The actual logical forms corresponding to the ten interpretations of the conditional

Las formas lógicas reales correspondientes a las diez interpretaciones del condicional

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Abstract

This paper explores the idea of a syntax of thought proposed by theories such as the one of the mental logic. Taking into account that syntax as it is described by the mental logic theory can have some problems, this paper offers a possible alternative syntax based only on conjunctions, disjunctions, and negations. This alternative syntax is mainly studied here in the case of conditional reasoning and follows the mental models theory, a framework that considers semantics to be more relevant than syntax, and that hence is, in principle, incompatible with the mental logic theory.

Keywords: *Conditional Reasoning, Iconic Models, Logical Forms, Semantics, Syntax*.

Resumen

Este trabajo explora la idea de una sintaxis del pensamiento propuesta por teorías como la de la lógica mental. Teniendo en cuenta que tal sintaxis, como es descrita por dicha teoría, puede tener algunos problemas, se ofrece aquí una posible sintaxis alternativa de ese tipo basada exclusivamente en conjunciones, disyunciones y negaciones. Esta sintaxis alternativa se estudia principalmente con respecto al caso del razonamiento condicional y sigue la teoría de los modelos mentales, esto es, de un marco que considera que la semántica es más relevante que la sintaxis y que, por tanto, es, en principio, incompatible con la teoría de la lógica mental.

Palabras clave: *formas lógicas; modelos icónicos; razonamiento condicional; semántica; sintaxis*.

Introduction

Based on several previous theoretical frameworks and experiments carried out by its proponents, the mental logic theory claims that human thought works following a particular syntax, which is very similar but not identical to that of classical logic (e.g., Braine & O'Brien, 1998a). In this way, that theory speaks about purely formal structures underlying the expressions in natural language to which inference schemata are applied in order to deduce conclusions (see also, e.g., Braine & O'Brien, 1998b). However, nowadays this idea seems to have several difficulties and, therefore, maybe it can be hard to accept a «syntax of thought» exactly such as the one proposed by the mental logic theory. And this is so because it appears that people do not always apply the formal rules indicated by it and individuals sometimes use other schemata explicitly rejected by that very theory (e.g., Orenes & Johnson-Laird, 2012).

Nevertheless, this paper tries to show that, while it is true that the specialized literature seems to challenge the thesis of the existence of the specific syntax of thought to which the mental logic theory refers, it is also true that the results reported in that literature do not completely remove the possibility that there is some other mental syntax with other features. In particular, following several works on this issue (e.g., López-Astorga, 2017a), a very simple possible syntax of that type will be described here. As accounted for below, it will be a syntax with only conjunctions, disjunctions, and negations, and one of its most interesting characteristics will be that those logical operators will work in that syntax in a manner very akin to that as they do in systems such as the one of Gentzen (1934, 1935).

The general idea of such works (another example in this regard can be the paper of López-Astorga, 2017b), whose overall framework will be called, from now on, using the acronyms MS (coming from the expression «Mental Syntax», which is used in order to

differentiate this syntax from the syntax of thought supported by the mental logic theory), really emanates from essential theses of other approach, the mental models theory (e.g., Johnson-Laird, Khemlani, & Goodwin, 2015). This last theory is actually a rival framework to the mental logic theory, the main reason of that being that the former explicitly rejects the idea that there is logical forms in the human mind corresponding to the sentences in natural language, and holds that people only think by analyzing models that represent reality and are iconic (e.g., Johnson-Laird, 2010). Nonetheless, the exact idea of MS is to go beyond the mental models theory and its iconic representations, and to transform those models into well-formed formulae similar to those of classical propositional calculus, with which it is possible to come, by means of exclusively formal operators, to the same conclusions predicted by the mental models theory.

In this way, the concrete goal of this paper is basically to argue that the proposal of MS can be accepted. And, to achieve that aim, the paper will focus mainly on the conditional and resort to a work supporting the mental models theory (Johnson-Laird & Byrne, 2002) in which ten possible interpretations of this last connective are distinguished. Those interpretations are given because, according to the mental models theory, there are ten different types of conditional that, although they include in natural language the words «if» and «then», are related to different combinations of iconic models. Thus, what will be done is to transform those ten combinations of those ten interpretations of the conditional into well-formed formulae following the general lines of MS. From there on, the purpose will be to show that the conclusions to which such formulae can lead in habitual reasoning tasks with the formal structures of Modus Ponendo Ponens, Modus Tollendo Tollens, Affirming the Consequent Fallacy, and Denying the Antecedent Fallacy are exactly the same as those of the initial iconic representations of the mental models theory. This will in turn allow raising that any experiment in this regard demonstrating that the mental models theory is right will also prove that MS is correct.

All of this will be done in the following way. Firstly, the basic framework on the conditional of the mental models theory and the ten interpretations this last theory assigns to that connective will be described. Then, from those ten interpretations, logical formulae with just conjunctions, disjunctions, and negations corresponding to them will be built. This part of the paper will not be absolutely new, since in the literature logical forms for some of such interpretations are to be found (e.g., López-Astorga, 2017b). However, the novelty will be here that all of the ten interpretations mentioned above will be given logical forms in a systematic way and that, as said, this will be done resorting only to conjunctions, disjunctions, and negations. Thus, thirdly, each of the ten logical forms detected will be considered separately and four reasoning tasks with the structures of Modus Ponendo Ponens, Modus Tollendo Tollens, Affirming the Consequent Fallacy, and Denying the Antecedent Fallacy will be constructed for each of them. In such tasks, the first premise will be a sentence in natural language with the words «if» and «then» of the kind exactly corresponding to the particular logical form that is being addressed in that specific moment. This, clearly, will enable to identify the conclusions that could be logically derived in those reasoning tasks. And that will in turn allow checking whether or not such conclusions are precisely those that the mental models theory predicts, and hence whether or not it is true that the fact that the mental models theory is correct necessarily implies that MS is so too.

Furthermore, as an illustrative datum, it can be added that, although, as stated, the literature seems to make it evident that the mental logic theory is not absolutely right, it has also been argued that, in certain particular aspects, it does work, and its predictions in connection to the ten types of sentences detected by the mental models theory have been analyzed using hypothetical problems with Modus Ponendo Ponens, Modus Tollendo Tollens, Affirming the Consequent Fallacy, and Denying the Antecedent Fallacy as well, the result of those analyses being that, equally, the predictions are the same (López-Astorga, 2016). So, it could also be said that, while there are some reasons to think that is not so, even if the mental logic theory correctly described human conditional thought, that

would not demonstrate that MS does not hold either, since the predictions match in this case too. However, the discussion on the validity of the mental logic theory is beyond the scope of this paper and, accordingly, it continues with a general description of the part of the mental models theory interesting here.

The general framework of the mental models theory and its conception about the conditional

As indicated, the mental models theory speaks about iconic mental representations or models describing reality (see also, e.g., Johnson-Laird, 2012). In the case of the conditional, those models are three, which can be expressed, resorting to a relatively recent way its proponents use (Khemlani, Hinterecker, & Johnson-Laird, 2017), as follows:

[I] Possible (p & q) & Possible (not-p & q) & Possible (not-p & not-q)

In [I], Possible (a) stands for a possible iconic scenario in which a is true. Nevertheless, two important points should be highlighted here. On the one hand, it is not always easy to detect the three possible scenarios in [I]. Some of them, especially the second one and the third one, are sometimes built after a certain mental effort. On the other hand, and this is more important for the aims of this paper, the scenarios can be modulated (e.g., Quelhas, Johnson-Laird, & Junos, 2010), which means that, in a set of possibilities such as [I], some models can be impossible and, for that reason, eliminated, and, in the same manner, certain scenarios that are not included can be possible and, for that reason, added. This can be clearly noted by means of the following example, which corresponds to the kind of conditional named «Enabling» in several works of the literature on the mental models theory (e.g., Johnson-Laird & Byrne, 2002):
[II] "If oxygen is present then there may be a fire" (Johnson-Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 281).

If these equivalences are assumed:

p: oxygen is present.
q: there is a fire.

It is obvious that the combination of possibilities of [II] is not the one of [I], but:

[III] Possible (p & q) & Possible (p & not-q) & possible (not-p & not-q)

As it can be noted and is explained in a number of works (e.g., Johnson-Laird & Byrne, 2002; López-Astorga, 2016), what happens is that the scenario Possible (not-p & q) that was in [I] has disappeared in [III], the reason of that being evident: it is not possible that there is a fire without oxygen. However, in the same way, another scenario has been added: Possible (p & not-q). And the reason of that is also clear, since oxygen can be present and that does not necessarily imply that there is a fire, that is, since p & not-q is obviously a possibility consistent with [II].

This is the manner modulation works in general in the mental models theory. Nevertheless, what is truly interesting for this paper in this regard is that, as it is also accounted for or can be derived from papers such as those indicated (i.e., Johnson-Laird & Byrne, 2002, and López-Astorga, 2016), all of this has consequences related to schemata such as Modus Ponendo Ponens, Modus Tollendo Tollens, Affirming the Consequent Fallacy, and Denying the Antecedent Fallacy. As it is well known, in classical propositional calculus, the two former are correct and the two latter are wrong, and that is easy to note if an habitual and non-modulated conditional is considered, that is, if a conditional to which [I] can be assigned is considered. Indeed, [I] reveals that Modus Ponendo Ponens ([p →q, p] ⊢ [q]) is right because, in the only possibility in which p is true (the first one), q is true as well. In the same way, it also shows that Modus Tollendo Tollens ([p → q, ¬q] ⊢ [¬p]) should be accepted, as the only scenario in which q is false (the third one) is a scenario in which p is also false. But, on the other hand, according to [I], Affirming the Consequent Fallacy and Denying

the Antecedent Fallacy are actually fallacies. Affirming the Consequent ([p → q, q] ⊭ [p]) is so because there are two scenarios in which q is true (the first one and the second one), and in one of them (the first one) p is true and in the other one (the second one) p is false. Likewise, Denying the Antecedent ([p → q, ¬p] ⊭ [¬q]) is also so because there is two scenarios in which p is false too (the second one and the third one), and, in a similar way, q is true in one of them (the second one) and false in the other one (the third one).

Nonetheless, all of this is different in the case of [III]. Now, Modus Ponendo Ponens cannot be admitted because p is true in two iconic possibilities (the first one and the second one), and q is true in one of them (the first one) and false in the other one (the second one). Equally, Modus Tollendo Tollens also has to be rejected, since q is false in two possibilities (the second one and the third one), and, however, p is true in one of those possibilities (the second one) and false in the other one (the third one). Nevertheless, neither Affirming the Consequent nor Denying the Antecedent are fallacies in this case. The former is correct because there is only one scenario in which q is true (the first one) and that is a scenario in which p is true too. As far as the latter is concerned, there is only one possibility in which p is false as well (the last one) and that is a possibility in which q is not true either.

But the most important point of these facts for this paper is that, as said, if sets of possibilities such as [I] and [III] are transformed into well-formed formulae of classical logic, it is possible to come, mainly following requirements of that logic, to the same conclusions as the mental models theory, and this applies both for those two interpretations of the conditional and for the eight remaining interpretations of this last connective it proposes. This will be shown below. However, before that, it seems to be necessary to indicate which those remaining interpretations are exactly. To do that, all of the interpretations of the conditional of the mental models theory (including the two ones mentioned) are presented now in the same way, using the names given to them in the cognitive science literature (e.g., Johnson-Laird & Byrne, 2002), and resorting to particular examples that are to be found in that very literature (all of the examples that will be considered here come from Johnson-Laird & Byrne, 2002).

The first one is «Tautology». This is an interpretation in which all the scenarios are possible, and an example of it can be as follows:

[IV] "If there are lights over there then there may be a road" (Johnson-Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 283).

As stated, all the scenarios are possible in Tautology and so its models can be akin to these ones:

[V] Possible (there are lights over there & there is a road) & Possible (there are lights over there & there is not a road) & Possible (there are not lights over there & there is a road) & Possible (there are not lights over there & there is not a road)

The second interpretation can be «Conditional», that is, the usual interpretation of the conditional, which has already been commented on above and corresponds to [I]. An example with thematic content can be the following:

[VI] "If the patient has malaria then she has a fever" (Johnson-Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 285).

As mentioned, the combination of scenarios that can be linked to a sentence such as this one is [I], which, in the case of this particular example, can also be presented in this way:

[VII] Possible (the patient has malaria & the patient has a fever) & Possible (the patient does not have malaria & the patient has a fever) & Possible (the patient does not have malaria & the patient does not have a fever)

Other interpretation is clearly Enabling. An example has already been given above ([II]), and it has already been explained that its set of possibilities has to be [III] too. Nevertheless, if the thematic

content of [II] is taken into account, another way to express [III] can be this one:

[VIII] Possible (oxygen is present & there is a fire) & Possible (oxygen is present & there is not a fire) & Possible (oxygen is not present & there is not a fire)

However, one more interpretation with three possible scenarios can be «Disabling». A Disabling sentence is, for example, the following:

[IX] "If the workers settle for lower wages then the company may still go bankrupt" (Johnson-Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 287).

And, clearly, the models of a sentence of this kind are:

[X] Possible (the workers settle for lower wages & the company goes bankrupt) & Possible (the workers settle for lower wages & the company does not go bankrupt) & Possible (the workers do not settle for lower wages & the company goes bankrupt)

Of course, there are also interpretations that only allow two scenarios. One of them often receives in the literature a name used in logic to refer to an operator: «Biconditional». An example of this kind of sentence can be as follows:

[XI] "If she owes money then she must repay it" (Johnson-Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 289).

Maybe what happens with this interpretation is the same as it occurs with Conditional: it is assigned a habitual logical denomination to it because its iconic possibilities match the combinations of possibilities in which the logical operator with the same name is true in a truth table. In any case, certainly, a sentence such as [XI] is related to these iconic representations in the mental models theory:

[XII] Possible (she owes money & she must repay it) & Possible (she does not owe money & she does not have to repay it)

Nevertheless, there are two more interpretations referring to only two scenarios. One of them is «Strengthen Antecedent», and a clear example of it is:
[XIII] "If there is gravity (which there is) then your apples may fall" (Johnson-Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 290).

Indeed, this sentence only admits two possibilities. Nonetheless, they are not similar to those of [XII] but undoubtedly:

[XIV] Possible (there is gravity & your apples fall) & Possible (there is gravity & your apples do not fall)

And the other interpretation with two possibilities is «Relevance», a kind of sentence to which this example corresponds:

[XV] "If you are interested in seeing Vertigo then it is on TV tonight" (Johnson-Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 291).

True, it is not hard to note that those are the possibilities that can be related to [XV]:

[XVI] Possible (you are interested in seeing Vertigo & Vertigo is on TV tonight) & Possible (you are not interested in seeing Vertigo & Vertigo is on TV tonight)

Finally, the three remaining interpretations are sets with just one possible scenario, since each of them can be linked to just one possibility. The name of the first one is «Tollens» and it can refer to sentences with a figurative sense such as this one:

[XVII] "If it works the I'll eat my hat" (Johnson- Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 292).

It is clear that the speaker does not mean by [XVII] that the possibility that he/she really eats his/her hat exists. What the speaker actually means is that the only possible scenario is as follows:

[XVIII] Possible (it is not working and I am not eating my hat)

Something similar is what happens in the case of the interpretation «Ponens», the main difference between them being that in Ponens none of the clauses is negated. A simple example for this last case can be this one:

[XIX] "If my name is Alex then Viv is engaged" (Johnson-Laird & Byrne, 2002: 663; see also, e.g., López-Astorga, 2016: 293).

Certainly, there is no doubt that the only possibility indicated by [XIX] is:

[XX] Possible (my name is Alex & Viv is engaged)

And the last interpretation keeps having to do with irony and figurative language. Its denomination is «Deny Antecedent and Affirm Consequent» and a clear example of it is the following:

[XXI] "If Viv has been so kind to Pat then Pat as a devout person must forgive Viv" (Johnson-Laird & Byrne, 2002: 663; see, also, e.g., López-Astorga, 2016: 294).

Of course, what [XXI] truly means is that Viv has not been kind and that, however, Pat should forgive Viv. Thus, it just indicates this possibility:

[XXII] Possible (Viv has not been kind to Pat & Pat must forgive Viv)

These are the ten interpretations provided by the mental models theory (in particular, by Johnson-Laird & Byrne, 2002) with the names that this same theory (in particular in, e.g., again, Johnson-Laird & Byrne, 2002) gives to them. In this way, before continuing, it only appears to be important to add, even if that is only as an illustration, that, as said, despite their apparent problems, theories such as the mental logic theory have the same predictions as the mental models theory with regard to Modus Ponendo Ponens, Modus Tollendo Tollens, Affirming the Consequent Fallacy, and Denying the Antecedent Fallacy for the ten interpretations described. The reasons why this is so are to be found in the literature (López-Astorga, 2016) and will not be repeated here. However, it can be opportune in this regard to acknowledge, beyond that, that the structure of this paper from now on will be very akin to the one of the main work in which such reasons are accounted for (i.e., to the one of López-Astorga, 2016). Truly, as in that work, each of the ten interpretations of the conditional will be addressed here separately (and in the same order as in that very work) to check predictions about the four schemata with their first premise being, in each of the cases, the corresponding type of conditional indicated above. The difference will be, as pointed out, that this paper will only use logical formulae such as those that can be got from the method proposed by MS and containing just conjunctions, disjunctions, and negations. So, following the order mentioned, the next section is devoted to Tautology.

Tautology

The proposal of MS is actually very simple. Based on truth tables of classical logic, as stated, well-formed formulae are built from the combinations of possibilities of the mental models theory. This last theory already claims that each possibility is internally a conjunction (e.g., Khemlani et al., 2017). Thus, respecting this point, the only change MS introduces is that it gives logical features to such conjunctions and considers each of possibilities to be really

disjuncts in a disjunction (the reason of this being precisely that they are possibilities). In this way, for example, given this set of possibilities,

[XXIII] Possible (α) & Possible (β)

The transformation of MS leads to this formula:

[XXIV] $\alpha \vee \beta$

Which means that, if [V] is taken into account, the formula that can be constructed is the following:

[XXV] $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

Obviously, now, «p» stands for «there are lights over there», «q» represents «there is a road», and «&» has been replaced by «∧» in order to highlight that it is the logical conjunction. However, perhaps the most important point here is that a possible objection against a formula such as [XXV] can be that it ignores modal logic and its symbols. Indeed, the fact that [IV] includes a word such as «may» can lead one to think about frameworks more or less similar to those described in works such as the ones of Kripke (1959, 1962, 1963a, 1963b, 1965). Nevertheless, although, undoubtedly, studies in this direction can be raised and clearly justified, this paper, both in the case of [IV] and in the one of some other examples already cited in the previous section and that will be dealt with below, is intended to work, as stated, only with conjunctions, disjunctions, and negations that behave in a similar way as those connectives do in classical logic. This is so because, as also said, the idea is essentially to check whether or not a syntax of thought as simple as the one that can be derived from the works supporting MS can be, at least a priori, admitted. So, while modal symbols and systems can be always suitable in logic, to use them here is to go beyond the aims of this paper.

Thus, what is interesting here is hence to identify the predictions that can be made for hypothetical reasoning tasks with the structures of Modus Ponendo Ponens, Modus Tollendo Tollens, Affirming the Consequent Fallacy, and Denying the Antecedent Fallacy in which the first apparent conditional premise really has a logical form such as [XXV]. Accordingly, it can be said that the predictions that have to be detected are the corresponding ones to hypothetical problems such as these ones:

[XXV]
p

Ergo q?

[XXV]
¬q

Ergo ¬p?

[XXV]
q

Ergo p?

[XXV]
¬p

Ergo ¬q

Evidently, these four schemata try to represent, respectively, tasks of Modus Ponendo Ponens, Modus Tollendo Tollens, Affirming the Consequent Fallacy, and Denying the Antecedent Fallacy in which the first premise can seem to be a material conditional (i.e., a conditional as understood in classical logic) because is expressed in natural language by means of words such as «if» and «then», but it is really a sentence with, as said, a true logical form such as [XXV]. In

this way, the predictions, following MS, on what hypothetical participants would respond to tasks such as the previous ones are evident too:

Modus Ponendo Ponens: it cannot be known whether or not q can be deduced, since $\{[XXV] \wedge p\} \nvdash [q]$. At most, $\{[XXV] \wedge p\} \vdash [(p \wedge q) \vee (p \wedge \neg q)]$. So, it is not possible to know the truth-value of q.

Modus Tollendo Tollens: it cannot be known whether or not $\neg p$ can be inferred, as $\{[XXV] \wedge \neg q\} \nvdash [\neg p]$. At most, $\{[XXV] \wedge \neg q\} \vdash [(p \wedge \neg q) \vee (\neg p \wedge \neg q)]$. So, it is not possible to know the truth-value of p.

Affirming the Consequent Fallacy: it cannot be known whether or not p can be concluded, because $\{[XXV] \wedge q\} \nvdash [p]$. At most, $\{[XXV] \wedge q\} \vdash [(p \wedge q) \vee (\neg p \wedge q)]$. So, it is not possible to know the truth-value of p.

Denying the Antecedent Fallacy: it cannot be known whether or not $\neg q$ can be derived, since $\{[XXV] \wedge \neg p\} \nvdash [\neg q]$. At most, $\{[XXV] \wedge \neg p\} \vdash [(\neg p \wedge q) \vee (\neg p \wedge \neg q)]$. So, it is not possible to know the truth-value of q.

Therefore, the predictions are clear in the case of Tautology. They are so in the case of the other nine interpretations as well, and that is shown below. The next section continues with the next interpretation.

Conditional

Now, the set of combinations that has to be transformed into well-formed formula is [VII], and, if something similar to what has been done in the previous section with [V] is done, the result is clear:

[XXVI] $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

Obviously, [XXVI] is a formula equivalent to $p \rightarrow q$ and, accordingly, the predictions here are very evident too, since they are exactly the same as those that, from classical logic, could be assigned to the standard material conditional. Certainly, if hypothetical tasks such as the ones of the last section in which [XXV] is replaced by [XXVI] are thought, such predictions are:

Modus Ponendo Ponens: the result should be considered to be correct, as $\{[XXVI] \wedge p\} \vdash [q]$.

Modus Tollendo Tollens: the result should be considered to be correct, because $\{[XXVI] \wedge \neg q\} \vdash [\neg p]$.

Affirming the Consequent Fallacy: it cannot be known whether or not p can be deduced, since $\{[XXVI] \wedge q\} \nvdash [p]$. At most, $\{[XXVI] \wedge q\} \vdash [(p \wedge q) \vee (\neg p \wedge q)]$. So, it is not possible to know the truth-value of p.

Denying the Antecedent Fallacy: it cannot be known whether or not $\neg q$ can be inferred, as $\{[XXVI] \wedge \neg p\} \nvdash [\neg q]$. At most, $\{[XXVI] \wedge \neg p\} \vdash [(\neg p \wedge q) \vee (\neg p \wedge \neg q)]$. So, it is not possible to know the truth-value of q.

But, as pointed out, the predictions are not hard to identify in the other interpretations either. The next two sections address, respectively, the cases of the other two interpretations with three possible scenarios.

Enabling

As indicated, the set of Enabling is [III] (or [VIII]). Hence its formula is this one:

$[XXVII](p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

And this formula, in the four hypothetical tasks, which would have [XXVII] as their first premise now, also leads to predictions difficult to reject:

Modus Ponendo Ponens: it cannot be known whether or not q an be concluded, because $\{[XXVII] \wedge p\} \nvdash [q]$. At most, $\{[XXVII] \wedge p\} \vdash [(p \wedge q) \vee (p \wedge \neg q)]$. So, it is not possible to know the truth-value of q.

Modus Tollendo Tollens: it cannot be known whether or not $\neg p$ can be derived, since $\{[XXVII] \wedge \neg q\} \nvdash [\neg p]$. At most, $\{[XXVII] \wedge \neg q\} \vdash [(p \wedge \neg q) \vee (\neg p \wedge \neg q)]$. So, it is not possible to know the truth-value of p.

Affirming the Consequent Fallacy: the result should be considered to be correct, as $\{[XXVII] \wedge q\} \vdash [p]$.

Denying the Antecedent Fallacy: the result should be considered to be correct, as $\{[XXVII] \wedge \neg p\} \vdash [\neg q]$.

Disabling

And, clearly, given that the possibilities of Disabling are those of [X], a well-formed formula for this last interpretation can be as follows:

$[XXVIII] (p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)$

And, once again, if [XXVIII] is the first premise of the previous hypothetical tasks, the predictions are evident as well:

Modus Ponendo Ponens: it cannot be known whether or not q can be deduced, since $\{[XXVIII] \wedge p\} \nvdash [q]$. At most, $\{[XXVIII] \wedge p\}$

$\vdash [(p \wedge q) \vee (p \wedge \neg q)]$. So, it is not possible to know the truth-value of q.

Modus Tollendo Tollens: the result should be considered to be incorrect, as $\{[XXVIII] \wedge \neg q\} \nvdash [\neg p]$. On the contrary, $\{[XXVIII] \wedge \neg q\} \vdash [p]$.

Affirming the Consequent Fallacy: it cannot be known whether or not p can be inferred, because $\{[XXVIII] \wedge q\} \nvdash [p]$. At most, $\{[XXVIII] \wedge q\} \vdash [(p \wedge q) \vee (\neg p \wedge q)]$. So, it is not possible to know the truth-value of p.

Denying the Antecedent Fallacy: the result should be considered to be incorrect, since $\{[XXVIII] \wedge \neg p\} \nvdash [\neg q]$. On the contrary, $\{[XXVIII] \wedge \neg p\} \vdash [q]$.

But, of course, the predictions are even clearer in the cases of the interpretations with only two possible scenarios. The three next sections deal with such interpretations.

Biconditional

From what has been argued, there is no doubt that the logical form corresponding to [XII] is the following:

[XXIX] $(p \wedge q) \vee (\neg p \wedge \neg q)$

And that the predictions for the four schemata that are being addressed are, in this case, these ones:

Modus Ponendo Ponens: the result should be considered to be correct, as $\{[XXIX] \wedge p\} \vdash [q]$.

Modus Tollendo Tollens: the result should be considered to be correct, because $\{[XXIX] \wedge \neg q\} \vdash [\neg p]$.

Affirming the Consequent Fallacy: the result should be considered to be correct, since {[XXIX] ∧ q} ⊢ [p].

Denying the Antecedent Fallacy: the result should be considered to be correct, as {[XXIX] ∧ ¬p} ⊢ [¬q].

Strengthen Antecedent

In the same way, the formula for Strengthen Antecedent, that is, for the iconic representations included in [XIV], is:

[XXX] (p ∧ q) ∨ (p ∧ ¬q)

So, the predictions here are also easy to derive:

Modus Ponendo Ponens: it cannot be known whether or not q can be concluded, because {[XXX] ∧ p} ⊬ [q]. At most, {[XXX] ∧ p} ⊢ [XXX]. So, it is not possible to know the truth-value of q.

Modus Tollendo Tollens: the result should be considered to be incorrect, since {[XXX] ∧ ¬q} ⊬ [¬p]. On the contrary, {[XXX] ∧ ¬q} ⊢ [p].

Affirming the Consequent Fallacy: the result should be considered to be correct, as {[XXX] ∧ q} ⊢ [p].

Denying the Antecedent Fallacy: the result should be considered to be incorrect, because {[XXX] ∧ ¬p} ⊢ [q] (via Ex Contradictione Quodlibet Sequitur principle). Of course, an objection against this could be that, while that is true, it is also so that {[XXX] ∧ ¬p} ⊢ [¬q] (via Ex Contradictione Quodlibet Sequitur principle too). Nonetheless, although this idea is, certainly, right, one might think that, in a real reasoning situation, people can come to the conclusion that, given that q can be derived from [XXX] and ¬p as well, they cannot admit that ¬q is even a possible conclusion in an inference

with those very formulae as premises, since, clearly, q is the opposite of ¬q. And this particular point can refer to a very important discovery of the mental logic theory, which, although, as said, maybe it can have certain problems related to other aspects of reasoning, claims that, when individuals find a contradiction such as q and ¬q, they do not tend to infer, as allowed by Ex Contradictione Quodlibet Sequitur principle, anything, but to simply assume that there is at least a false datum in the information given to them (e.g., Braine & O'Brien, 1998c). Therefore, from this point of view, it can be said that what can be expected in an hypothetical Denying the Antecedent Fallacy task whose first premise is an «if…then» sentence with a real logical form such as [XXX] is that the participants will think that the inference describes an impossible situation in which q and ¬q can be true at the same time and in which hence neither ¬q nor anything else should be concluded.

Relevance

And one more interpretation with two possibilities is, as indicated, Relevance. In particular, its possibilities are those of [XVI] and, accordingly, its form is:

[XXXI] $(p \wedge q) \vee (\neg p \wedge q)$

Thus, evidently, this last formula allows one to think about the following predictions with regard to the four hypothetical reasoning problems indicated if it is their first premise:

Modus Ponendo Ponens: the result should be considered to be correct, since $\{[XXXI] \wedge p\} \vdash [q]$.

Modus Tollendo Tollens: the result should be considered to be incorrect, as, in a similar way as in the case of Denying the Antecedent Fallacy with Strengthen Antecedent, that is, with [XXX], it is true

that {[XXXI] ∧ ¬q} ⊢ [¬p], but it is also so that {[XXXI] ∧ ¬q} ⊢ [p]. So, it can be expected that it will be thought that nothing can be deduced from those premises.

Affirming the Consequent Fallacy: it cannot be known whether or not p can be inferred, as {[XXXI] ∧ q} ⊬ [p]. At most, {[XXXI] ∧ q} ⊢ [XXXI]. So, it is not possible to know the truth-value of p.

Denying the Antecedent Fallacy: the result should be considered to be incorrect, because {[XXXI] ∧ ¬p} ⊬ [¬q]. On the contrary, {[XXXI] ∧ ¬p} ⊢ [q].

Lastly, the three last interpretations, that is, the interpretations that only enable one possible scenario, will be addressed. This will be done in the next three sections.

Tollens

In this case, only the possibility contained in [XVIII] can be admitted. So, the formula corresponding to it is just:

[XXXII] ¬p ∧ ¬q

Given the simplicity of this formula, the predictions are even easier to note than in the last three previous interpretations:

Modus Ponendo Ponens: the result should be considered to be incorrect, since, in a manner similar to other cases above, while {[XXXII] ∧ p} ⊢ [q], {[XXXII] ∧ p} ⊢ [¬q] too.

Modus Tollendo Tollens: the result should be considered to be correct, as {[XXXII] ∧ ¬q} ⊢ [¬p].

Affirming the Consequent Fallacy: the result should be considered to be incorrect, because the situation is very akin to the one of Modus Ponendo Ponens: although {[XXXII] ∧ q} ⊢ [p], {[XXXII] ∧ q} ⊢ [¬p] as well.

Denying the Antecedent Fallacy: the result should be considered to be correct, since {[XXXII] ∧ ¬p} ⊢ [¬q].

Ponens

Now, the possibility is the one of [XX]. Therefore, its well-formed formula is as follows:

[XXXIII] p ∧ q

Undoubtedly, the predictions for a formula as simple as this one can also be found without difficulties:

Modus Ponendo Ponens: the result should be considered to be correct, as {[XXXIII] ∧ p} ⊢ [q].

Modus Tollendo Tollens: the result should be considered to be incorrect, because the premises lead to a contradiction again: {[XXXIII] ∧ ¬q} ⊢ [¬p], but {[XXXIII] ∧ ¬q} ⊢ [p] too.

Affirming the Consequent Fallacy: the result should be considered to be correct, since {[XXXIII] ∧ q} ⊢ [p].

Denying the Antecedent Fallacy: the result should be considered to be incorrect, as, once again, a contradiction is found. It is true both {[XXXIII] ∧ ¬p} ⊢ [¬q] and {[XXXIII] ∧ ¬p} ⊢ [q].

Deny Antecedent and Affirming Consequent

Finally, what can be said with regard to [XXII] is that the logical form suitable for it is the following:

[XXXIV] $\neg p \wedge q$

And, obviously, this is a formula that also leads to clear predictions:

Modus Ponendo Ponens: the result should be considered to be incorrect, as this is a new case of contradiction. Both $\{[XXXIV] \wedge p\} \vdash [q]$ and $\{[XXXIV] \wedge p\} \vdash [\neg q]$ are true at the same time.

Modus Tollendo Tollens: the result should be considered to be incorrect, because, as in the previous case, the participants would come to an inconsistency. It is evident that $\{[XXXIV] \wedge \neg q\} \vdash [\neg p]$. However, it is so that $\{[XXXIV] \wedge \neg q\} \vdash [p]$ as well.

Affirming the Consequent Fallacy: the result should be considered to be incorrect, since $\{[XXXIV] \wedge q\} \nvdash [p]$. On the contrary, $\{[XXXIV] \wedge q\} \vdash [\neg p]$.

Denying the Antecedent Fallacy: the result should be considered to be incorrect, as $\{[XXXIV] \wedge \neg p\} \nvdash [\neg q]$. On the contrary, $\{[XXXIV] \wedge \neg p\} \vdash [q]$.

Conclusions

Nevertheless, the most important point here is that, as it has been pointed out above, can be derived from what has been explained about Enabling, and can be checked in the cognitive science literature (e.g., Johnson-Laird & Byrne, 2002; López-Astorga, 2016), all of these predictions exactly match those of the mental models theory. Thus, as also said, any experiment on these issues demonstrating that the predictions of the mental models theory are

right should demonstrate that MS is correct too. And, in the end, what this really shows is that it is very possible that conditional reasoning in general, or, at least, the hypothetical conditional reasoning tasks used here, are not useful at all to help prove anything in favor of or against those two approaches.

Certainly, the experiments in the extensive literature on the mental models theory (as it can also be seen in works such as, e.g., those cited above) seems to confirm its predictions, which, as claimed, can be understood as a confirmation of the predictions that can be raised from an approach such as the one of MS too. Nonetheless, the problem can be even greater. As also stated, the predictions of the mental logic theory about tasks with the structure of Modus Ponendo Pones, Modus Tollendo Tollens, Affirming the Consequent Fallacy, and Denying the Antecedent Fallacy in which the first premise is a sentence of one of the ten kinds analyzed in this paper are also exactly the same. And this means that, actually, it is not true that conditional reasoning may not remove the doubts about which of two approaches (the mental models theory and MS) is the correct one, but that, strictly speaking, it may not eliminate the doubts about three approaches. As commented on, it is possible that the mental logic theory has problems to pay attention to. However, it also has to be acknowledged that it holds theses that are hard to challenge. Some of such theses have been indicated above, and a very important fact in this way appears to be that its predictions for tasks such as those mentioned are not different from those of the mental models theory either (López-Astorga, 2016), which implies that, at least as far as topics such as the one studied here are concerned, the mental logic theory cannot be absolutely ignored either.

However, beyond all of these debates, what seems to be evident is that this paper, by means of the arguments above (which, in some cases, can appear to be obvious and trivial but, as it can be noted, they are essential to achieve the goal of the paper), shows that a syntax of the human mind such as the one that can be derived from the general framework of MS (which is straightforward and only includes conjunctions, disjunctions, and negations) is an acceptable and feasible possibility. And this can be so even in the case that, as

argued by the mental models theory, people do not identify all the iconic possible scenarios of a particular sentence.

This last one is a very important point because, truly, the proponents of the mental models theory often argue, for example, that, as made it explicit by the literature, Modus Tollendo Tollens is more difficult than Modus Ponendo Ponens for a sentence of the kind Conditional because the former requires individuals to identify the third possibility of that type of conditional such as it is in [I]. Nevertheless, to apply the latter, what has to be done is to detect the first possibility of that set. According to them, the reason of this is clear. To apply Modus Tollendo Tollens to Conditional, it is necessary to see a possibility in which the consequent is false, that is, a possibility such as the third one in [I] (in the other two possibilities the consequent is true), which leads to the fact that the antecedent is also false (it is also false in that possibility). Nonetheless, to apply Modus Ponendo Ponens, it is needed to note a possibility such as the first one, which is the only one in which the antecedent is true and the consequent is true in it too. In this way, the key is that, following the mental models theory, to detect the first possibility in [I] is easy. However, to detect the second one and the third one implies effort, and, for this reason, these two last possibilities are not always identified in the case of sentences of the type Conditional and Modus Tollendo Tollens is harder in this kind of «if…then» sentences (e.g., Byrne & Johnson-Laird, 2009).

But, as said, this explanation does not have an influence on MS, which, as also accounted for, is based on the mental models theory. In fact, MS has already addressed this problem (e.g., López-Astorga, 2017a). From this last perspective, an individual that makes the mistake of falling to note the three possibilities in [I] is just an individual that makes the mistake of falling to see that the logical form of the sentence is [XXVI]. If that individual only identifies that first possibility, according to what has been indicated, he/she can come only to a formula such as [XXXIII], and this alone can already explain the difference in the degree of difficulty, since as equally commented on, while $\{[XXVI] \wedge \neg q\} \vdash [\neg p]$, $\{[XXXIII] \wedge \neg q\} \vdash [p \wedge \neg p]$.

Nonetheless, two more issues remain outstanding, which are also raised, for example, in works corresponding to the literature

on MS such as those cited above. On the one hand, if there are relationships between the iconic models of the mental models theory and the logical forms of MS, one might ask whether or not it can be thought that they are compatible perspectives and that each of them refers to a different aspect (semantics and syntax, respectively) of language and thought. Maybe the problem is that what is being done is to try to decide between two limited accounts that only describe (each of them) one particular aspect of the same global reality.

On the other hand, and this is something that is acknowledged in the works supporting MS as well, from what is presented in papers such as this one, it can be easily concluded that, although it is possible to recover the logical forms corresponding to the ten interpretations of the conditional dealt with, it seems that, to do that, a previous imperative condition is to identify the iconic scenarios that the mental models theory attributes to those interpretations. In this way, it appears that it is necessary to previously detect sets such as [I], [III], [V], [X], [XII], [XIV], [XVI], [XVIII], [XX], and [XXII], and that, without combinations of possibilities such as those ones, it is impossible to come to well-formed formulae such as, respectively, [XXVI], [XXVII], [XXV], [XXVIII], [XXIX], [XXX], [XXXI], [XXXII], [XXXIII], and [XXXIV]. However, if this is so, to obtain these last formulae, as claimed, firstly the iconic scenarios have to be taken into account and then they can be transformed into disjuncts (which are internally conjunctions) of disjunctions. Thus, and, as also stated, MS explicitly acknowledges it, this can be interpreted as a primacy of the semantic processes described by the mental models theory, at least at the cognitive level, since it seems that such processes happen before in time. Consequently, another point to clarify is whether or not that means a primacy of semantics over syntax in general in the human mind too.

So, if what is wished is to keep researching the real possibilities of existence of a simple syntax of the human mind such as that proposed here and in the works on MS, and its actual status in the human cognitive activity, aspects, questions, and problems such as these ones cannot be ignored. This paper has been intended to give

further evidence with regard to such issues. Nevertheless, there is no doubt that there is still a lot of work to do following this line of study and analysis.

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