

# FPGA Implementation of Fuzzy Mamdani System with Parametric Conjunctions Generated by Monotone Sum of Basic t-Norms

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**Abstract**—The paper presents the results of FPGA implementation of fuzzy Mamdani system with parametric conjunctions generated by monotone sum of basic t-norms. The system is implemented on the DE2 Altera development board using VHDL language. The system contains reconfigurable fuzzy Mamdani model with parametric membership functions and parametric operations that gives possibility to adjust the system to specific application.

**Index terms**— Fuzzy Mamdani model, parametric conjunction, t-norm, FPGA, Altera.

## I. INTRODUCTION

THE fuzzy models and soft computing systems based on them have wide applications in the solution of real world problems [1, 2]. For this reason it is increasing the need in hardware implementation of highly reconfigurable fuzzy systems that can be easy adapted to various applications or to change of environment where fuzzy system is operated. Such reconfigurable fuzzy systems can be developed on two levels: on the level of fuzzy model and on the level of hardware implementation. This paper presents a method of hardware implementation of Mamdani fuzzy systems that reconfigurable on both levels. On the level of fuzzy model we consider fuzzy systems with parametric membership functions and parametric operations [3-5]. Such parameterization of fuzzy systems gives possibility to construct highly reconfigurable fuzzy systems with high adaptive possibilities. On the level of hardware we consider FPGA (Field Programmable Gate Array) implementation of fuzzy systems, i.e. an easily reprogrammable integrated circuit [6] that can be adapted to the change of parameters of fuzzy system and moreover to the change of the structure of fuzzy system.

We consider Mamdani fuzzy models with two input variables with fuzzy values given by triangular membership functions and one output variable with fuzzy values given by singletons. Such models are very popular in applications of

fuzzy systems [2, 3]. Fuzzy models with parametric operations have been considered in various papers [4, 5, 7-10]. The methods of FPGA implementation of fuzzy systems have been studied in [11-16]. FPGA implementation of parametric conjunctions based on generator functions have been proposed in [17, 18]. In [19] it was proposed the method of generation of parametric conjunctions based on (p)-monotone sum of basic t-norms that has simpler FPGA implementation [20] than the method based on generator functions. In our paper we extend results obtained in [20] and we propose the method of FPGA implementation of Mamdani fuzzy models with parametric conjunctions based on (p)-monotone sum of basic t-norms.

The paper has the following structure. In the section II we discuss Mamdani fuzzy models that we consider in this work. The (p)-monotone sum of basic t-norms is discussed in Section III. Section IV presents logic diagrams of modules used in FPGA implementation of fuzzy system with parameterized membership functions and operations. Section V contains conclusions and directions of future work.

## II. MANDANI FUZZY MODELS

Mamdani fuzzy models with two inputs consist of the following rules [3]:

$$R_i: \text{If } x \text{ is } A_i \text{ AND } y \text{ is } B_i \text{ then } z \text{ is } C_i$$

where  $x, y$  are input variables,  $A_i$  and  $B_i$  are fuzzy sets defined on domains  $X$  and  $Y$  of  $x$  and  $y$  respectively,  $z$  is the output variable and  $C_i$  is its fuzzy value or singleton. In this paper we suppose that  $C_i$  is a singleton. Fuzzy sets are defined by their membership functions  $A_i: X \rightarrow L$ ,  $B_i: Y \rightarrow L$  where  $L$  is a set of membership values. In traditional fuzzy systems it is used the set of membership values  $L = [0, 1]$ . In digital representation of membership values with  $m$  bits as in [19] we use the set of membership values  $L = \{0, 1, 2, \dots, 2m-1\}$  with maximal value  $2m-1$  denoted as  $I$ . This value will represent the full membership corresponding to the value 1 in traditional set of membership values  $[0, 1]$ . For example,  $I = 15$  if  $m = 4$  and  $I = 255$  if  $m = 8$ . Many concepts of fuzzy systems have straightforward extension on digital case when we replace the set of membership values  $[0, 1]$  by  $L = \{0, 1, 2, \dots, 2m-1\}$  and maximal membership value 1 by  $I = 2m-1$ .

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In our implementation we use  $m=8$  bits for representation of membership values and hence  $I=255$ . Input variables  $x, y$  have three fuzzy values  $\{XS, XM, XL\}, \{YS, YM, YL\}$  respectively with parameter values  $P_x, P_y$  defined as shown in Fig. 1 for variable  $x$ . These fuzzy values can be considered as formalizations of linguistic values *SMALL, MIDDLE, LARGE* of variables  $x, y$  respectively. For simplicity of fuzzy system implementation we use also  $m=8$  bits for representation of domains  $X, Y$  such that  $X = Y = L$ . Generally it can be used more bits for such representation, but in any case we can consider such  $x,y$  as normalized inputs of fuzzy system. The fuzzy sets  $A_i, B_i$  in the rules  $R_i$  take values from the sets  $\{XS, XM, XL\}, \{YS, YM, YL\}$  respectively and the system generally contains 9 rules corresponding to all possible combinations of fuzzy values  $A_i$  and  $B_i$ .

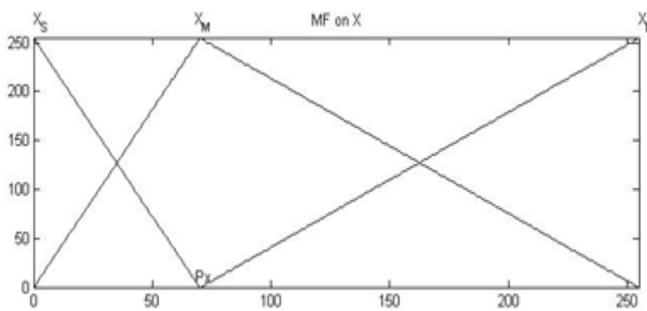


Fig. 1. The membership functions of fuzzy values  $X_S, X_M, X_L$  defined by parameter value  $P_x$ .

The details of inference, aggregation and defuzzification procedures for Mamdani fuzzy systems can be found in [3]. As a conjunction operation AND in Mamdani fuzzy model we use parametric conjunction defined by (p)-monotone sum of basic t-norms discussed in the following section.

### III. (P)-MONOTONE SUM OF BASIC t-NORMS

Fuzzy conjunction operation is a function  $T : L \times L \rightarrow L$  satisfying on  $L$  conditions:

$$T(x, I) = x, \quad T(I, y) = y, \quad (\text{boundary conditions})$$

$$T(x, y) \leq T(u, v) \text{ if } x \leq u, y \leq v. \quad (\text{monotonicity})$$

Commutative and associative conjunctions are called t-norms [21]. Usually in fuzzy systems as conjunction operation they are used the simplest conjunctions such as min or product operations. As parametric conjunction operations in fuzzy models it is possible to use parametric t-norms [21] or parametric fuzzy conjunctions introduced in [4,5] and having more simple form. But both types of parametric conjunctions are complicated for hardware implementation. In [17,18] it has been introduced a wide class of parametric fuzzy conjunctions based on simple generator functions and suitable for hardware implementation. In [19] it was proposed more simple class of parametric conjunctions called (p)-monotone sum of basic t-norms. We consider here the following basic t-norms:

$$T_M(x, y) = \min\{x, y\}$$

(minimum)

$$T_L(x, y) = \max\{x+y-I, 0\},$$

$$T_D(x, y) = \begin{cases} x, & \text{if } y = I \\ y, & \text{if } x = I \\ 0, & \text{if } x, y < I \end{cases}$$

(drastic t-norm)

Note that any fuzzy conjunction  $T$  satisfies the following inequalities:

$$T_D(x, y) \leq T(x, y) \leq T_M(x, y).$$

We say that  $T_1 \leq T_2$  if  $T_1(x, y) \leq T_2(x, y)$  for all  $x, y$  from  $L$ , e.g. we have:  $T_D \leq T_L \leq T_M$ .

Suppose  $a, b \in L$  and  $a \leq b$ . The set of all elements  $c \in L$  such that  $a \leq c \leq b$  is denoted as  $[a, b]$ . Suppose  $p \in \{0, 1, \dots, 2^m-2\}$  is a parameter. (p)-monotone sum of basic t-norms is defined as follows [19]. Define a partition of  $L$  on two sets:  $X_1 = [0, p]$  and  $X_2 = [p+1, I]$ . These sets define a partition of  $L \times L$  on four sections:  $D_{ij} = X_i \times X_j, i, j \in \{1, 2\}$  as shown in Fig. 2. Select a sequence of fuzzy conjunctions  $(T_{11}, T_{21}, T_{12}, T_{22})$  ordered as follows:  $T_{11} \leq T_{12} \leq T_{22}, T_{11} \leq T_{21} \leq T_{22}$ . Define a function  $T$  on  $L \times L$  by  $T(x, y) = T_{ij}(x, y)$  if  $(x, y) \in D_{ij}, i, j \in \{1, 2\}$ . This function  $T$  is called a (p)-monotone sum of fuzzy conjunctions  $T_{ij}, i, j \in \{1, 2\}$  and it will be a fuzzy conjunction [19]. It is clear that this fuzzy conjunction  $T$  can be defined as follows:

$$T(x, y) = \begin{cases} T_{11}(x, y), & \text{if } x \leq p, y \leq p, \\ T_{21}(x, y), & \text{if } x > p, y \leq p, \\ T_{12}(x, y), & \text{if } x \leq p, y > p, \\ T_{22}(x, y), & \text{if } x > p, y > p, \end{cases}$$

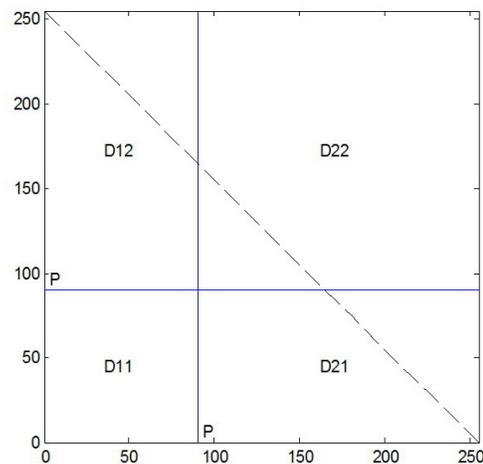


Fig. 2. Partition of  $L \times L$  on sections  $D_{ij} = X_i \times X_j$  defined by parameter value  $p$ .

As conjunctions  $(T_{11}, T_{21}, T_{12}, T_{22})$  in the definition of (p)-monotone sum we will use basic t-norms  $T_D, T_L, T_M$ . For example, (p)-monotone sum defined by basic t-norms  $(T_D, T_L, T_L, T_M)$  will be denoted as conjunction  $T_{DLLM}$  and defined as follows:

$$T(x,y) = \begin{cases} T_D(x,y), & \text{if } x \leq p, \quad y \leq p, \\ T_L(x,y), & \text{if } x > p, \quad y \leq p, \\ T_L(x,y), & \text{if } x \leq p, \quad y > p, \\ T_M(x,y), & \text{if } x > p, \quad y > p. \end{cases}$$

Fig. 3 depicts on the left the locations of basic t-norms used in the construction of  $T_{DLLM}$  in sections  $D_{11}, D_{21}, D_{12}, D_{22}$ . On the right it is shown the shape of this digital fuzzy conjunction when the membership scale  $L$  is defined by  $m= 4$  bits with parameter value  $p = 9$ .

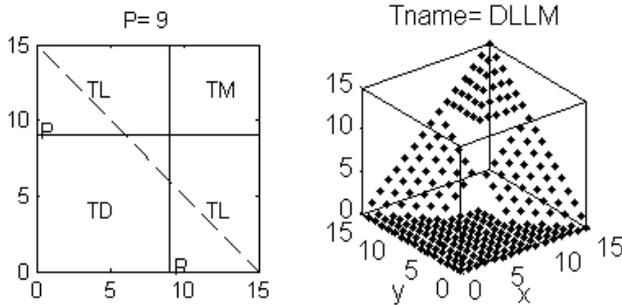


Fig. 3. Parametric conjunction TDLLM obtained by (p)-monotone sum of basic t-norms: TD- drastic, TL- Lukasiewicz and TM- minimum t-norms.

(p)-monotone sum will be commutative if  $T_{21} = T_{12}$ . By means of basic t-norms it can be constructed 7 different non-trivial (p)-monotone sums defined by the following sequences of basic t-norms: DDDL, DDDM, DLLL, DLLM, DMMM, LLLM, LMMM. All of them have been implemented in [20] in FPGA. In the following section we will propose the method of FPGA implementation of these operations in Mamdani model.

#### IV. FPGA IMPLEMENTATION OF MAMDANI MODEL

The paper presents the results of 8-bit FPGA implementation of nine rule Mamdani fuzzy system developed by means of VHDL language [22] in Quartus II with ModelSim software of Altera [23,24]. FPGA implementation of Mamdani fuzzy system consists of various modules. First, for each input values of  $x$  and  $y$  we need to calculate their membership values in corresponding three fuzzy sets  $\{X_S, X_M, X_L\}$  and  $\{Y_S, Y_M, Y_L\}$  respectively. Fig. 4 shows a diagram of module calculating membership values of input variable  $x$  for all fuzzy sets  $\{X_S, X_M, X_L\}$  by means of one divisor.

Fig. 5 presents the logic diagrams for implementing the basic t-norms, which are used to generate parametric conjunction defined by (p)-monotone sum of basic t-norms developed in [20].

As mentioned above, there are 7 possible combination operators generated by (p)-monotone sum, which are shown in Table I.

The methodology used to implement the parametric operators is based on the principle of partial reconfiguration, so that the implementation of the operators will be done according to the block diagram shown in Fig. 6, where a parametric operator consists of five basic modules: the modules of drastic, Lukasiewicz and minimum t-norms

described above that connected to a multiplexer, which controls the output through the comparator module, which is responsible for comparing the values of the weights that generate the membership functions, with parameter  $p$ , and determine the combination used to obtain the result.

TABLE I  
(p) PARAMETRIC CONJUNCTION OPERATOR

Code	Operator
1	DDDL
2	DDDM
3	DLLL
4	DLLM
5	DMMM
6	LLLM
7	LMMM

As an example the logic diagram that implements the Comparator module to DLLM operator implementation is shown in Fig. 7, consists of two comparators and a multiplexer. The implementation of the other operators generated by (p)-monotonic sum method are made in much the same way, varying only the allocation of the output and the code of comparators shown in Table 1.

The output crisp value calculation in Mamdani fuzzy system with output membership functions given by singletons  $z_i$  is shown in equation

$$z_c = \frac{\sum_{i=1}^n w_i z_i}{\sum_{i=1}^n w_i}, \tag{1}$$

where  $w_i$  are true values of antecedents of the rules. The implementation of this module can be divided in two parts. First, the module shown in Fig. 8a multiplies the nine outputs of the parametric conjunction operators with the corresponding singletons of consequents of the rules  $z_i$ ,  $F_i = w_i * z_i$ . Second, the module shown in Fig. 8, calculates the rest of the formula (1).

Mamdani system implementation, with nine rules is shown in Fig. 9. The modules Fuzzy\_X, Fuzzy\_Y implement the membership functions of fuzzy sets. The basic operations are presented by modules Drastic, Lukasiewicz and Minimum. Comparator determines the sector of monotone sum where the operands of parametric conjunction are located and defines what outputs of basic t-norms should be used in multiplexor for all nine rules in parallel. The last module calculates the output of Mamdani model using singletons that stores in registers  $A_i$ .

#### V. CONCLUSION

The paper presents FPGA implementation of 8-bit parameterized fuzzy Mamdani system with two input variables and one output variable with singleton values. The fuzzy variables have three linguistic values used in antecedents of nine rules and presented by parameterized triangular membership functions. The system is implemented on the DE2 Altera development board using VHDL language. The system contains reconfigurable fuzzy Mamdani model with parametric membership functions and parametric operations that gives possibility to adjust the system to specific

application. In the future we plan to extend the class of generated parametric operations by including associative operations (t-norms), and to join together Mamdani y Sugeno models in one architecture.

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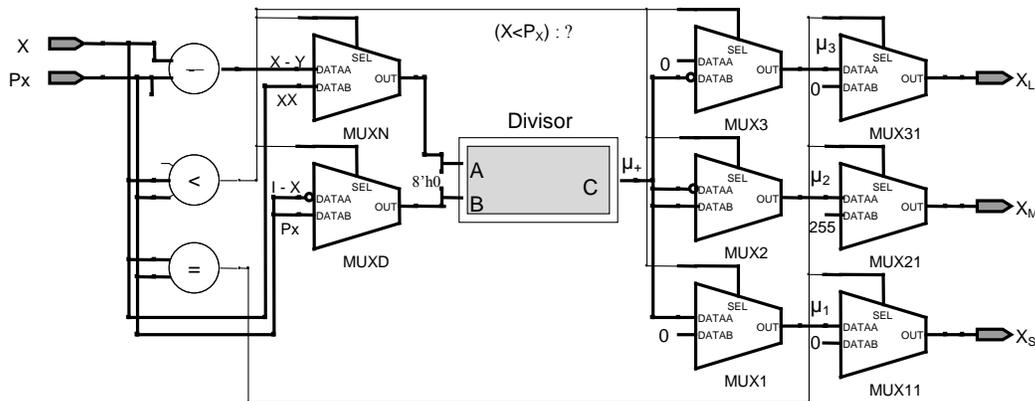
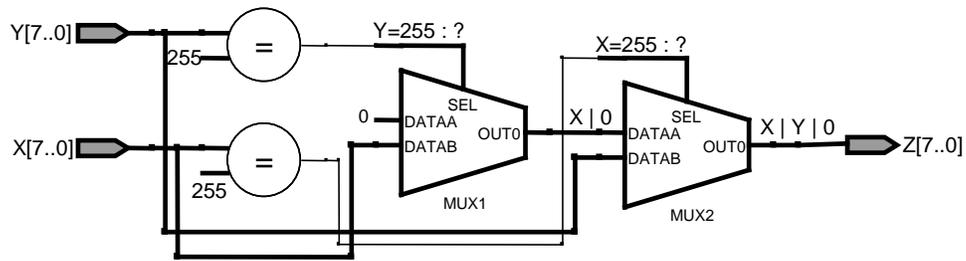
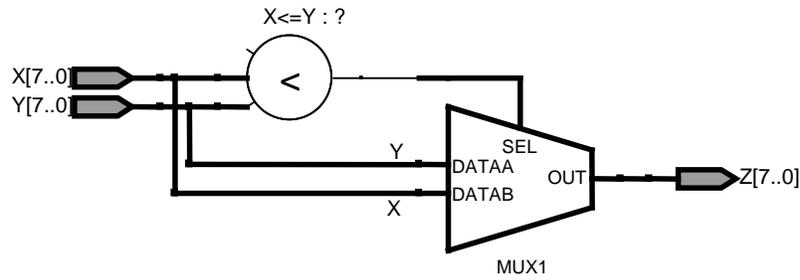


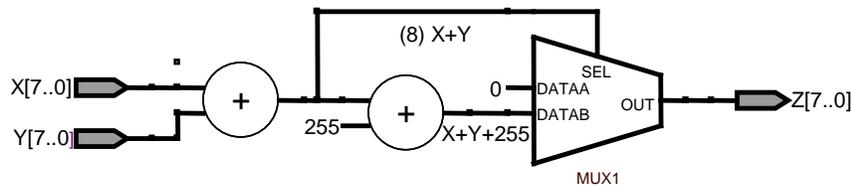
Fig. 4. Diagram of calculation of membership values of input variable x.



a) Drastic operator.



b) Min operator.



c) Lukasiewicz operator.

Fig. 5 Diagrams of basic t-norm calculation: a) Drastic, b) Lukasiewicz y c) min.

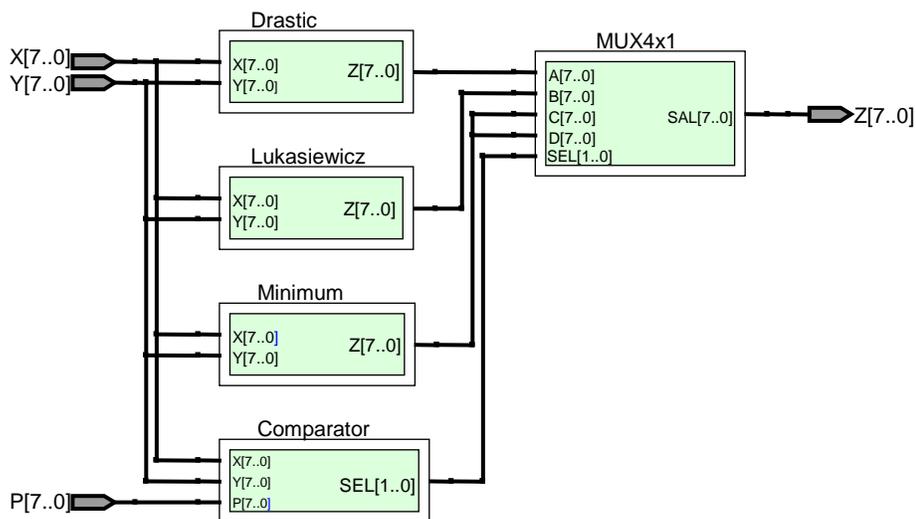


Fig. 6 Diagram of calculation of parametric conjunction used by (p) monotonic sum.

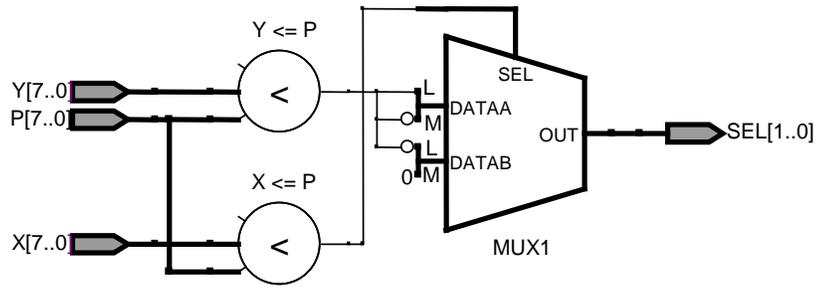
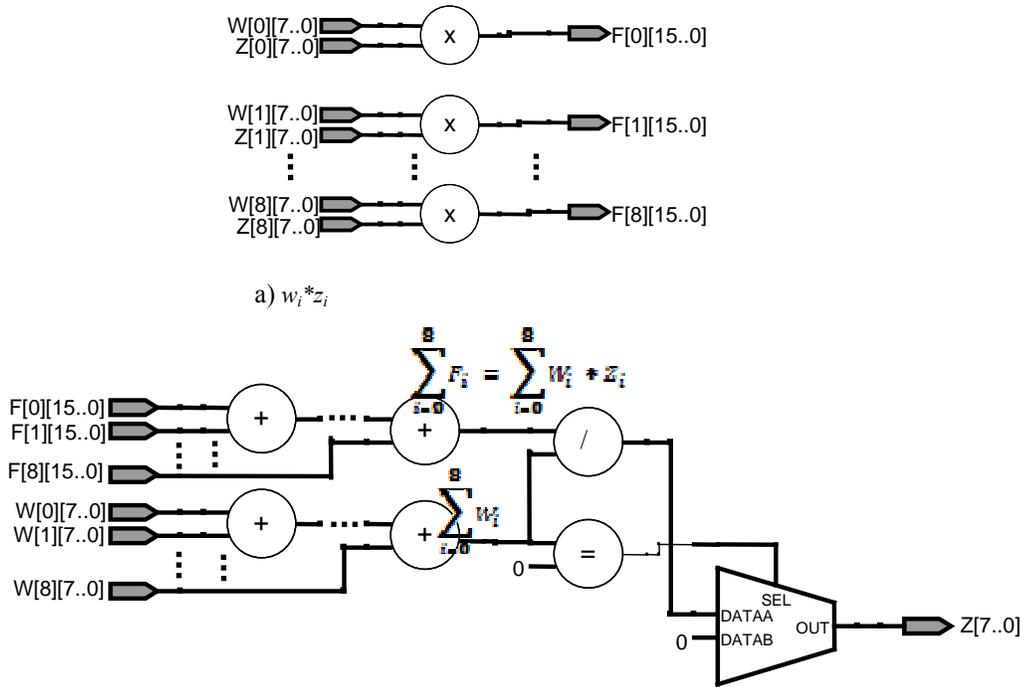


Fig. 7. Diagram of comparator module.



b) implementation of the rest of the formula (1).  
Fig. 8 Diagrams of calculation of the output of the system.

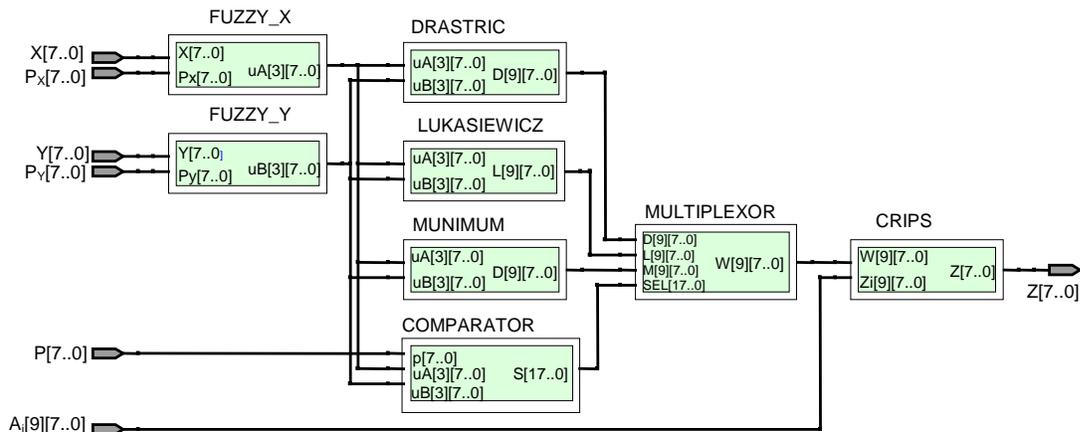


Fig. 9. General diagram of Mandani system.