merge and split. But there is generally no justification for that assumption in fictional contexts, for the reasons we previously mentioned. Therefore, two world lines can share the same manifestation in one world and not in another; $(\forall x)(\forall y)((x = y) \rightarrow [F](x = y))$ is invalidated, too.

5. Existential Generalization and Fictionally Individuated World Lines

WLs provides the means to explain how it is possible to quantify over individuals that manifest themselves in various circumstances. Fictional individuals are world lines that connect different objects of several worlds, without being reducible to them. Quantification in intensional contexts presupposes that world lines have been drawn, since they are the entities over which we quantify. How they are drawn is a very complex matter we cannot deal with exhaustively, but which deserves further comments. In general, they are drawn...
by an agent and they are context relative (i.e., relative to a modality and a relevant modal framework). The set of world lines available to an agent \( \alpha \) and relative to her set of beliefs need not be the same as the set of world lines available to another agent \( \beta \) (Hintikka 1967, p. 416). They do not constitute by themselves means to re-identify individuals, but they are drawn on the basis of criteria of cross-identification albeit not necessarily descriptive. Since criteria are not infallible, world lines may split or merge. Obviously, the author’s intentions are also crucial in the context of fictionality. This is sufficient to move from a Kripkean setting to WLs.

Even if the drawing of world lines presupposes that criteria of identification have been given, these criteria cannot be expressed in the language. Hintikka and Sandu (1995, p. 249) argue that their expression in the language would involve cross-world quantification. But cross-world quantification presupposes that world lines have already been drawn. That is, cross-world identity would be explained by presupposing cross-world identity, and the proposal would be doomed to circularity. As stressed by Hintikka (1969, pp. 109–110) in a very Kantian way, world lines are themselves human artifacts, which depend on modes of identification, have an objective reality, and make possible our transaction with reality. Nevertheless, the semantic role of world lines must be carefully distinguished from epistemic and cognitive considerations that would explain how they are drawn. From a semantic perspective, world lines are nothing more but a precondition for first order modal languages and cross-world quantification. This is, according to Tulenheimo (2017, p. 20), the ‘transcendental interpretation’ of world lines, by contrast with the rejected ‘epistemic interpretation’.\(^{17}\)

In this paper, we have tackled fictionality in a modal framework. In accordance with the ATF, it must be assumed that, together with the content of the literary work, world lines corresponding to the relevant fictional characters must have been drawn across the worlds compatible with the actual world fiction. For example, *The Adventures of Pinocchio* is a story about Pinocchio, and the character created by Collodi manifests itself in the worlds compatible with the fiction. Things might have been easy if Existential Generalization—a classically valid inference rule that allows the conclusion of \((\exists x)\phi(x)\) from \(\phi(k)\)—was valid in WLs. Indeed, we would be able

\(^{17}\) About this distinction, and the rejection of the epistemic interpretation (i.e., the interpretation of world lines as means of recognizing an individual), see Tulenheimo 2017 (pp. 20–24).
to infer the existence of a character that is a wooden marionette in *The adventures of Pinocchio*—say \((\exists x)[F]M(x)\) from \([F]M(p)\)—where \(p\) holds for “Pinocchio”. However, given the relative independence of the system of reference and the system of individuals—i.e. the fact that the interpretation of individual constants may behave independently from the world lines (Hintikka and Hintikka 1989, pp. 156–160)—such an inference is not guaranteed in WLs. In order to validly draw that conclusion, we must add an extra-premise of the form \((\exists x)[F](x = p)\) by means of which it is made explicit that \(p\) is associated with a well-identified individual. Under which conditions can we admit such an extra-premise?\(^{18}\)

WLs has nothing more to say in this respect. Again, we are concerned with a presupposition of quantification in a modal framework for fictionality. We may expect that world lines of fictional individuals are drawn on the basis of an acceptable criterion of individuation, even though such a criterion could not be expressed in the language. In order to understand Thomasson’s criterion, let us come back to the phenomenological tradition according to which we must distinguish between three components of intentional relations: the conscious act, the object towards which it is directed and the content of the act. The content is the way the object appears to consciousness. In the ATF, if there is no preexisting object, the intention creates it (Thomasson 1999, p. 88). The first time Conan Doyle thought of Holmes, he created it. Sometimes, creation is not instantaneous; it is the result of a process that can be diffuse (Thomasson 1999, p. 7). Creation can even occur through a creative illocutionary act (acts like marriages, contracts or promises). Emma Woodhouse is created by Jane Austen when she writes the following sentence: “Emma Woodhouse, handsome, clever, and rich, with a comfortable home and happy disposition, seemed to unite some of the best blessings of existence, and had lived nearly twenty-one years in the world with very little to distress or vex her” (Thomasson 1999, p. 12). Given that a fictional character might have been described otherwise by its author, its identity is warranted not by its internal properties but...
its origin (rigid historical dependence) and its generic dependence upon a literary work (Thomasson 1999, p. 39). In other words, the content, which corresponds to an internal description of the object as it appears to its author, is not relevant for the identity of a fictional character. But what are the identity conditions of the creative act, fundamental to the identity of fictional characters, if the content is not relevant? An inability to answer the question would undermine the ATF.

Voltolini (2006, p. 59) argues that Thomasson’s account does not even provide a sufficient condition for the individuation of fictional characters. When was Pinocchio created? Is it a creation of Collodi? Or is it a fictional character resulting from a tradition and very long creative process? Is the creation of Pinocchio actually finished? Let us assume, like Voltolini, that there is a possible world in which Collodi only writes (or merely thinks) “Mastro Cherry happened to come across a thing”, and then stopped writing (or thinking). Is it sufficient for generating Pinocchio, the same as in our world? Why not? Should we expect from the author a more substantial act of creation, for example by writing (or thinking) “how it happened that Mastro Cherry, carpenter, found a piece of wood that wept and laughed like a child”? Would it be sufficient? Why? What are the limits of the creative process? These difficulties led Voltolini to defend a moderate creationism, a syncretistic ontology, in which a set-theoretical element (a set of properties) is combined with a game-theoretical one (a game of make-believe). The properties are those mobilized in a game of make-believe in which the author is engaged while writing a certain text. A fictum is created when the author herself recognizes in a reflexive stance that such a set of properties is ascribed to a given individual.19 Without committing themselves to make-believe, Fontaine and Rahman (2014, p. 513) also put forward a criterion of individuation in which an external and an internal component are taken into account. We rephrase it for the sake of clarity:

[D1][LITERARY FICTIONAL INDIVIDUAL] An individual I is a literary fictional individual iff it satisfies the following two conditions:

19 “[A] reflexive stance in which such a [make-believe] process is taken as mobilizing a certain set of properties, the properties ascribed to a given pseudoindividual within that process. […] That stance manifests itself in one’s engaging in a piece of extrafictional discourse of the sort ‘FC is a fictional character’, where ‘FC’ is a singular term standing for a fictional entity. Once that reflexive stance occurs, a certain fictum arises” (Voltolini 2012, p. 563).
(i) it is an abstract artifact satisfying the relevant ontological dependencies in the worlds compatible with its creation (external aspect),

(ii) it has the characterizing properties in the worlds (partially) described by the relevant story (internal aspect).

The characterizing properties are the properties that describe the fictional character in the relevant fiction. Ontological dependencies are defined over worlds compatible with creation. The notion of a world compatible with creation is relative: from the actual world perspective, they are the worlds compatible with the creation of artifacts in the actual world. For example, if Holmes is a creation of Conan Doyle in the actual world, in every world compatible with Holmes’s creation accessible from the actual world, Holmes is also a creation of Conan Doyle. By contrast, in the worlds compatible with *A Study in Scarlet*, Holmes is described as a human being and has not been created by Conan Doyle. Indeed, worlds compatible with *A Study in Scarlet* are determined by the content of the literary work and need not be worlds compatible with Holmes’s creation. This criterion of individuation therefore assumes a multimodal framework, with accessibility relations of two kinds: one —say $R_C$— relates to the worlds compatible with creation, the other —say $R_F$— relates to the worlds compatible with fiction.

More concretely, in Fontaine and Rahman (2014), ontological dependencies are defined in a modal-temporal framework with standard Kripkean domains (i.e. where objects can pertain to the domain of several possible worlds). They first define a relation of modal requirement: $X$ modally requires $Y$ in $w$ iff in every $w'$ such that $w' \in R_C(w)$, if $X \in \text{dom}(w')$, then $Y \in \text{dom}(w')$. Ontological dependencies assume the fulfillment of the modal requirement plus other conditions (in order to avoid reflexive or symmetric relations for example). These relations apply to world lines by taking into account their manifestations in every world compatible with creation. For example, the individual $I$ modally requires $J$ in $w$ iff for every $w'$ such that $w' \in R_C(w)$, if $I(w') \in \text{dom}(w')$, then if $J(w') \in \text{dom}(w')$. Thus, $I$ is an abstract artifact if it satisfies the relevant ontological dependencies in the worlds compatible with creation. If in addition it satisfies the characterizing properties in the worlds compatible with the relevant fiction, $I$ is a literary fictional individual. As previously

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20 For the sake of simplicity, we can omit the temporal parameter.
explained, these worlds are accessible from worlds compatible with creation thanks to the existence of relevant copies of the fictional work and a competent readership. Thus, a literary fictional individual is a world line drawn from the actual world across the worlds compatible with creation and the worlds compatible with fiction in accordance with this syncretistic criterion of individuation.

The content of the creative act, fixed in the fictional work, is now essential to the individuation of fictional characters. As a consequence, if the content of Collodi’s creative act recorded in The Adventures of Pinocchio had been different, a different character would have been created. Such a criterion is assumed when a world line is drawn. That a fictional world line must have been drawn is a precondition of quantification in fictional contexts. As such, we should not expect an expression of this criterion in the formal language. Moreover, other considerations (e.g. pragmatic) could be taken into consideration, too. Nonetheless, we now have at our disposal a sufficient ground to accept the additional premise \((\exists x)[F](x = p)\) which allows, in accordance with the ATF, to infer \((\exists x)[F]φ(x)\) from \([F]φ(p)\). That is, internal fictional discourse is about a created fictional character, the same character that appears in the plurality of worlds compatible with the relevant fiction.

6. Weak Impermeability

If our criterion is sufficient to admit the presupposition that a fictionally individuated world line has been drawn, then each new creative act with its proper content yields a new character. Indeed, if Collodi’s creative act had been different, Pinocchio would not have been the same character. Nothing strange here. This is due to the relativity of world lines, not a will of overpopulating worlds with fictionally individuated individuals. However, does this become problematic if we consider that there are several fictions about (the same) Faust? Or that Maurice Leblanc’s Arsene Lupin Vs. Herlock Sholmès is a parody involving Holmes? How can we make sense of the notion of cross-fictional identity?

Sameness relation, understood as a cross-world relation, is now explained in terms of world lines connecting world-bounded objects. Impermeability must therefore be thought otherwise. Let \(w_◊\) be the actual world. For any fiction \(α\) and for every individual \(I ∈ J\), if \(I(w_◊) ∈ dom(w_◊)\), then for every \(w ∈ RFX_α(w_◊)\): \(I(w) \notin dom(w)\). This is clearly too strong for the ATF, given that fictional individuals created in the actual worlds also appear in the worlds compatible with
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We could weaken the proposal by restricting the impossibility to individuals which are not ontologically dependent in the actual world. As a consequence, *A Study in Scarlet* could be about an individual created by Conan Doyle in the actual world, but *War and Peace* would be about a fictional character different from the historical Napoleon. On the basis of Genette’s arguments, if impermeability is accepted we should probably conclude that cross-fictional identity is not possible either. That is, if an individual $I \in J$ is such that given a fiction $\alpha$, $I \in \text{dom}(w)$ for at least one $w \in R_{F_{\alpha}}(w)$, then for every fiction $\beta$ such that $\alpha \neq \beta$ and for every $w \in R_{F_{\beta}}(w)$: $I(w) \notin \text{dom}(w)$. Cross-fictional identity should be explained in terms of another kind of relation, possibly Lewis’s (1986) counterpart relation. However, in the spirit of the ATF, we wish to quantify over created fictional individuals that manifest themselves under different perspective, instead of quantifying over individuals that have counterparts in other possible worlds.

Proponents of the ATF usually see no problem in cross-fictional reference. In fiction, we can refer to both elements of reality and of immigrant fictional characters. This is perfectly compatible with our criterion. First, an author like Tolstoy may have had the intention to import the historical Napoleon as the character called “Napoleon” in *War and Peace*. In our account, the author creates a new world line. Then, in the worlds compatible with fiction, the two world lines share the same manifestations. Here, it is worth noting that the criterion given in [D1] only states the conditions under which a world line is drawn in its context of creation. It says nothing concerning its behavior across other contexts. Indeed, world lines are relative to a context, a modality, an agent and probably a number of other criteria. That is why, outside the worlds compatible with *War and Peace*, these world lines may split. In the actual world, they have different manifestations. This can also be the case in worlds compatible with an agent’s beliefs. Similarly, in the worlds compatible with *Animal Farm*, three world lines merge but they could split in other worlds.

Second, an author may have the intention to import a previously created character. For example, Maurice Leblanc intends to import Sherlock Holmes as Herlock Sholmès in *Arsene Lupin Vs. Herlock Sholmès*. Again, the world lines of Sherlock Holmes and Herlock Sholmès may split in other worlds. Let us consider that Maurice does not think that Holmes is as clever as he appears in Conan Doyle’s stories. The world line of Holmes is drawn beyond the original fiction, across the worlds compatible with what Maurice thinks. In these worlds, Holmes can behave differently from how he does in the

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doi:10.22201/iifs.18704905e.2020.1174
fiction. Nonetheless, Holmes’s origin remains the same: it is still the character created by Conan Doyle as he appears in *A Study in Scarlet*. Once created, Conan Doyle has no power on what happens to Holmes and its world line can be drawn on the basis of other criteria. However, as we have already explained, cross-fictional identity of that sort cannot be explicitly stated in the content. The expression of cross-fictional identity statements would be immediately fictionalized, and cross-fictional reference would be annihilated.

We might, of course, admit implicit and pragmatic reasons, but this would involve reading hypotheses and interpretations. For example, there are discussions on what is referred to by “Napoleon” in *Animal Farm*. Literary critics need not agree about the identity of the characters involved. The story is compatible with a reading according to which it is a satire of the Soviet political regime after the Russian Revolutions, a reading according to which it is a satire of the tyrannical governance of Napoleon after the French Revolution, and even with a reading according to which it is a satire of revolutions in general. This surely influences the way we interpret the name “Napoleon” in the novel. Even though there might be a *good* interpretation, defended by a majority of literary critics on the basis of accepted rules of inference, this does not mean that the content prescribes a uniquely acceptable interpretation. We thus suggest tackling cross-fictional identity not from the author’s perspective and his intentions, but from the reader’s perspective and his interpretation. If identity is left undetermined by the content, then various interpretations are compatible with the story.

Our proposal is that domains of fictions are weakly impermeable; that is, real and immigrant characters can appear in some (but not all) worlds compatible with a fiction, depending on certain interpretive efforts of the reader. The point can be clarified in relation to two modes of predication distinguished by Tulenheimo (2017, p. 36). The *extensional mode of predication* is the mode of predication assumed in the semantics previously defined (see section 4). It is first-order predication, relative to a possible world. In addition, for any n-ary predicate $Q$, the semantics induces an $(n + 1)$-ary relation $R_Q$ as follows: $<I_1, \ldots, I_n, w> \in R_Q$ iff all world lines $I_1, \ldots, I_n$ are realized in $w$ and $<I_1(w), \ldots, I_n(w)> \in Int(Q, w)$. Formulas $Q(x_1, \ldots, x_n)$ with $n$ free variables can be seen as n-ary intentional predicates, by means of which the *intentional mode of predication* is defined. An n-tuple $<I_1, \ldots, I_n>$ satisfies the intentional predicate $Q(x_1, \ldots, x_n)$ in a world $w$ iff $<I_1(w), \ldots, I_n(w)> \in Int(Q, w)$. The
predicate \( \phi(x_1,\ldots,x_n) \) applies in \( M \) in \( w \) to those \( n \)-tuples of world lines that satisfy it in \( M \) at \( w \).\(^{21} \)

In a modal framework for fictionality, intentional predicates can be built from complex formulas, such as \([F](P(x) \rightarrow Q(x))\), that can be applied to world lines such that in every \( w \) in which its realization satisfies \( P(x) \), it also satisfies \( Q(x) \). Usually, fictional characters are determined with respect to their intentional properties. Does Holmes wear underpants? If \textit{A Study in Scarlet} is silent about this property, then the world line representing Holmes in the modal framework does not satisfy the intentional predicate built from \([F] \text{Wear underpants}(x)\). However, if the question is addressed from the perspective of a given interpretation, in terms of extensional predication, then there are worlds compatible with the fiction in which the manifestation of Holmes satisfies \( \text{Wear underpants}(x) \) and other worlds compatible with the fiction in which it does not. Therefore, it is not correct that fictional characters are incomplete entities, even if their authors have not fully described them.

Identity can be conceived as an intentional relation too; that is, by considering it is built from \([F](x = y)\). It holds trivially for every character of the fiction, in particular between Dr. Jekyll and Mr. Hyde. However, it does not generally apply to cross-fictional identity, as in the examples of Pinocchio, Faust or Herlock Sholmes. It does not apply to Napoleon and Stalin in \textit{Animal Farm} either, since Orwell’s work admits different interpretations. Nonetheless, once the matter is put in terms of extensional predication, identity can hold between the manifestations of various world lines in some (but not all) worlds compatible with the fiction. This is what is meant when it is said that the world lines of different characters can merge and split.

Let \( I,J,K \in \mathcal{J} \) be the three different world lines representing the historical Napoleon, the historical Stalin and the fictional Napoleon, respectively. Let \( w@ \) be the actual world, \( I(w@) \neq J(w@) \), \( J(w@) \neq K(w@) \) and \( I(w@) \neq K(w@) \). The content says nothing more than \([F] \text{Animal Farm}(3x)(x = \textit{Napoleon})\). A model relevant to describe \textit{Animal Farm} will thus give the following:

1. \( M, w@, g[x/K] \models [F] \text{Animal Farm}(x = \textit{Napoleon}) \)
2. \( M, w@, g[x/I] \not\models [F] \text{Animal Farm}(x = \textit{Napoleon}) \)

\(^{21}\) By means of the notion of semantic value, given in Appendix [D7], a general formal definition of intentional predication is given in Appendix [D8].
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(3) $M, w_@, g[x/J] \not\models [F]_{\text{Animal Farm}}(x = \text{Napoleon})$

Which is perfectly compatible with:

(1) $M, w_@, g[x/I] \models <F>_{\text{Animal Farm}}(x = \text{Napoleon})$

(2) $M, w_@, g[x/J] \models <F>_{\text{Animal Farm}}(x = \text{Napoleon})$

The distinction between intentional and extensional modes of identity makes it possible to offer a more precise definition of weak impermeability for the ATF. Given a fiction $\alpha$, weakly impermeable domains are such that the world lines of real (Napoleon, Stalin, . . .) or immigrant characters (Faust, . . .) cannot satisfy intentional identity relations — whose semantic content is defined over $R_{\text{FA}}(w_@)$ — but can share the same manifestation in some (but not all) $w \in R_{\text{FA}}(w_@)$. By contrast, whereas permeability imposes no restriction on intentional identity, impermeability precludes extensional identity as well. Weak impermeability seems nevertheless better suited for the version of ATF we have defended, making sense of cross-fictional identity relations despite a stronger criterion of identity.

7. Splitting and Merging in Other Formal Frameworks

The problem of cross-fictional identity should not be restricted to the use of proper names or individual concepts (see section 4). As stressed by Fitting and Mendelsohn (1998, p. 206), non-rigidity invalidates Kripke’s necessity of identity between proper names ($(k_1 = k_2) \rightarrow \Box (k_1 = k_2)$) but not the necessity of identity of objects ($(\forall x)(\forall y)((x = y) \rightarrow \Box (x = y))$. World lines should not be confused with the linguistic notion of individual concept either. Indeed, following Carnap (1947), individual concepts are usually defined as functions that select for every context an individual as the referent of a given singular term. By contrast, the value of a world line in a possible world is a local object, not an individual; i.e. they cannot be values of quantified variables. Moreover, the mere introduction of a new individual concept does not entail the creation of a new individual; i.e. a new individual available for quantification.

In fact, rigidity of proper names does not preclude splitting and merging. Priest (2016, pp. 43ff.) rejects necessary identity in open worlds by means of a similar semantics in which proper names rigidly refer to individual functions whose values are identities (or roles) in the different worlds of the modal framework. As a consequence,
Existential Generalization, which in our setting assumes an extra-premise (see section 5), turns out to be valid. But, in the version of the ATF we defend, proper names are not always associated with the apparitions of a creation or a well-identified individual. Actually, Existential Generalization would have the undesirable consequence that everything that could be named in fiction should be the manifestation of a created individual. But, as stressed by Voltolini (2006, p. 209), who defends a similar view, we could name several Uruk-hai in The Lords of the Rings without being able to identify them precisely.

An appealing theory that handles the difficulties we have been confronted with is Lewis’s counterpart theory. According to Lewis (1986), cross-world quantification is explained in terms of counterpart relations between world-bounded individuals. However, this is not really compatible with our syncretistic criterion of individuation of literary fictional characters (see [D1]). Our individuals are not world-bounded entities; they are world lines, which cannot be reduced to their apparitions. In the worlds compatible with Animal Farm, the pigs called “Napoleon” are manifestations of one and the same literary fictional character created by Orwell. They are not mere counterparts of an abstract artifact created in the actual world. Although they can be defined in different manners, counterpart relations are usually determined by qualitative similarities in such a way that Napoleon in the worlds compatible with Animal Farm could be the counterpart of the real Napoleon or the real Stalin. But it is difficult to see how the same relation could hold between them and an actually existing abstract artifact that has almost none of the properties by means of which it is described in the fiction. It is perhaps possible to accommodate Lewis’s counterpart theory in order to solve these difficulties, but it seems more natural to account for the ATF in terms of created fictional individuals conceived as world lines.²²

8. Conclusion

In this paper, we have identified a number of requisites, in accordance with the ATF. First, in fictional discourse, we must be able to refer to and quantify over created entities that appear under different perspectives. Second, these individuals may split and merge depending on the author’s intentions. Third, an author may have the intention to import real or immigrant characters to a new fiction.

²² Tulenheimo (2017, p. 50) explains how Lewis’s counterpart theory can be simulated in WLs.
After having clearly set out the problem in a modal framework, in which the question was addressed in terms of cross-fictional identity and cross-fictional quantification, we were faced with a choice. On the one hand, Kripkean semantics dissolves the problem of cross-fictional identity and allows for quantification over individuals that form part of the domains of different worlds. However, it cannot explain how fictional characters sometimes merge and split. WLs solves this difficulty, but the price to pay is to acknowledge the problematic nature of cross-fictional identity.

In accordance with the ATF, the content should refer to a created entity. In WLs, unless an extra-premise is assumed, Existential Generalization is not valid and such an ontological inference requires further justification. However, the criteria usually offered by the proponents of the ATF, Thomasson in particular, are not sufficient for the individuation of literary fictional characters. We have thus strengthened the definition of fictionally individuated world lines by incorporating an internal component, namely the properties a fictional character is described as having in the story in which it originally arises. We finally had to meet the third requisite and explain how cross-fictional identity was possible given this syncretistic criterion of individuation. This we did by making a distinction between intentional and extensional identity, on the basis of which we defended a weakly impermeable conception of the domains. Depending on the interpretive efforts of the readers, two fictional characters can share the same manifestations in some (but not all) worlds.

To conclude, much more should be said on the inference rules at stake in interpretation. This would explain the notion of a world compatible with fiction in a more constructive way. It would also constitute a basis for understanding more precisely what is presupposed when world lines are drawn across worlds compatible with fiction. Nonetheless, we are confident that our framework provides a sufficiently general foundation in which various kinds of precisions could be implemented.

9. Appendix

[D2][LANGUAGE] Let Var be a set of variables and \( \tau \) be a relational vocabulary (a set of predicate symbols). For all \( n \geq 0 \), \( \tau_n \) is the set of \( n \)-ary predicate letters. Constant symbols are elements of \( \tau_0 \) (nullary predicate letters). The set Term is the set \( \text{Var} \cup \tau_0 \).

The quantified modal language \( L[\tau] \) of vocabulary \( \tau \) is built according to the following syntax:

doi:10.22201/ifs.18704905e.2020.1174

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\[ \phi ::= Q(t_1, \ldots, t_n)|t_1 = t_2|\neg\phi \wedge \phi|\phi \lor \phi|\phi \rightarrow \phi|[\mathcal{F}]\phi <\mathcal{F}> \]

where \( n \geq 1, Q \in \tau_n, < t_1, \ldots, t_n > \in Term \) and \( x \in Var \). 

Let \( L[\tau] \) be a quantified modal language of vocabulary \( \tau \) defined as before. It is now interpreted with respect to a model defined as follows:

[D3] [MODEL] A model is a structure \( M =< W, R, \mathcal{J}, Int > \). \( W \) is a non-empty set of worlds \( w \), each \( w \) having its own non-empty set \( dom(w) \) of local objects. \( R \) is a set of relations \( R_{\mathcal{F}} \) on \( W \). \( R_{\mathcal{F}}(w) \) is the set \( \{ w' : R_{\mathcal{F}}(w, w') \} \). \( Int \) is a function assigning to every \( n \)-ary predicate \( Q \) of \( \tau \) and element \( w \) of \( W \) a subset \( Int(Q, w) \) of \( dom(w)^n \), and to every individual constant \( c \) of \( \tau_0 \) and world \( w \) an element of the set \( dom(w) \cup \{ \ast \} \), where \( \ast \notin \cup_{w \in W} dom(w) \) (\( Int(c, w) = \ast \) indicates that \( c \) has no referent in \( w \)). \( \mathcal{J} \) is a collection of world lines, and each element \( I \in \mathcal{J} \) is a non-empty partial function on \( W \), assigning an element of \( dom(w) \) to every \( w \) on which this partial function is defined. 

[D4] [ASSIGNMENT] An assignment in \( M \) is a function of type \( Var \rightarrow \mathcal{J} \). If \( g \) is an assignment defined on \( x \), then \( g(x) \) is a world line. If this world line is realized in \( w \), the result \( g(x)(w) \) of applying the function \( g(x) \) to the world \( w \) is a local object belonging to \( dom(w) \). If \( g \) is an assignment and \( I \) is a world line, \( g[x := I] \) stands for an assignment that differs from \( g \) at most in that it assigns \( I \) to \( x \). 

[D5] [VALUE OF A TERM] Value \( t^{M,w,g} \) of term \( t \) in model \( M \) at world \( w \) under assignment \( g : Var \rightarrow \mathcal{J} \):

\[
\begin{align*}
t^{M,w,g} & = \quad Int(t, w) \text{ if } t \in \tau_0 \text{ and } Int(t, w) \neq \ast. \\
& = \quad g(t)(w) \text{ if } t \in Var \text{ and } g(t) \text{ is realized in } w. 
\end{align*}
\]

[D6] [TRUTH IN A MODEL] Truth is defined with respect to a model \( M \), a world \( w \) and an assignment \( g \) as follows:

- \( M, w, g \models Q(t_1, \ldots, t_n) \) iff for all \( 1 \leq i \leq n \), the value \( t_i^{M,w,g} \) of the term \( t_i \) in \( M \) at \( w \) under \( g \) is defined, and the tuple \( < t_1^{M,w,g}, \ldots, t_n^{M,w,g} > \) belongs to \( Int(Q, w) \).
- \( M, w, g \models t_1 = t_2 \) iff for all \( i \in \{1, 2\} \), the value \( t_i^{M,w,g} \) of the term \( t_i \) in \( M \) at \( w \) under \( g \) is defined and \( t_1^{M,w,g} \) equals \( t_2^{M,w,g} \).
\( M, w, g \models \neg \phi \text{ iff } M, w, g \not\models \phi. \)

\( M, w, g \models \phi \land \psi \text{ iff } M, w, g \models \phi \text{ and } M, w, g \models \psi. \)

\( M, w, g \models \phi \lor \psi \text{ iff } M, w, g \models \phi \text{ or } M, w, g \models \psi. \)

\( M, w, g \models \phi \rightarrow \psi \text{ iff } M, w, g \not\models \phi \text{ or } M, w, g \models \psi. \)

\( M, w, g \models [\mathcal{F}]\phi \text{ iff for all } w' \in R_{\mathcal{F}}(w): M, w', g \models \phi. \)

\( M, w, g \models \triangleleft \mathcal{F} \phi \text{ iff there is at least one } w' \in R_{\mathcal{F}}(w) \text{ such that } M, w', g \models \phi. \)

\( M, w, g \models \forall x \phi \text{ iff for all } I \in \mathcal{J} \text{ such that } I(w) \in \text{dom}(w): M, w, g[x := I] \models \phi. \)

\( M, w, g \models \exists x \phi \text{ iff there is } I \in \mathcal{J} \text{ such that } I(w) \in \text{dom}(w) \text{ and } M, w, g[x := I] \models \phi. \)

[D7] [SEMANTIC VALUE] Let \( M \) be a model, and let \( \phi(x_1, \ldots, x_n) \) be a formula of the language \( L_0 \). The semantic value \( [\phi(x_1, \ldots, x_n)]^M \) of \( \phi \) in \( M \) is the set of all \((n+1)\)-tuples \(<w, I_1, \ldots, I_n> \in \text{dom}(M) \times \mathcal{J}^n\) such that: \( M, w, x_1 := I_1, \ldots, x_n := I_n \models \phi(x_1, \ldots, x_n) \).

If \( \phi \) is a sentence, then \( [\phi]^M \) is a (possibly empty) subset of \( \text{dom}(M) \) —namely, the set of worlds \( w \) at which \( \phi \) is true in \( M \). 

[D8] [INTENTIONAL PREDICATION] Ascribing \( \phi(x_1, \ldots, x_n) \) to the tuple of world lines \(<I_1, \ldots, I_n> \) in \( w_0 \) under the intentional mode relative to a state \( i \) is to affirm that \(<w, I_1, \ldots, I_n> \in [\phi]^M \) for all worlds \( w \in R_{\iota}(w_0) \cap \bigcap_{1 \leq j \leq n} \text{mar } g(I_j) \). 

REFERENCES


The domain of the partial function \( I \) is its modal margin, denoted \( \text{mar } g(I) \).

I acknowledge financial support of the postdoctoral program of the FCT-Portugal (Grant Number SFRH/BPD/116494/2016) and the FCT CFCUL UID/FIL/00678/2019. I warmly thank anonymous reviewers and Bruno Jacinto for their helpful comments and suggestions.


