Article

Estimación del crecimiento de dos especies de *Pinus* de la Región Centro de Guerrero, México

Growth estimation of two pine species from the Central Region of *Guerrero* state México

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Resumen

La aplicación de modelos de crecimiento para árboles individuales en bosques mezclados permite realizar estimaciones a nivel de la unidad de manejo. El objetivo del presente estudio fue evaluar los modelos de crecimiento en diámetro normal, área basal, altura total y volumen fustal de Chapman-Richards, Schumacher, Hossfeld I y Weibull para árboles individuales de Pinus pseudostrobus y Pinus oocarpa de Guerrero, México. Mediante muestreo selectivo se recolectaron 27 árboles dominantes y 28 codominantes para reconstruir los perfiles de árboles ordenados en grupos de diez años, por medio de la técnica de análisis troncales. La selección de los mejores modelos para cada variable se realizó con base en el coeficiente de determinación ajustado, la raíz del error medio cuadrático, las propiedades de los parámetros y las tendencias lógicas de crecimiento. Los resultados indican que el modelo de Schumacher fue el mejor para estimar el crecimiento en diámetro normal y la altura en ambas especies, así como el área basal de Pinus pseudostrobus y el volumen de Pinus oocarpa; mientras que, el modelo de Chapman-Richards fue el mejor para estimar el área basal de Pinus oocarpa y el volumen para Pinus pseudostrobus. Las edades estimadas del turno para volumen en Pinus oocarpa fueron de 62 años y para Pinus pseudostrobus de 82 años.

Palabras clave: Altura total, área basal, crecimiento en diámetro, modelos de crecimiento, Pinus, volumen

Abstract

The application of growth models for individual trees in mixed forests allows estimates to be made at the management unit level. The objective of the present study was to evaluate the growth models in normal diameter, basimetric area, total height and stem volume of Chapman-Richards, Schumacher, Hossfeld I and Weibull for individual Pinus pseudostrobus and Pinus oocarpa trees from the state of Guerrero, Mexico. Through selective sampling, 27 dominant and 28 codominant trees were collected to reconstruct the profiles of trees arranged in groups of ten years, by means of the stem analysis technique. The selection of the best models for each variable was made based on the adjusted coefficient of determination, the root mean square error, the properties of the parameters and the logical growth trends. The results indicate that the Schumacher model was the best to estimate growth in normal diameter and height in both species, the basal area of Pinus pseudostrobus and the volume of Pinus oocarpa, while the Chapman-Richards model was the best for estimate the basal area for Pinus oocarpa and the volume for Pinus pseudostrobus. The estimated turn ages for volume in Pinus oocarpa were 62 years and for Pinus pseudostrobus, 82 years.

Key words: Total height, basimetric area, diameter growth, growth models, *Pinus*, stem volume.

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Introduction

The timber volume to be harnessed is a function of the growth and yield of the species that make up the forest stands. In this context, it is common practice to predict the normal diameter, basimetric area, total height or the volume as a function of predictor variables such as age, density, station quality, competition indexes or others that allometrically or biological explain tree growth (Briseño *et al.*, 2020).

For several decades, several models have been generated that simulate in a logical and precise way the growth at the individual or stand level over time (Clutter *et al.*, 1983; Corral and Návar, 2005; McCullagh *et al.*, 2017); Chapman-Richards, Schumacher, Weibull and Hosffeld I models in particular, have the property of simulating the sigmoidal growth of timber forest species and reflect the antagonistic dynamics that occur between the intrinsic physiological factors that stimulate growth and those of the environment that limit it (Kiviste *et al.*, 2002). From a practical point of view, the equations that result from the adjustment of the parameters of these models are used to estimate the growth of trees, an essential knowledge in making efficient forestry decisions that ensure sustainable timber harvesting (Santiago-García *et al.*, 2013; Vargas-Larreta *et al.*, 2013).

The modeling and prediction of stand growth in mixed and irregular forests is a difficult task, since these forests, in addition to gathering a variety of species, are irregular in structure. In them, each species has a different rate of biological growth in response to genetic inheritance and adaptation to the environmental conditions where it develops (Diéguez *et al.*, 2009; Fien *et al.*, 2019). Based on the above, growth models for individual trees facilitate prediction at the unit area level since the sum of the growths of the individual trees that make up a unit area results in total growth (Torres and Magaña, 2001). The technique used to obtain information and the type of model to be used depends on the type of forest: pure, mixed,

contemporary, non-even aged, homogeneous or heterogeneous, and the projection interval (Diéguez *et al*., 2009; Hernández *et al*., 2018; Briseño *et al.,* 2020).

Pinus pseudostrobus Lindl. and *Pinus oocarpa* Schiede ex Schltdl. are widely distributed throughout the forest territory of the central region of the state of *Guerrero*; they form patches of a single species or of several species of the *Pinus* genus associated with oaks and some broadleaves and are considered within the six species of the genus of greatest commercial importance. They have good characteristics to be used in the manufacture of plywood, sheet metal, packing boxes, moldings, construction and furniture manufacturing (Conafor, 2015).

In order to generate tools that help decision-making to contribute to achieving sustainable use of timber species in the central region of the state of *Guerrero*, the aim of this study was to assess the goodness of fit of four growth models in normal diameter, basimetric area, total height and stem volume of *Pinus pseudostrobus* and *Pinus oocarpa*, and select the most efficient one.

Materials and Methods
Study area

The study area was Forest Management Unit 1205 (UMAFOR-1205), which is located within the *Sierra Madre del Sur* mountain system in the central region of the state of *Guerrero* (Figure 1).
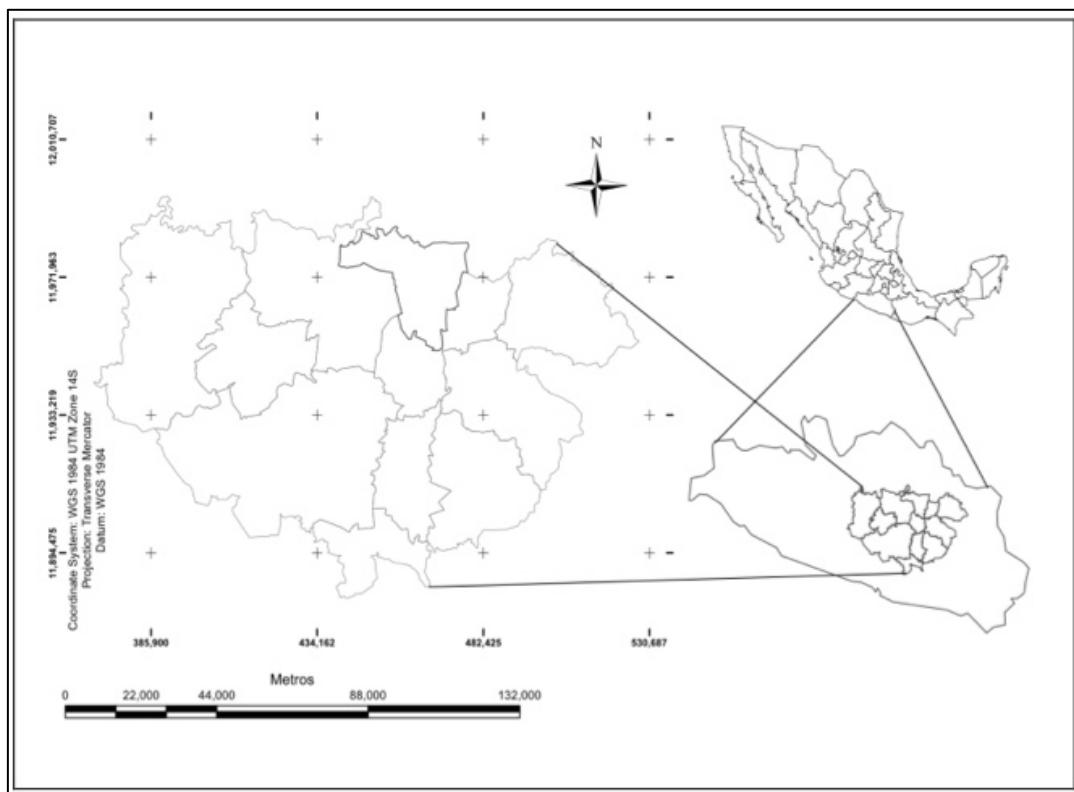
Figure 1. Location of the study area.

Climate types in coniferous forest areas vary from temperate to semi-warm with different humidity graduations. However, the predominant type of climate is (A) C (w) (García, 2004).

Soils are classified as Leptosols, Luvisols, Regosols and Cambisols, of which Leptosols have the highest distribution within the study area (Inegi, 2016).

Sampling

The selective sampling method was applied in the low, medium and high season qualities of UMAFOR-1205 to collect 27 dominant trees and 28 codominant trees, without physical damage, with ages from 20 to 100 years. Slices were obtained from the stem of each tree at 0.30 m, 0.60 m and 1.30 m high and subsequently at every 2.60 m until reaching the tip to reconstruct 102 and 152 profiles of *Pinus pseudostrobus* and *Pinus oocarpa* trees, respectively, starting from the collected trees and applying the methodology to perform the trunk analysis described by Klepac (1983).

Variables

The initial measurements that were taken per tree of each species were normal diameter with bark obtained at 1.30 m height with a 283D Lufking™ 5M diametric tape; barkless diameter of each slice measured with a Lufking™ rule and estimated as the average of two measurements taken in the cross. The rings from each slice were divided into groups of ten to measure radial growth at different heights. The age of each of the reconstructed trees was determined on the largest diameter of the slice obtained at 30 cm high of the stem. To measure the normal diameter and estimate the basimetric area of each reconstructed tree, growth rings were grouped in ages of 10 years from the center to the periphery (Klepac, 1983). The estimation of the true height was carried out using the Carmean (1972) formula modified by Newberry (1991), whose expression is:

$$H_{ij} = h_i + \frac{(h_{i+1} - h_i)}{[2(r_i - r_{i+1} - 0.5)]} + (j - 1)\frac{(h_{i+1} - h_i)}{(r_i - r_{i+1} - 0.5)}$$

Where:

H_{ij} = Estimated total height in the i-th slice, corresponding to j growth rings

h_i = Cumulative height to top cut of each i -th slice

h_{i+1}= Cumulative height to top cut of each next i -th slice

r_i = Number of growth rings of the i -th slice

r_{i+1} = Number of growth rings of the next i -th slice

j = Number of growth rings from the core of each slice

The basimetric area was determined by the following equation:

$$AB = \frac{\pi}{4} dn^2$$

Where:

AB = Basimetric area

π =3.14159

dn = Normal diameter measured at 1.30 high from the surface of the ground

The volumes of each log were estimated with the Smalian equation and that of the cone, respectively.

$$Vi = \frac{\frac{\pi}{4}(D_M^2 + D_m^2)}{2} l$$

$$Vp = \frac{\left(\frac{\pi}{4} D_b{}^2\right) l}{3}$$

Where:

D_M = Largest diameter of each log

D_m = Smallest diameter of each log

D_b = Diameter at the base of the tip (m)

l = Log's length (m)

The sum of the volumes of the logs of each tree was equal to the total volume without bark of the stem.

Table 1 shows the basic descriptive statistics of the variables analyzed by species.

Table 1. Descriptive statistics of the analyzed variables.

Species	Variable	N	Minimum	Average	Maximum	Standard deviation
Pinus pseudostrobus Lindl.	Diameter (cm)	102	1.3	19.84	41.5	10.65
	Basimetric area (m^2)	102	0.00013	0.03976	0.13526	0.03584
	Height (m)	282	0.17	12.26	38.28	8.84
	Volume (m^3)	102	0.00042	0.45	2.32	0.5075
Pinus oocarpa Schiede ex Schltdl.	Diameter (cm)	152	1.3	20.23	45.5	10.0061
	Basimetric area (m^2)	152	0.00013	0.03999	0.16259	0.03305
	Height (m)	257	0.17	10.05	28.3	6.90207
	Volume (m^3)	152	0.00022	0.37325	1.84337	0.35941

Analysis

To the growth data in normal diameter, basal area, height and volume, the growth models of Chapman-Richards (M1), Hossfeld I (M2), Weibull (M3) and Schumacher I (M4) mentioned by Kiviste *et al.,* (2002) using nonlinear regression (Table 2). In addition, their respective expressions were generated to determine the annual current increase (*ICA*, for its acronym in Spanish) and the average annual increase (*IMA*, for its acronym in Spanish). The adjustments were made by applying the PROC MODEL procedure of the SAS statistical program (SAS, 2002). The analytical criterion to select the goodness of fit of the models considered the root mean square error (*REMC*), the adjusted coefficient of determination (R^2_{adj}) and the statistical properties of the estimated parameters.

$$R^2_{adj} = 1 - \left[\frac{(n-1)\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-p)\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \right]$$

$$RECM = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-p}}$$

Where:

$y_i, \hat{y}_i, \bar{y}_i$ = Valor observado y valor predicho i de cada variable, así como el promedio estimado de la variable observada

n = Number of observations

p = Number of the parameters of the mode

Table 2. Assessed growth models and their *ICA* and *IMA* expressions.

Model	Expression	ICA	IMA
Chapman-Richards	$y = \beta_0(1 - exp^{-b_1 E})^{b_2} + \varepsilon$	$ICA = \beta_0 b_1 \beta_2 (1 - exp^{-b_1 E})^{b_2 - 1} exp^{-b_1 E}$	$IMA = \dfrac{y}{E}$
Hossfeld I	$y = \dfrac{t^2}{(\beta_0 + b_1 E)^2} + \varepsilon$	$ICA = \dfrac{2\beta_0 E}{(\beta_0 + b_1 E)^3}$	$IMA = \dfrac{y}{E}$
Weibull	$y = \beta_0\left(1 - exp^{-\beta_1 E^{\beta_2}}\right) + \varepsilon$	$ICA = \beta_0 \beta_1 \beta_2 E^{\beta_2 - 1} exp^{-\beta_1 E^{\beta_2}}$	$IMA = \dfrac{y}{E}$
Schumacher	$y = \beta_0 exp^{-b_1\left(\frac{1}{E}\right)} + \varepsilon$	$ICA = \dfrac{\beta_0 b_1}{E^2} exp^{\frac{-b_1}{E}}$	$IMA = \dfrac{y}{E}$

y = Normal diameter growth (cm), basimetric area (m^2), total height (m) and barkless stem volume (m^3); E = Age (years); β_1 β_2 and β_2 = Parameters of the model; ICA = Annual current increase of the normal diameter (cm), basimetric area (m^2), total height (m) and stem volume (m^3); IMA = Average annual increase of the normal diameter (cm), basimetric area (m^2), total height (m) and stem volume (m^3).

Finally, as a determining factor to select the best model, the graphical analysis of the growth curves was considered.

Since the growth measurements over time of each assessed variable within each tree are correlated, the significance of the autocorrelation of the selected models was evaluated as the best using the Durbin-Watson (*DW*) test (Sharma *et al.*, 2011; Quiñonez *et al.,* 2018).

$$d = \frac{\sum(e_t - e_{t-1})^2}{\sum e_t^2}$$

Where:

e_t = t residual of each observation

e_{t-1} = residual of each observation before the e_t residual

When this test was significant, the CAR (X) autoregressive model (Zimmerman *et al.,* 2001) was applied to correct the autocorrelation problem and generate parameters and standard errors consistent with the models. The structure of the autoregressive model is:

$$e_{ij} = \sum_{k=1}^{k=x} I_k \rho_k^{h_{ij}-h_{ij-k}} e_{ij-k} + \varepsilon_{ij}$$

Where:

$e_{ij} = j - th$ ordinary residual in observation i

$I_k = 1$ for $j > k$ and 0 for j=k (k=1, 2, 3,…, x)

ρ_k = Autoregressive parameter of the k order

$h_{ij} - h_{ij-k}$, = Separation distances of the j th to the $j - k$ *ésim* observation

ε_{ij} = Independent error that follows a normal distribution with zero mean and constant variance

Once the best model was selected, the maximum *ICA* (*ICA$_{máx}$*) and maximum *IMA* (*IMA$_{máx}$*) were estimated for each of the variables and species. Both increases correspond to the highest values estimated with the *ICA* and *IMA* equations.

Results

The estimation of the parameters of the models without considering the autoregressive structure of the error for *Pinus pseudostrobus* presented *DW* values for the diametric growth between 0.9518 and 1.0028, basimetric area 0.6188 and 0.9390, height 0.3057 and 0.3324 and volume 0.9896 and 1.0343; while for *Pinus oocarpa*, the *DW* values for diameter growth ranged from 0.5095 to 0.5912, basimetric area from 0.5814 to 0.6621, height 0.3437 to 0.3655 and volume 0.6211 to 0.6966, figures that indicate autocorrelation. The *DW* values estimated after adjusting the growth models with the autoregressive model CAR (1), CAR (2) and CAR (3) allow us to deduce that the autocorrelation of the residuals improved significantly (Table 3). In turn, the estimators of the parameters ρ_k ($k = 1, 2, 3$) were significant for the growth models in diameter, height and volume in both species (*Pr*> 0.0001), from which it is inferred that the estimate of the parameters and standard errors are consistent in these growth models, but not for those with basal area (*Pr*> 0.05) (tables 4 and 5). The independence of the errors favors the estimation of efficient and unbiased parameters and prevents the underestimation of the covariance matrix of the parameters (González *et al*., 2016).

Table 3. Adjustment statistics of the growth models in the analyzed variables.

Model	*Pinus pseudostrobus* Lindl.			*Pinus oocarpa* Schiede ex Schltdl.		
	R^2_{adj}	*RCME*	*DW*	R^2_{adj}	*RCME*	*DW*
Normal diameter						
M1	0.7403	5.427	1.653	0.8654	3.671	1.162
M2	0.7230	5.605	1.746	0.9001	3.162	1.649
M3	0.7831	4.960	1.480	0.8640	3.690	1.182
M4	0.7400	5.431	1.654	0.8648	3.679	1.117
Basimetric area						
M1	0.7063	0.0194	1.489	0.9154	0.0096	1.590
M2	0.6962	0.0197	1.591	0.9096	0.0096	1.660
M3	0.7019	0.0196	1.514	0.9120	0.0097	1.509
M4	0.7105	0.0193	1.500	0.9079	0.0100	1.515
Height						
M1	0.9650	1.6093	1.661	0.9460	1.603	1.220
M2	0.9338	2.2315	1.770	0.9441	1.631	1.300
M3	0.9645	1.6224	1.692	0.9688	1.231	1.597
M4	0.9275	2.3167	1.652	0.9695	1.204	1.598
Volume						
M1	0.8544	0.1977	1.735	0.9321	0.0936	1.635
M2	0.7426	0.2574	1.626	0.9240	0.0990	1.621
M3	0.9338	0.0924	1.361	0.9338	0.0924	1.361
M4	0.8500	0.1964	1.737	0.9305	0.0947	1.608

M1= Chapman-Richards; M2=Hossfeld I; M3=Weibull; M4=Schumacher; R^2_{adj} = Adjusted coefficient of determination; *RCME* = Mean square root of error; *DW* = Durbin-Watson.

Table 4. Estimators of the parameters and standard errors of the evaluated models for *Pinus pseudostrobus* Lindl*.*

Pinus pseudostrobus		Parameters					
Model	Statistic	β_0	β_1	β_2	ρ_1	ρ_2	ρ_3
Diameter							
M1	Estimator	32.96137	0.07249	3.197013	1.00368		
	E. E.	2.86740	0.01540	0.83570	0.01100		
M2	Estimator	2.81260	0.12419		0.99036		
	E. E.	0.33320	0.00931		0.01450		
M3	Estimator	30.8810	0.00095	2.09730	1.01299	0.97076	0.32513
	E. E.	2.58110	0.00063	0.2283	0.00954	0.01740	0.21780
M4	Estimator	49.14015	24.38972		0.99762		
	E. E.	4.18750	2.49430		0.01210		
Basimetric area							
M1	Estimator	0.08711	0.06955	4.85097	1.72976		
	E. E.	0.01440	0.02710	2.73200	8.73230		
M2	Estimator	92.12886	1.84175		0.09097		
	E. E.	15.33660	0.34970		0.45990		
M3	Estimator	0.081866	0.000242	2.37926	0.878077		
	E. E.	0.0111	0.000388	0.51580	4.26000		
M4	Estimator	0.16539	40.21302		3.05171		
	E. E.	0.0267	5.8208		15.43200		
Height							
M1	Estimator	26.26966	0.06274	1.77583	1.146894	0.97393	
	E. E.	1.01290	0.00689	0.16550	0.01310	0.01190	
M2	Estimator	2.221376	0.15547		1.03211		
	E. E.	0.11780	0.00404		0.01310		
M3	Estimator	25.82468	0.01108	1.43804	1.14903	0.97225	
	E. E.	0.95930	0.00222	0.07600	0.01310	0.0119	
M4	Estimator	33.64891	16.64891		1.00452		
	E. E.	1.17250	0.83290		0.01420		
Volume							
M1	Estimator	2.20766	0.02989	4.80571	4.203409		

	Statistic	β_0	β_1	β_2	ρ_1	ρ_2
	E. E.	1.54900	0.0185	2.27560	0.4316	
M2	Estimator	-46.1913	-0.14897		1.95681	
	E. E.	8.03150	0.14610		0.42170	
M3	Estimator	0.81488	8.30×10^{-8}	4.12430	10.62855	3.36962
	E. E.	0.06390	1.15×10^{-7}	0.36980	1.33610	0.34480
M4	Estimator	3.66476	82.53542		4.06032	2.38687
	E. E.	0.90860	11.30020		0.39320	0.27590

M1 = Chapman-Richards; M2 = Hossfeld I; M3 = Weibull; M4 = Schumacher; E.E. = Standard error; β_k = Parameters of the model; ρ_k = Parameters of the autoregressive model CAR(x).

Table 5. Estimators of the parameters and standard errors of the evaluated models for *Pinus oocarpa* Schiede ex Schltdl.

Pinus oocarpa		Parameters					
Model	**Statistic**	β_0	β_1	β_2	ρ_1	ρ_2	ρ_3
Diameter							
M1	Estimator	33.30371	0.05272	2.78622	0.97376		
	E. E.	1.91760	0.00869	0.54060	0.01280		
M2	Estimator	3.53210	0.12302		0.98023	0.97631	0.93747
	E. E.	0.25640	0.00556		0.01490	0.01180	0.01910
M3	Estimator	32.19344	0.00184	1.77252	0.97084		
	E. E.	1.71310	0.00092	0.16090	0.01300		
M4	Estimator	48.59111	30.57237		0.97336		
	E. E.	2.46550	2.04540		0.01310		
Basimetric area							
M1	Estimator	0.11168	0.03658	3.81223	182509.8	1700.225	0.03670
	E. E.	0.01520	0.00706	0.72260	642673.0	4436.2	0.12610
M2	Estimator	134.7345	1.65843		1.8093×10^8	101.28700	3.93×10^{-10}
	E. E.	14.3358	0.23650		5.1302×10^8	278.70000	2.79×10^{-9}

M3	Estimator	0.08803	0.00010	2.36525	1.1768×10^{8}	13.08743	
	E. E.	0.00915	0.00008	0.23610	3.07×10^{8}	34.40770	
M4	Estimator	48.59111	30.57237		0.97336		
	E. E.	2.46550	2.04540		0.01310		
Height							
M1	Estimator	19.2500	0.01592	1.34003	1.05532	0.97638	0.90647
	E. E.	0.54760	0.00334	0.07240	0.02130	0.01850	0.02060
M2	Estimator	2.17484	0.18839		0.96258		
	E. E.	0.10580	0.00343		0.01970		
M3	Estimator	19.25000	0.01592	1.34003	1.05532	0.97638	0.90647
	E. E.	0.54760	0.00334	0.07240	0.02130	0.01850	0.02060
M4	Estimator	23.34795	14.04075		1.06527	0.99418	0.93053
	E. E.	0.66200	0.69910		0.01990	0.01670	0.01650
Volume							
M1	Estimator	0.90389	0.05058	6.86763	6.84513	4.00005	1.26232
	E. E.	0.10200	0.00685	1.33010	1.32240	0.64170	0.33180
M2	Estimator	-55.2519	-0.39983		10.48295	2.96807	0.33751
	E. E.	6.34390	0.09880		1.63760	0.46260	0.16290
M3	Estimator	0.81488	8.3×10^{-8}	4.12430	10.62855	3.36962	0.33751
	E. E.	0.06390	1.15×10^{-7}	0.36980	1.33610	0.34480	0.16290
M4	Estimator	2.33633	80.82759		7.09319	4.19346	1.35311
	E. E.	0.30240	6.66230		1.35920	0.67930	0.36270

M1= Chapman-Richards; M2=Hossfeld I; M3=Weibull; M4=Schumacher; E.E.= Standard Error; β_k = Parameters of the model; ρ_k = Parameters of the autoregressive model CAR(x).

Goodness of fit of models

The R^2_{adj} and *RCME* statistics indicated that in *Pinus pseudostrobus*, the Weibull model was the one that best adjusted to estimate growth in diameter and volume and those of Chapman-Richard and Weibull in height; in turn, all models were adjusted in a similar way in estimating the basal area. In *Pinus oocarpa*, the Hossfeld I model was the one that registered the best fit in diameter, those of Chapman-Richard and Weibull in basimetric area, those of Schumacher and Weibull in height, as well as those of Chapman-Richard, Weibull and Schumacher in volume (Table 3).

However, when considering the level of significance of the parameter estimators (α <0.05), the Weibull model was rejected to predict growth in diameter, basimetric area, height and volume in *Pinus pseudostrobus*, and growth in basimetric area. and volume for *Pinus oocarpa*. In a similar way, the Chapman-Richards and Hossfeld I volume growth models showed non-significant parameters in *Pinus pseudostrobus*. For this species, only the Schumacher model recorded estimators of the significant parameters in the four growth variables studied. In relation to *Pinus oocarpa*, all the estimators of the Chapman-Richard, Hossfeld I and Schumacher models were significant (*Pr* <0.0001) in the four variables studied. Based on the previous context, the graphical assessment in the estimation of the growth of each variable by species carried out with the models whose parameters were all significant, allowed to define the best model.

Growth curve trends

Figure 2 shows that both the Chapman-Richards and Schumacher models predict very similar normal diameter growths for both species up to 50 and 65 years. From these ages, the growth rate predicted with the Chapman-Richards model tends to zero, while, with the Schumacher model, growth continues upward until reaching the asymptote. The maximum growth in diameter estimated with the Chapman-

Richards model in *Pinus pseudostrobus* and *Pinus oocarpa* were lower than those predicted with the Schumacher model; the latter presented more plausible growth rates at ages over 60 years and was selected to make the predictions of the growth in diameter of both species.
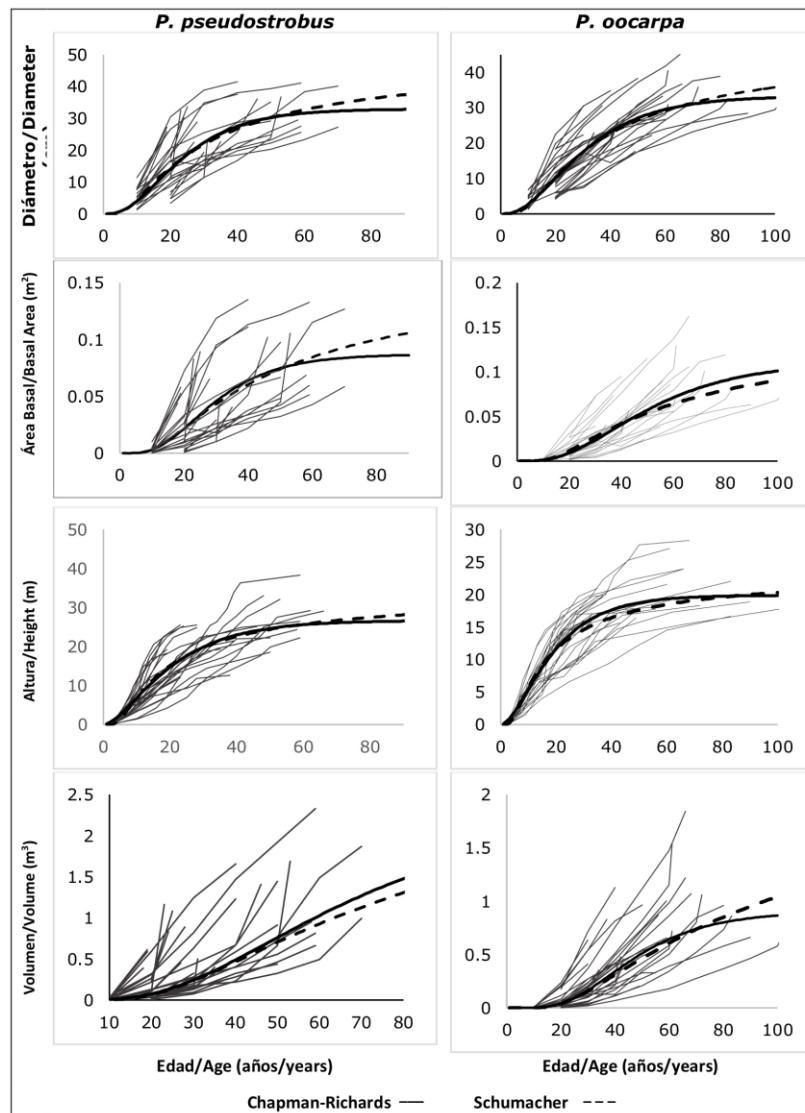
Figure 2. Comparison of the trends of the growth curves generated by the Schumacher and Chapman-Richards models with respect to the growth series observed for the variables analyzed.

In reference to the basimetric area, the Schumacher model was the one that best showed the trend in *Pinus pseudostrobus*, while the Chapman-Richards model did so in *Pinus oocarpa* during the evaluated period. In the first species, the Schumacher model predicted minor growths in the basimetric area up to an approximate age of 70 years, while in *Pinus oocarpa*, the same happens over the entire growth period evaluated.

In the graphical analysis of the growth in total height, the Schumacher model is the one that best predicts this growth. The Hossfeld I model exhibits a higher growth rate after 40 years in *Pinus pseudostrobus* and after 60 years in *Pinus oocarpa*, while the Chapman-Richards and Weibull models decrease it drastically after 50 years, and showed the asymptotes at an early age.

On the other hand, the predictions of volume growth with the Schumacher model in *Pinus pseudostrobus* and *Pinus oocarpa* are lower than those calculated with the Chapman-Richards model throughout the evaluation period, in which the intermediate fellings are programmed, Therefore, if it is desired to avoid overestimating the usable volume, the Schumacher model is considered the most appropriate for such predictions. In turn, the Chapman-Richards model presents the asymptote after 60 years of age, and reflects the lower estimates after that age.

Maximum Annual and Shift Current Increases

The estimates made with the equation derived from the Schumacher normal diameter growth model indicated that the $ICA_{máx}$ in diameter in *Pinus pseudostrobus* and *Pinus oocarpa* were recorded at 12 (1.09 cm) and 15 years (0.86 cm), while those in the area baseline predicted with the equation derived from the Chapman-Richards model were obtained at 30 and 37 years (0.0017 cm), respectively. The $ICA_{máx}$ in total height determined with the equation derived from the Schumacher model occurred at 8 (1.15 m) and 7 years (0.90 m). In turn, the $ICA_{máx}$ in stem volume predicted with the equations derived from the Chapman-Richards and

Schumacher models were apparent at 38 (0.024 m³) and 41 years old (0.018 m³) in *Pinus oocarpa* and *Pinus pseudostrobus*, respectively. The age of the physical shift of the volume estimated with the Schumacher equation was determined at 82 years in *Pinus pseudostrobus* and 62 years in *Pinus oocarpa* (Figure 3).
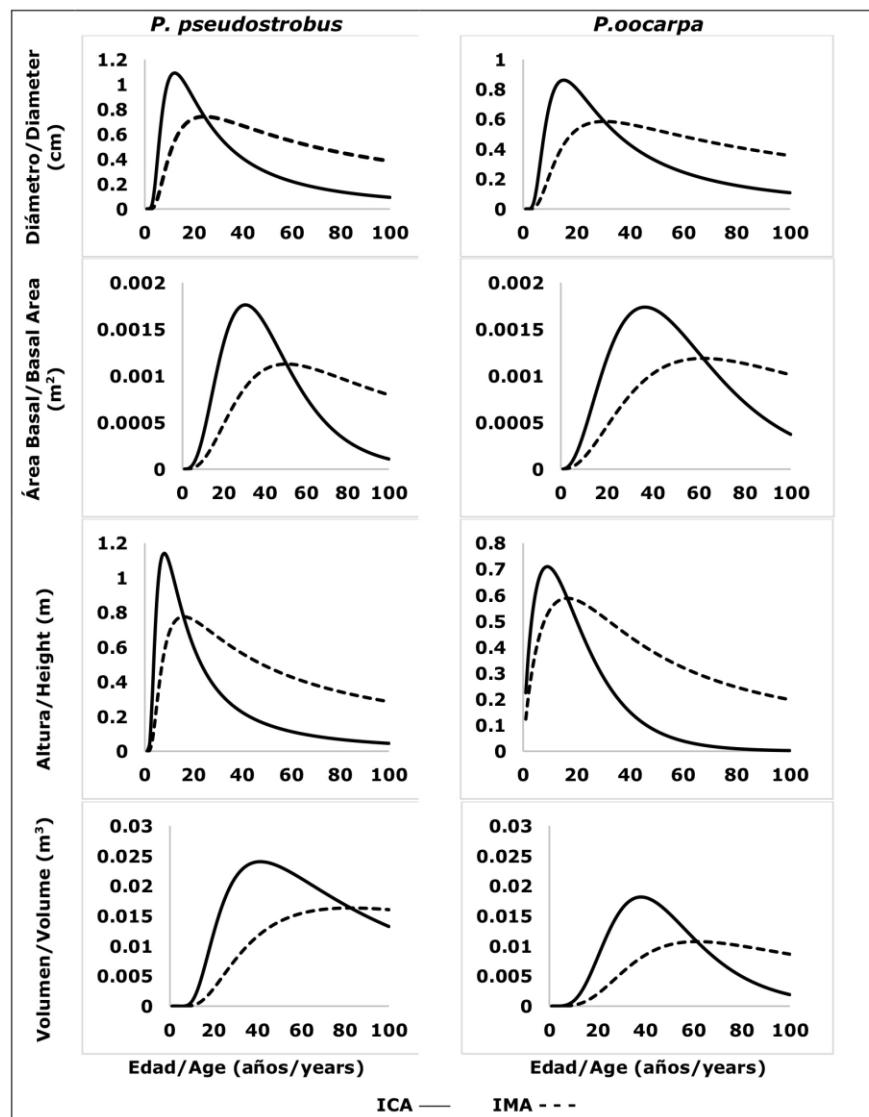
Figure 3. Trends of the *ICA-IMA* curves generated by the evaluated models in relation to the growth series.

If both species are associated in the same type of forest, the difference in growth patterns, the time in which the maximum increases are recorded and the turn, it is possible to assume that these species respond differently to the same environmental conditions and levels. productivity prevailing in this ecosystem. The growth predictions made with the selected models suggest that the *Pinus pseudostrobus* individuals reached greater dimensions in diameter, height and volume than those of *Pinus oocarpa* during the studied period.

Discussion

The R^2_{adj} and *RCME* statistics were very similar between the adjusted models, so the analysis of the trends in the growth curves that each model generates compared to the observed growth trends was essential to select the model that best represented growth. of normal diameter, basimetric area, total height and stem volume in both studied species. The Chapman-Richards and Schumacher models, selected as the best, in addition to being simple and parsimonious (Vanclay, 1994; Vanclay and Skovsgaard 1997), have asymptotic behavior and represent biologically realistic curves (Alder, 1980; Salas *et al*., 2016).

Similar to the results of this study, Arteaga (2000) recorded that the Schumacher model had the best fit and represented the growth in height and volume of *Pinus pseudostrobus* while the Chapman-Richards model did for the diameter.

Monárrez and Ramírez (2003) indicated that the best model to estimate diametric growth for *Pinus durangensis* Martínez was Schumacher's and for volume that of Chapman-Richards, while Corral and Návar (2005), when evaluating the behavior of several models, determined that the Chapman-Richards model was properly adjusted to estimate growth in diameter, height and volume in five Pinus species from the state of *Durango*.

Martínez *et al.* (2014), when adjusting the Chapman-Richards and Schumacher models to assess *Pinus hartwegii* Lindl. diameter growth, height and volume

reported that, although both models presented good fit, Schumacher's had the best predictive quality. Pacheco *et al.* (2016), when adjusting the Schumacher, Chapman-Richards and Weibull models, decided to apply the second one to estimate growth, *ICA* and *IMA* in height of *Pinus montezumae* Lamb.

Quiñonez *et al.* (2015) established the Chapman-Richards model as the most suitable for generating families of diameter growth curves of *Pinus lumholtzii* Robins & Ferns, while De Almeida *et al.* (2019) reported that the Chapman-Richards, together with the Weibull model, showed the best adjustments to generate diameter growth curves of five pine species. In turn, Hernández *et al.* (2018) indicated that the Chapman-Richards growth model was the best fit to generate site index curves in *Pinus ayacahuite* Ehren.

Recently, Hernández *et al.* (2020), when working with the same models that were applied in this study to three pine species from the state of *Chihuahua*, concluded that the Chapman-Richards and Schumacher models best represented growth in diameter, basimetric area, height and volume.

From the standpoint that the normal diameter is an indicator of the growth phases of the studied species (Imaña and Encinas, 2008), the age at which the $ICA_{máx}$ value was calculated suggests that the juvenile period of *Pinus pseudostrobus* and of *Pinus oocarpa* culminated naturally at 12 and 14 years, and reached diameters of 7.5 and 6.3 cm, respectively. From these ages, both species begin their maturity period, which reaches 24 and 31 years, temporalities related to average growth in diameter of 17.5 and 18.1 cm and intersections of the *ICA-IMA* of 0.752 and 0.575 cm, respectively. The senile stage begins when the last age of maturity is accomplished and extends until the individuals achieve their maximum growth, which is equal to the value of the asymptote. When projecting the diameter growth estimates with the Schumacher model, both *Pinus pseudostrobus* and *Pinus oocarpa* would reach maximum growth (49 and 48 cm, respectively) after 200 years.

The intersection of the *ICA* curves with the *IMA* indicates the time when the best returns occur. Based on what was mentioned by Martínez *et al.* (2014), the ages at

which the $ICA_{máx}$ of the diameter and height are recorded serve as a reference to start the intermediate treatment program, which ends a cutting cycle before the application of regeneration cutting, age of intersection of the *ICA* and *IMA* curves in volume (physical shift). Therefore, if you want to apply a regular management system, it is suggested to start the application of intermediate fellings when the individuals of the studied species are on average between 10 and 15 years old in order to promote the redistribution of water, light and nutrients. in residual trees to favor their growth and initiate the regulation of the dimensional structure of the forest (Novák, *et al.,* 2017).

Conclusions

The results of the study indicated that the Schumacher model was the best to predict the growth in diameter and height of both, *Pinus pseudostrobus* and *Pinus oocarpa*, the basimetric area of *Pinus pseudostrobus* and the volume of *Pinus oocarpa*, while the Chapman- Richards was the best predictor of *Pinus oocarpa* basimetric area growth and *Pinus pseudostrobus* volume growth. In turn, *Pinus pseudostrobus* has better growths in diameter, height and volume than *Pinus oocarpa*. The physical turn of the volume predicted in *Pinus oocarpa* and *Pinus pseudostrobus* was 65 and 85 years, respectively.

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Conflict of interest

The authors declare no conflict of interest.

Contribution by author

Francisco Javier Hernández, Brenda Mireya Bretado Medrano and Ezequiel Márquez Bernal: study design, preparation and quantitative analysis of the data and preliminary and final writing of the manuscript; Juan Abel Nájera Luna and Benedicto Vargas Larreta: review of the quantitative analysis as well as the final writing of the manuscript.

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