Formalization of Basic Semiotic Notions in Set Theoretic Terms

Alisa Zhila

***Abstract*—Many handicaps to effective introduction of semiotic achievements into software development in the spheres related to human perception such as interfaces, computer languages, DB queries, arise due to the absence of analysis and modeling methods adequate to the complicity of the investigated objects. A formal model that expresses basic semiotic notions in species of structures is presented in the paper. The generality of initial assumptions and restrictions allows the model to describe a vast variety of communication situations from the semiotic point of view. The effectiveness of the approach is demonstrated by its stricter definitions of basic semiotic notions (sign, signified, signifier, sign types) and by the example description of sign situations (sign reading, sign reproduction).**

***Index terms*—Formal modeling, set theory, species of structures, semiotics, sign, signifier, signified, sign vehicle.**

I. Introduction

SEMIOTICS has a great potential for practical application in the filed of information technologies from better computer-to-human interface design to development of a framework for new computer languages to DB queries. Yet a number of problems with basic semiotic notions definition prevents semiotics as a filed of research from being formalized and quantified to allow its conscious usage in computer science projects and numerical evaluation of its benefit.

It was advertised in [1], [11], and [6] that one semiotic term stands for different notions in works of different authors, that definitions of notions, traditionally given in the descriptive form in a natural language, are vague, ambiguous and might imply contradictory interpretation. Although semiotics offers a series of descriptive models representing the notion of a sign, they are not suitable for direct formalization and application for any evaluations for the same reason of the lack of strict definition of their constituents [15], [16].

In the paper we offer an approach to formalization of basic semiotic notions that is capable of modeling such sign situations as common sharing of a sign between members of a communicative group, designation and reading. We assume that more complex notions and situations can be strictly described and formalized through consistent application of this approach.

This approach is based on the methods of Conceptual Analysis and Design (CAandD) [4], which allows explication of social phenomena in the terms of Species of Structures [9, 17]. Consecutive application of the steps prescribed by the CAandD methodology will ensure consistent formal modeling of the domain at hand. Basic sets needed as a starting point of formalization are selected after thorough study of a domain to be modeled. The effectiveness of the approach is demonstrated by its stricter definitions of basic semiotic notions, e.g. sign, signified, signifier, Peirce's sign categories, unique signs, individual signs, collective signs, tokens, sign types, by its enhanced descriptions of semiotic phenomena, e.g. processes of signification, processes of reading and reproducing of a sign, and by the discovery of such semiotic properties as absence of reproducing process for unique signs.

We use the term "sign situations" for notions and phenomena involving at least one of the elements of a sign as a system of multiple elements. These include signs, sign classes, signifieds, sign processes, etc. By writing *basic* sign situations we imply the notions and phenomena that are traditionally referred to as "basic" in the semiotics domain as well as those that we regard as important for further investigation using this approach.

The conceptual model consists of three parts or three conceptual schemes (further CS).The name of part 3 coincides with the name of the whole model.

In the paper we will provide representation in species of structures only for a limited number of elaborated notions, the enumeration being kept as in the original source where more entities and phenomena have been elaborated [16]. The representation of described notions in species of structures is given in a table after the respective paper subsection. For the expressions of axioms and terms mentioned in the text, please, refer to the correspondent table.

The following nomenclature will be used in this paper. The sign *B* stands for *Boolean* operator that being applied to a set A gives the set of all subsets of the set A. The pattern Dx.y, where x and y are numbers, stands for a species constant also referred to as "species structure" or "structure" in the text, the pattern Trx.y signifies terms in the sense of species of structures and Ax x.y.z, where z is also a number, stands for axioms.

II. Description of "Basic Sign Situations" Model

A. General Assumptions

The conceptual model describing the notions of a sign and sign situations in the sense discussed above is constructed in accordance with the assumptions:

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The materialistic approach: objects of the material world are assumed to be primary against abstract objects, an individual (a subject) perceives an object through sensual perception, etc.

Time is not considered in the model, e.g. material objects are not regarded before or after existence, individuals do not perceive new objects, etc. (more specific assumptions are discussed at the corresponding paragraphs).

We do not take into consideration complex cognitive processes of learning, change of subject's states of knowledge about an object, etc. The corresponding sets of the conceptual schemes contain only "final (for each subject) versions" of images, associations, etc.

1) Basic sets:

To specify basic sets we took in considerations the sign definitions by Saussure (i.1908-1911) and Peirce (i.1866-1913). Both Saussure and Peirce define a sign as a complex entity: the dyadic model of a signified and a signifier, by Saussure and the triunity of an object sign vehicle, a sense and a referent by Peirce ([1]). Although the intensional definitions of the sign elements in both cases do not allow strict assertion of identity between a signified and a referent or a signifier and a sign sense, commonly they are considered to be similar or comparable ([1]). G.P. Melnikov offers to consider an image of a signifier and an image of a signified as the entities similar to discussed above [5].

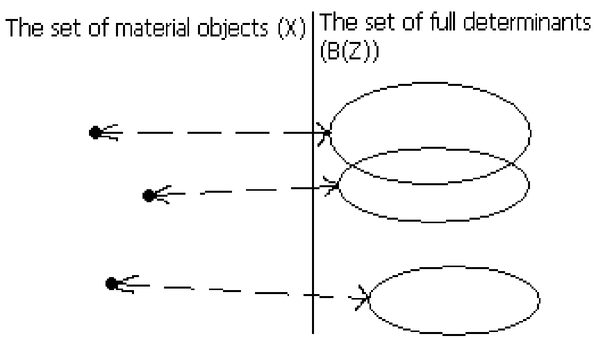

Fig.1. On the left part of the picture the black dots represent objects, and on the right part the white ovals represent objects' full determinants.

Taking into consideration prerequisite given in [5] and in [7], we concluded that the basic sets necessary and sufficient for construction of our theory with given assumptions and restrictions are: 1) the set of material objects (X) (further "objects"); 2) the set of subjects of a communicative group (Y) (further "subjects"); 3) the set of various values of various properties, that can belong to objects (Z).

For the research goal achievement we imposed the following outerscheme (non-explicated, stated only in the verbal form) restrictions:

– We consider only objects of the material world, which are perceptible through sensual perception (the restriction in the frames of the materialistic approach).
– We assume that all subjects (all members of the set of subjects of a communicative group) have equal access to all objects, have equal perceptive abilities, form object and abstract images equally, etc.
– Subjects are not considered as objects of signification (the restriction can easily be eliminated).
– The set of various values of various properties includes the results of evaluation of any properties (i.e. «objective»: size, weight, color, etc. as well as «subjective»: fear, beauty, etc.) within any scales.

TABLE I. BASIC SETS

Name	Type	***Interpretation***
X	B(X)	the set of material objects (in the sequel, objects)
Y	B(Y)	the set of subjects of a communicative group (in the sequel, subjects)
Z	B(Z)	the set of all possible values of all properties

2) Description of Part 1 «Properties and structural composition of material objects»:

The investigation of the mutual influence of the objective structure of the world and a sign, or in particular, language imprint of the world on the subject's perception of the objective and language reality [5], [14], [1] revealed that one cannot describe the perception of the world through signs without making assumptions on its "material" origin. Therefore, we introduced the mapping of material objects into imaginative entities of cognitive reality. We imposed assumptions and restrictions on the material objects as well on the mapping. Some of them are considered to be outerscheme, others are explicated in species of structures (see Table II):

– Material objects and sets of values of their properties are independent;
– Each object has a single set of values of all its properties which uniquely defines the object and is defined uniquely by it (full determinant of an object);
– For each object some subsets of values of its properties can be singled out from the set of values of all object's properties (full determinant), which uniquely determine the object (object determinants);
– For each object at least one minimal set of values of object's properties exists, which is a subset of the full determinant and defines the object uniquely (a primary key of an object);
– Existence of abstract objects is assumed to be secondary to the objects of the material world and is assumed to be related to the perception of subjects (see subsection 3) below).

Further, we postulate the structure "object and its full determinant" (D1.II), which is expressed as a binary relation on the basic sets: the set of pairs of an object and the set of values of all its properties: B(X x B(Z)) (Fig.1). It is postulated axiomatically that each object has one and only full determinant, a full determinant is not an empty set, and that a full determinant mutually and uniquely defines an object:

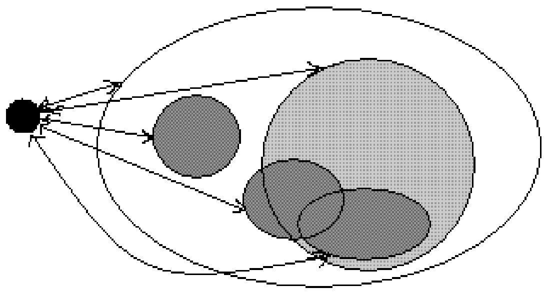

Fig. 2. The black dot stands for an object. The big white oval is object's full determinant. The light gray oval is object's *determinant*. The dark grey ovals are object's primary keys.

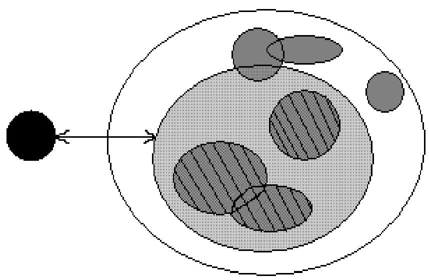

Fig. 3. The black dot stands for an object; the white oval is the object's full determinant; the light grey oval is the object's perceptible determinant; the dark grey ovals are the object's primary keys; the dark grey shaded area is object's perceptible

We introduce structures and derive terms that will be used further to derive the main terms explicating basic semiotic notions. For these purposes the structure defined by the binary relation on the basic set of objects and the set of sets of values of properties, «an object and the set of all its determinants»: B(X x B(B(Z))), is introduced. An object's determinant is a non-empty subset of the full determinant, which uniquely defines the object (Fig.2). A primary key of an object is its minimum determinant (a minimum set of values of properties) that uniquely determines the object. A material object can have more than one primary key (Fig. 2). We also derive term Tr12.II describing the set of pairs «an object and its perceptible determinant» (B(XxB(Z))) due to the requirements imposed by the assumptions relevant to part 2 «Subjectivied images and subjects as their bearers» (see below). Perceptible determinant is a determinant of an object, which contains at least two object's primary keys in the case when there are more than two or otherwise the single object's primary key (Fig. 3). For the same purposes the term Tr16.II was derived, which describes the set of triplets «an object, its perceptible determinant and its perceptible kernel». Perceptible kernel corresponding to a perceptible determinant is a union of all primary keys included into the determinant (Fig. 3). The term Tr22.II describes a notion of object's full kernel that is a union of all its primary keys (Fig. 4).

See Table II in Appendix.

3) Description of Part 2 "Subjectivied images and subjects as their bearers":

We imposed the following restrictions (axiomatic as well as stated in the verbal form):

- Objects exist objectively (independent from subjects' existence).
- We assume that there is a preexisting classification (the process of formation is not considered in the work at hand) of the objects within a communicative group ("*usual classification*"). The classification is shared among all subjects of a communicative group (the process is not considered in the research). This usual classification (D0.I) is the projection of "language cognition" of a communicative group to our model as it is described in [14], [5].
- Subjects form their own independent images of objects (*subjectivied images of certain objects*). The objects and the corresponding images mutually and uniquely determine each other for the particular subject.
- An image of an object is assumed to be a set of values of properties, which possesses certain features (see below).
- We do not consider processes of cognition, education, etc. We presume that a subject possesses a set of unchangeable images of objects (D1.I) and abstract images (Tr9.I, Tr11.I, Tr23.I) (The restriction is imposed in concordance with the assumption of absence of time flow. Introduction of these processes into the model goes far beyond the frames of the research.). The set is sufficient for communication within a communicative group.
- Abstract objects are assumed to be secondary to material objects, and an abstract object itself is regarded to coincide with its image (hereby the terms "abstract image" and "abstract object" signify the same notion, therefore only the term "*abstract image*" is used).
- All subjects form abstract images in the same way (see below).
- Subjects are able to compare a set of properties of objects, images of which they do not possess, with their abstract images, the compared abstract image and the set of possessed images not being changed in spite of the comparison process (the assumption corresponds with the absence of complex cognition processes).
- Subjects are bearers of processes and are not regarded as an input or an output of a process. We assume that any process performed by a subject does not change him or her. Thus, in the terms of process models, a subject is an «index» of a process.

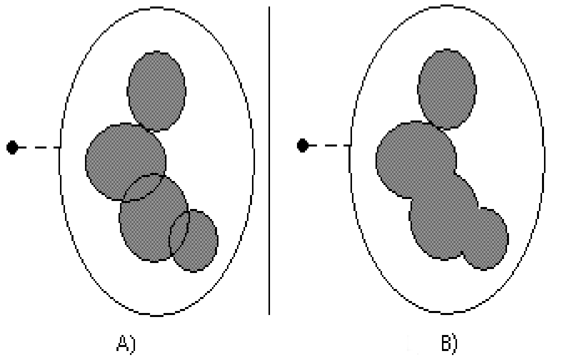

Fig. 4 The black dots are the object. The white ovals are its *full determinant*. In the left part the gray ovals represent all *object's primary keys*. In the right part the grey area represents *the full kernel of the object*.

In accordance with the made assumptions we introduce a species of structure D0.I on the set of sets of objects and the set of sets of values of properties, which is a binary relation between these two sets (B(B(X)xB(Z))). The structure models *usual classes of objects* and *usual class-forming sets of values of properties*. The classes are necessary for processes of image forming. Axioms postulate that a usual class of objects, and these classes are considered preexisting in the model, is uniquely determined by a corresponding usual class-forming set of values of properties (Ax0.1.I) and that each object belongs to at least one usual class of objects (Ax0.2.I).

The structure D1.I is introduced upon the set of subjects, the set of objects and the set of sets of values of properties. The structure represents a set of formation processes of a subjectivied image of a certain object by a subject. The axiom states that *subjectivied image of a certain object* (further, *subjectivied image of an object*) is a union of perceptible determinant (Tr12.II) with a union of all usual class-forming sets of values of properties corresponding to the usual classes, which the object belongs to (Ax1.1.I) (Fig. 5). A formed image of an object and the corresponding object mutually and uniquely determine each other for the bearing subject (Ax1.2.I).

See Table III and Fig. 5 in Appendix.

We introduce a tree of terms on the basis of the structure D1.I, which describes relations between subjects, some images, and objects (Tr2.I-Tr7.I) as well as processes of cognitive manipulating with objects by a subject (Tr8.I, Tr23.I) (see [16] for more detail).

4) Description of Part 3 «Basic sign situations»:

We analyzed the sign models of Saussure (i.1908-1911), Peirce (i.1866-1913), and Melnikov (1978). Taking into consideration the comments to the models made in [1], [11], [13], [3], and [12], the following assumptions were derived:

– A subject should possess an association (presumably in his or her cognition) between signified and signifying images.
– All signs are regarded as conventional in the research, i.e. for a sign to exist either a single subject should consciously make an association between a signified and signifying or an association should exist as a result of convention among all subjects of a communicative group (the forming of the convention is not considered in the frames of the research). Therefore, causes in cause-effect relations and sources of logic speculations are not considered to be signs in the model.
– We do not consider sign motivation, i.e. we do not consider *why* an association between a signified and a signifying is established.
– A sign associates signified and signifying images and has a material aspect (sign vehicle), i.e. a signifying perceptible material object.
– We do not distinguish between oral, written and other forms of a sign vehicle.
– We assume existence of *collective signs* common for all members of a communicative group.
– Due to the frames of the research, sign systems, notions of “text”, “context” are not explicated. Nevertheless, the model may be capable of their description through proper continuation of the term tree.
– Subjects are bearers of processes and are not regarded as an input or an output of a process. We assume that any process performed by a subject does not change him or her. Thus, in the terms of process models, a subject is an «index» of a process.
– For processes of designation with a *reproducible* signifying, it is objective properties that are to be relevant for the signifying. This assumption is made since a reproduced (for example, written) signifying object is to be objectively similar to a “template”. We do not introduce objective properties directly to the model. Yet we offer an abstract analogy of *kernel properties* (see section 2)).

To facilitate the reading of part 3 of the conceptual scheme one-valued factors of Cartesian product may be omitted if they are defined earlier. For example, in the term describing processes of reading of individuals unique signifying (Tr1.III) an image of signifying is omitted, although it should appear as a subsidiary step of the process.

Further we describe main results of the formalization.

a) Individual unique sign situations

Structure D1.III, describing the processes of *individual designation of a subjectivied image of an object with a unique signifying image* (i.e. other subjectivied image of an object), is introduces upon the set of subjects (basic set) and the set of sets of values of properties (B(Y×B(Z)×B(Z))). The input of a process is a full variety of pairs of images; the output is a chosen associated pair of a signified and a signifying object. The axiom states that signifying and signified objects are different (Ax1.1.III), i.e. they correspond to different material objects (in the frames of our model).

An example of this sign situation is a process of designation of The Statue of Liberty (an individual signified object) with a knotted handkerchief (an individual unique signifying object).

Due to the restriction of the model we presume that a handkerchief had been knotted before the process of designation and a subject already had an image of the knotted handkerchief.

On the basis of this structure term Tr1.III describing processes of reading of the unique individual signifying object is explicated. An example of the process is “reading” of a signifying knot.

Due to the construction of the term it becomes obvious that the process of reproduction of a unique signifying object is impossible (thus, the word “unique” is used).

Further, a series of derived notions is explicated in species of structures, such as specific processes of sign reading (Tr2.III, Tr11.III), designation (Tr10.III).

Term Tr5.III is the set of pairs “a signified image and an individual unique signifying image”, which is a projection of Saussure’s definition of a sign as “the whole that results from the association of the signifier with the signified” ([10]: p. 78) into the model.

Term Tr7.III explicates the set of *individual unique signs* as a set of triplets “a signified image, an individual unique signifying image, and a corresponding individual unique signifying object” (B(B(Z) ×B(Z) ×X)). The described set corresponds to Peirce’s definition of a sign as “something which stands to somebody for something in some respect or capacity” ([8]: 228). The modeling approach used in the paper shows the three part nature of a sign [2] as a relation, a Cartesian product rather than an object or an image. Additionally, the approach allows strict definitions of constituent elements hereby allowing distinguishing of individual and unique signs.

b) Individual sign situations with a signifying object that is an instance of a class of signifying objects

Structure D3.III is introduces on a set of subjects and two sets of sets of values of properties (B(Y×B(Z) ×B(Z))) and it explicates *processes of individual designation of an abstract image with an abstract image*. The input of a process is a full variety of abstract image pairs. The output is a select pair of an *individual abstract signified* and an *individual signifying class-forming image*, where an individual signified (we omit the word “image”) is a subjectivied abstract image and an individual signifying class-forming image is an abstract kernel image (Ax3.1.III). We do not impose any restrictions on signified and signifying images for this structure.

An individual signifying class-forming image, being a subjectivied kernel abstract image, corresponds to an *individual class of signifying objects* (further the word “object” is omitted), i.e. a set of objects the elements of which correspond to subjectivied images the abstract image was formed from. The relation between an individual signifying class-forming image and a corresponding class of signifyings for a certain subject is explicated in term Tr14.III.

A notion of an individual signifying class-forming image may be interpreted as a person’s notion about two parallel incline segments (such as «//») which stands for other notion, while a corresponding individual class of signifyings may be interpreted as the set of objects which the person has already seen or otherwise perceived to form this notion.

c) Collective sign situations

Structure D4.III is constructed over the set of sets of values of properties and the set of sets of objects and describes pairs “*a collective signified* and *a class of collective signifyings*” (B(B(Z) ×B(X))). Axioms state that a collective signified is a collective class-forming set of values of properties (Ax4.1.III), a class of collective signifyings is a collective strict class (see Tr22.II) of objects (Ax4.2.III). We also presume that classes of collective signifying do not intersect (Ax4.3.III).

A collective signified (a set of values of properties) is associated with a set of objects here rather than with a set of values of properties for convenience. The option of omitting one-valued factors of Cartesian products is specified above.

Term Tr21.III represents *a set of classes of collective signifyings*. The term may be interpreted as follows: a set of sets like {a, *a*, *a*, *a*, *a*, a, a, a…}, {b, *b*, b, b, …}.

Structure D5.III is a binary structure over a set of sets of values of properties and the set of objects that explicates pairs “a *collective signified* and *a unique collective signifying*” (B(B(Z) ×X)). We assume through axioms that a collective signified is a collective class-forming set of values of properties (Ax5.1.III) and any member of a communicative group has a subjectivied image of a collective unique signifying (Ax5.2.III). Interpretation of this type of signifyings may be The Statue of Liberty as a unique signifying for the communicative group of Americans, Moscow Kremlin for the communicative group of Russians, Caaba for the communicative group of Muslims, etc.

d) Other interpretations of classic semiotic notions

For the purpose of pure demonstration of descriptive possibilities of the offered model, we derive several terms interpreting notions of classic semiotics. In the paper we include only term Tr33.III representing general explication of the Saussure’s sign. It is derived through union of terms Tr5.III, Tr19.III, and Tr31.III representing particular sets, which may be interpreted into Saussure’s definition of a sign. I.e. any element of the set Tr33.III may be interpreted as a “dyadic” sign (with no sign vehicle) independently from other sign classification proposed (individuality-collectiveness, uniqueness – belonging to a class, etc.).

Other notions including those of Peircian approach can be found in [16].

See Table IV in Appendix.

III. Conclusion

The paper introduces the model of sign situations that includes significant and most common semiotic terminology and processes. Formalized definitions in the form of Species of Structures are given to basic semiotic notions, e.g. a sign, a unique sign, an individual sign. Different semiotic notions that were signified by the same semiotic term are clearly distinguished and correspondingly designated, e.g. a signifying object and a signifying image, a signified object and a signified image, individual and collective signs, unique signs and classes of signs. The classification of sign types by the number of their users (individual or collective) and by the type of signified objects is introduced. Additionally common phenomena that were not formally considered before such as processes of designation, reading processes for unique and collective signifyings, processes of reproducing of a signifying, are explicitly described in set theoretic form. The three part nature of a sign is represented as a Cartesian product of sets that emphasize a relational nature of a sign.

The novelty of the research is determined by the approach to definitions of semiotic concepts using the methods of conceptual analysis and design, which allow notion definition and phenomena representation in set theoretic terms.

Since our main result is a formal model of semiotic notions and phenomena, the work offers an approach for algorithmization and computer simulation of sign situations that can be exploited in human-to-computer communication applications as well as in communication between intelligent agents.

This approach bridges semiotic studies and formal modeling, broadening semiotics' role as a pan-scientific field.

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APPENDIX

TABLE II. PART 1 "PROPERTIES AND STRUCTURAL COMPOSITION OF MATERIAL OBJECTS"

Name	Type	Expression	***Interpretation***
X	B(X)		sets of material objects (in the sequel, objects)
Y	B(Y)		sets of subjects of communicative group (in the sequel, subjects)
Z	B(Z)		sets of various values of various properties
Objects and sets of values of their properties			
Objects and their full determinant			
D1.II	B(X × B(Z))		sets of pairs "object and its full determinant (the set of values of all its properties)"
Ax1.1.II		PR1(D1.II) = X	Full determinants are defined for all objects
Ax1.2.II		$\forall d \in D1.II(pr2(d) \neq \varnothing)$	There is no object with an empty full determinant
Ax1.3.II		$\forall d1 \in D1.II, \forall d2 \in D1.II((pr1(d1) = pr1(d2)) \Leftrightarrow \Leftrightarrow (pr2(d1) = pr2(d2)))$	any object is mutually and uniquely defined by its full determinant
Objects and their determinants			

Name	Type	Expression	***Interpretation***
D2.II	B(X × B(B(Z)))		sets of pairs «an object and a set of all its determinants (object determinant is a set of values of object's properties which uniquely defines an object)»
Ax2.1.II		$\forall d1 \in D2.II, \forall d2 \in D2.II((pr2(d1)=pr2(d2)) \Leftrightarrow$ $\Leftrightarrow (pr1(d1)=pr2(d2))))\}$	mutual and unique correspondence of an object and the set of *all* its determinants
Ax2.2.II		$\forall d1 \in D2.II, \forall d2 \in D2.II$ $(\forall E1 \in pr2(d1), \forall E2 \in pr2(d2)((E1 = E2) \Rightarrow$ $\Rightarrow (d1 = d2))$	unique correspondence of each object's determinant to the object
Ax2.3.II		$\forall d \in D2.II, \forall E \in pr2(d), (E \neq \varnothing)$	object determinants are not empty
Ax2.4.II		$\forall d \in D2.II, \forall d1 \in D1.II((pr1(d) = pr1(d)) \Rightarrow$ $\Rightarrow (\forall E \in pr2(d)(E \subseteq pr2(d1)))$	multiplicity of all object's determinants into object's full determinant
Tr2.II	B(X×B(Z))	$\{t \in X \times \mathrm{B}(Z) \mid \exists d \in D2.II, (pr1(t) = pr1(d))\,\&$ $\&(pr2(t) \in pr2(d))\}$	sets of pairs «object and some its determinant»
Objects and their primary keys (minimal sets of values of properties that uniquely define objects)			
Tr3.II	B(X×B(Z))	$\{t \in X \times B(Z) \mid \exists d \in D2.II((pr1(t) = pr1(d))\,\&$ $\&(pr2(t) \in pr2(d))\,\&$ $\&(\forall E \in pr2(d)(((E \neq pr2(t))\,\&$ $\&(pr2(t) \subset E)) \vee (E = pr2(t)))))$	sets of pairs «object and its primary key in the case when the primary key is unique » (object and its determinant, which is included for all other object's determinants)
Tr4.II	B(X)	Tr4.II=Pr1(Tr3.II)	set of objects, that have a single primary key
Tr5.II	B(X×B(B(Z))	$\{t \in \mathrm{X} \times \mathrm{B(B(Z))} \mid \forall \mathrm{E1} \in \mathrm{pr2(t)}, \forall E2 \in \mathrm{pr2(t)}$ $(((\mathrm{pr1(t),E1}) \in \mathrm{Tr2.II})\&((\mathrm{pr1(t),E2}) \in \mathrm{Tr2.II})\&$ $\&(((\mathrm{E1} \neq \mathrm{E2}) \Rightarrow ((\mathrm{E1} \not\subset \mathrm{E2})\&$ $\&(\mathrm{E2} \not\subset \mathrm{E1})))\&(\mathrm{card(pr2(t))} \geq 2))$ $\&(\exists d \in D2.II(pr1(t) = pr1(d))\,\&$ $\&(\forall E3 \in pr2(d) \setminus pr2(t), \exists E4 \in pr2(t)$ $((E4 \subset E3) \vee (E3 = \varnothing)))))\}$	set of pairs: «an object and the set of all its primary keys in the case when there are more than one primary key (sets of all minimal sets of values of its properties that uniquely determine the object)» *(none of these sets is a subset of another primary key, they are "simple", although they can have common elements")*
Tr6.II	B(X×B(B(Z)))	$\{t \in X \times B(B(Z)) \mid$ $(pr1(t), debool(pr2(t))) \in Tr3.II\}$	sets of pairs «an object that has a single primary key and the set consisting of this primary key" *(auxiliary term)*
Tr7.II	B(X×B(B(Z)))	Tr7.II = Tr5.II ∪ Tr6.II	sets of pairs «object and the set of all its primary keys»
Tr10.II	B(X×B(B(Z)))	$\{t \in X \times B(B(Z)) \mid \exists d \in D2.II, \exists t1 \in Tr7.II$ $((pr1(t) = pr1(d))\,\&\,(pr1(t) = pr1(t1))\,\&$ $\&(pr2(t) = pr2(d)\,/\,pr2(t1)))$	sets of pairs «an object and a set of all its excessive determinants», i.e. sets of pairs «an object and a set of all its determinants that are not its primary keys»
Tr11.II	B(X×B(Z))	$\{t \in X \times B(Z) \mid \exists t1 \in Tr10.II$ $((pr1(t) = pr1(t1))\,\&\,(pr2(t) \in pr2(t1)))$	sets of pairs «an object and one of its excessive determinants»

Name	Type	Expression	***Interpretation***
Tr12.II	B(X×B(Z))	$\{t \in X \times B(Z) \mid (t \in Tr11.II)\ \&$ $(((pr1(t) \notin Tr4.II)\ \&$ $\&(\exists t1 \in Tr5.II$ $(pr1(t) = pr1(t1))\ \&\ (\exists E1 \in pr2(t1),$ $\exists E2 \in pr2(t1)((E1 \neq E2)\ \&$ $\&(E1 \subset pr2(t))\ \&\ (E2 \subset pr2(t)))))) \vee$ $\vee(pr1(t) \in Tr4.II))\}$	sets of pairs «an object and one of its perceptible determinants», i.e. sets of pairs «an object and its excessive determinant, which embeds at least two object's primary keys if the object has more that one, or an object and some its excessive determinant, if the object has a single primary key». *(An auxiliary term for P3)*
Strict classes of objects and strict class-forming sets of values of properties			
Tr22.II	B(XxB(Z))	$\{t \in X \times B(Z) \mid \exists t1 \in Tr7.II$ $((pr1(t) = pr1(t1))\ \&$ $\&(pr2(t) = red(pr2(t1))))\}$	sets of pairs "an object and its full kernel (a union of all its primary keys)"
Tr23.II	B(B(X)xB(Z))	$\{t \in B(X) \times B(Z) \mid \forall x1 \in pr1(t), \forall x2 \in pr1(t)$ $\exists t1 \in Tr22.II, \exists t2 \in Tr22.II$ $((x1 \neq x2)\ \&\ (x1 = pr1(t1))\ \&\ (x2 = pr1(t2))\ \&$ $\&(\forall z \in pr2(t)((z \in pr2(t1))\ \&$ $\&(z \in pr2(t2))))\ \&\ (pr2(t) \neq \varnothing))\}$	sets of pairs "a subset of a strict class of objects and the strict class forming set of values of properties, corresponding to the class, i.e. a set of values of properties that is an intersection of objects' full kernels for all objects of the set "
Tr24.II	B(B(X)xB(Z))	$\{t \in B(X) \times B(Z) \mid (\forall t1 \in Tr23.II$ $((pr2(t) = pr2(t1)) \Rightarrow (pr1(t1) \subset pr1(t)))\ \&$ $\&(\forall x \in pr1(t), \exists t2 \in Tr23.II$ $(pr2(t) = pr2(t2))\ \&\ (x \in pr1(t2)))\}$	sets of pairs "a strict class of objects and a strict class-forming set of values of properties" *(mutual and unique correspondence between a class and the class forming set of values of properties)*

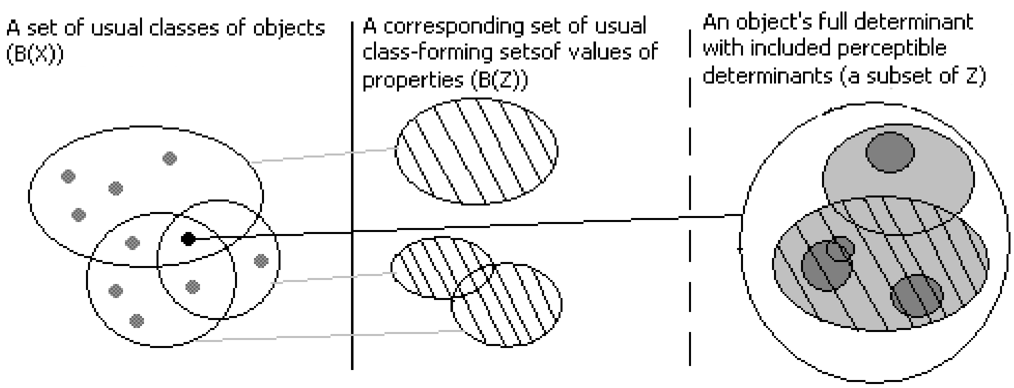

Fig. 5. The broken area with diagonal lines represents *a subjectivied image of an object*. The object is a black dot on the left. A subject is not shown.

TABLE III. PART 2 "SUBJECTIVIED IMAGES AND SUBJECTS AS THEIR BEARERS"

Preexisting classes of objects and class forming sets of property values in communicative group			
Name	Type	Expression	Interpretation
D0.I	B(B(X)×B(Z))		Sets of pairs "a preexisting class of objects and a preexisting class forming set of values of properties"

Preexisting classes of objects and class forming sets of property values in communicative group			
Name	Type	Expression	Interpretation
Ax0.1.I		$\forall d1 \in D0.I, d2 \in D0.I$ $((pr1(d1) = pr1(d2)) \Leftrightarrow$ $\Leftrightarrow (pr2(d1) = pr2(d2)))$	a preexisting class is uniquely and mutually determined by the preexisting class forming set of values of properties
Ax0.2.I		$\forall x \in X, \exists d \in D0.I (x \in pr1(d))$	each object is a member of some preexisting class
Tr1.I	B(X×B(B(Z)))	$\{t \in X \times B(B(Z)) \mid \forall E \in pr2(t), \exists d \in D0.I$ $((E = pr2(d)) \,\&\, (pr1(t) \in pr1(d)))\}$	sets of pairs "an object and a set of preexisting class forming sets of values of properties that correspond to the preexisting classes containing the object" *(an object determines uniquely a corresponding set)*
Object images			
D1.I	B(Y× (X×B(Z)))		sets of image forming processes, i.e. sets of triplets «a subject, an object and a subjectivied image of the object (otherwise object image)». *Input*: an object, being percepted by a subject; *Output*: subjectivied image of the object; a subject is a process bearer.
Ax1.1.I		$\forall d \in D1.I, \exists t1 \in Tr12.II, \exists t2 \in Tr1.I$ $((pr2(d) = pr1(t1)) \,\&\, (pr2(d) = pr1(t2)) \,\&$ $\&(pr3(d) = pr2(t1) \cup red(pr2(t2)))$	A subjectivied image of an object is a union of a perceptible determinant of all preexisting class forming sets of values of properties that correspond to the preexisting classes, containing the object.
Ax1.2.I		$\forall d1 \in D1.I, \forall d2 \in D2.I$ $(pr1(d1) = pr1(d2)) \,\&$ $\&((pr2(d1) = pr2(d2)) \Leftrightarrow$ $\Leftrightarrow (pr3(d1) = pr3(d2)))$	A subjectivied image of an object determines mutualy and uniquely the percepted object for a subject
Tr2.I	B(B(Z) ×Y)	$\{t \in (B(Z) \times Y) \mid \exists d \in D1.I,$ $((pr1(t) = pr2(d)) \,\&$ $\&(pr2(t) = pr3(d)))\}$	sets of pairs "subjectivied image of an object and a bearer of the image"
Abstract images			
Tr8.I	B(Y×B(B(Z)) × ×B(Z)))	$\{t \in Y \times B(B(Z)) \times B(Z)) \mid \forall E1 \in pr2(t),$ $\forall E2 \in pr2(t)$ $((E1 \neq E2) \,\&\, ((E1, pr1(t)) \in Tr2.I) \,\&$ $\&(E2, pr1(t)) \in Tr2.I) \,\&$ $\&((\forall z \in pr3(t)((z \in E1) \,\&\, (z \in E2))) \,\&$ $\&(pr3(t) \neq \varnothing) \,\&\, (card(pr2(t)) \geq 2))\}$	sets of formation processes of a subjectivied abstract image (or briefly an abstract image), i.e. sets of triplets "a subject, a set of subjectivied images of objects (no less than two), and a formed through subjectivied images intersection abstract image". *Input*: a set of subjectivied images of objects. O*utput*: an abstract image based on an input set of images. A subject is a process bearer.

Preexisting classes of objects and class forming sets of property values in communicative group			
Name	Type	Expression	Interpretation
Tr9.I	B(Y×B(B(Z))× ×B(Z))	$\{t \in Y \times B(B(Z)) \times B(Z) \mid \forall t1 \in Tr8.I$ $((pr1(t) = pr1(t1)) \,\&\, (pr3(t) = pr3(t1)) \Rightarrow$ $\Rightarrow (pr2(t1) \subset pr2(t)))\,\&$ $\&(\forall E \in pr2(t), \exists t2 \in Tr8.1$ $(pr1(t) = pr1(t2))\,\&\,(pr3(t) = pr3(t2)\,\&$ $(E \in pr2(t2))))\}$	sets of triplets “a subject, a set of all its subjectivied images of objects corresponding to a given abstract image, and an abstract image”
Tr10.I	B(Y×B(Z) × ×B(Z))	$\{t \in B(Z) \times B(Z) \times Y \mid \exists t1 \in Tr9.I$ $((pr1(t) = pr1(t1))\,\&\,(pr3(t) = pr3(t1))\,\&$ $\&(pr2(t) \in pr2(t1)))\}$	sets of triplets “a subject, an image of an object corresponding to a given image, and an abstract image”
Tr11.I	B(Y × B(B(Z)))	$\{t \in Y \times B(B(Z)) \mid (\forall t1 \in Tr8.I$ $(pr1(t) = pr1(t1)) \Rightarrow (pr2(t1) \in pr2(t)))\,\&$ $\&(\forall E \in pr2(t), \exists t2 \in Tr8.I$ $((pr1(t) = pr1(t2))\,\&\,(E = pr3(t2))))\}$	sets of pairs “a subject and a set of all its abstract images”
Tr15.I	B(Y×B(Z) × ×B(X))	$\{t \in Y \times B(Z) \times B(X) \mid \exists t1 \in Tr8.I$ $((pr1(t) = pr1(t1))\,\&\,(pr2(t) = pr3(t1))\,\&$ $\&(\forall x1 \in pr3(t), \exists E1 \in pr2(t1)$ $((pr1(t), x1, E1) \in Tr2.I))\,\&$ $\&(\forall E2 \in pr2(t1), \exists x2 \in pr3(t)$ $((pr1(t), x2, E2) \in D1.I)))\}$	sets of triplets “a subject, one of his or her abstract images, and a set of objects images of which belong to the subject and that correspond to the abstract image”
Tr16.I	B(Y×B(Z) ×X)	$\{t \in Y \times B(Z) \times X \mid \exists t1 \in Tr15.I$ $((pr1(t) = pr1(t1))\,\&\,(pr2(t) = pr2(t1))\,\&$ $\&(pr3(t) \in pr3(t1)))\}$	sets of triplets “a subject, one of his or her abstract images, and an object an image of which the subject possess and that corresponds to the abstract image”
Kernel abstract images			
Tr20.I	B(Y×B(Z) × ×B(Z))	$\{t \in Y \times B(Z) \times B(Z) \mid ((pr2(t), pr1(t)) \in Tr2.I)\,\&$ $\&(\exists t1 \in Tr12.II(pr2(t1) \subset pr2(t)))\}$	sets of triplets “a subject, a subjectivied image of an object, and a corresponding perceptible determinant”
Tr21.I	B(Y×B(Z) × ×B(Z))	$\{t \in Y \times B(Z) \times B(Z) \mid \exists t1 \in Tr20.I,$ $\exists t2 \in Tr16.II$ $((pr1(t) = pr1(t1))\,\&\,(pr2(t) = pr2(t1))\,\&$ $\&(pr3(t1) = pr2(t2))\,\&\,(pr3(t) = pr3(t2)))\}$	sets of triplets “a subject, a subjectivied image of an object, and a corresponding perceptible kernel”
Tr22.I	B(Y×B(Z))	$\{t \in Y \times B(Z) \mid \exists t1 \in Tr21.I$ $((pr1(t) = pr1(t1))\,\&\,(pr2(t) = pr3(t)))\}$	sets of pairs “a subject and a perceptible kernel of an object’s”

Preexisting classes of objects and class forming sets of property values in communicative group			
Name	Type	Expression	Interpretation
Tr23.I	B(Y×B(B(Z)) × ×B(Z))	$\{t \in Y \times B(B(Z)) \times B(Z) \mid \forall E1 \in pr2(t), \forall E2 \in pr2(t)((E1 \neq E2) \,\&\, \&((pr1(t), E1) \in Tr22.I) \,\&\, \&((pr1(t), E2) \in Tr22.I) \,\&\, \&(\forall z \in pr3(t)((z \in E1) \,\&\, (z \in E2))) \,\&\, \&(pr3(t) \neq \emptyset) \,\&\, (card(pr2(t)) \geq 2))\}$	sets of formation processes for abstract kernel image, i.e. sets of triplets “a subject, a set of perceptible image kernels, a corresponding abstract kernel object (a nonempty intersection of the perceptible kernel set elements)”
Tr25.I	B(Y× (Z) × ×B(B(Z)))	$\{t \in Y \times B(Z) \times B(B(Z)) \mid \exists t1 \in Tr24.I ((pr1(t) = pr1(t1)) \,\&\, (pr2(t) = pr3(t1)) \,\&\, \&(\forall E1 \in pr3(t), \exists E2 \in pr2(t1) (pr1(t), E1, E2) \in Tr21.I) \,\&\, \&(\forall E3 \in pr2(t1), \exists E4 \in pr3(t) (pr1(t), E4, E3) \in Tr21.I))\}$	sets of triplets “a subject, his or her abstract kernel image, and a set of images of corresponding objects”
Tr27.I	B(Y×B(Z) × ×B(X))	$\{t \in Y \times B(Z) \times B(X) \mid \exists t1 \in Tr25.I ((pr1(t) = pr1(t1)) \,\&\, (pr2(t) = pr2(t1)) \,\&\, \&(\forall x1 \in pr3(t), \exists E1 \in pr3(t1) ((pr1(t), x1, E1) \in D1.I)) \,\&\, \&(\forall E2 \in pr3(t1), \exists x2 \in pr3(t) ((pr1(t), x2, E2) \in D1.I)))\}$	sets of triplets “ a subject, an abstract kernel image, and a set of all objects, subjectivied images of which correspond to the abstract kernel image”
Collective classes of objects and collective class-forming sets of values of properties			
Tr32.I	B(B(X) ×B(Z))	$\{t \in B(X) \times B(Z) \mid (t \in Tr24.II) \,\&\, \&(\forall y \in Y, \exists E \subset pr1(t), \exists t1 \in Tr27.I ((y = pr1(t1)) \,\&\, (E = pr3(t1)) \,\&\, \&(pr2(t1) \subseteq pr2(t))))\}$	sets of pairs “a collective strict class of objects and a collective strict class-forming set of value properties” *(an auxiliary term for Tr33.I)*
Tr33.I	B(Y×B(Z) × ×B(Z) ×B(X))	$\{t \in Y \times B(Z) \times B(Z) \mid \exists t1 \in Tr32.I ((pr3(t) = pr2(t1)) \,\&\, \&((pr1(t), pr2(t)) \in Tr31.I) \,\&\, \&(pr2(t) \subseteq pr3(t)) \,\&\, \&((pr4(t), pr3(t)) \in Tr32.I)\}$	sets of quadruples “a subject, his or her subjectivied image, a corresponding collective strict class-forming set of value properties, and a corresponding collective strict class of objects” *(an auxiliary term for Tr33.III)*
Tr34.I	B(B(B(Z)) × ×B(Z))	$\{t \in B(B(Z)) \times B(Z) \mid \forall y1 \in Y, \exists E1 \in pr1(t) ((y1, E1) \in Tr12.I) \,\&\, \&(\forall E2 \in pr1(t), \exists y2 \in Y((y2, E2) \in Tr12.I)) \,\&\, \&(card(pr1(t)) \leq card(Y)) \,\&\, \&(\forall E3 \in pr1(t), \forall E4 \in pr1(t), \forall z \in pr2(t) (z \in E3) \,\&\, (z \in E4)) \,\&\, (pr2(t) \neq \emptyset)\}$	sets of pairs “a set of subjectivied abstract images belonging to each subject of a communicative group (one element for each subject) and a corresponding collective class-forming set of values of properties (a nonempty intersection of all elements from the former set)”

Preexisting classes of objects and class forming sets of property values in communicative group			
Name	Type	Expression	Interpretation
Tr35.I	B(Y×B(Z) × ×B(Z))	$\{t \in Y \times B(Z) \times B(Z) \mid ((pr1(t), pr2(t)) \in Tr12.I) \& \&(\exists t1 \in Tr34.I((pr3(t) = pr2(t1) \& \&(pr2(t) \in pr1(t1))))\}$	sets of triplets “a subject, his or her abstract image, and a collective strict class-forming set of value properties” *(an auxiliary term for Tr33.III)*

TABLE IV. PART 3 “BASIC SIGN SITUATIONS”

Individual sign situations			
Individual sign situations involving individual unique signifying objects			
Name	Type	Expression	Interpretation
D1.III	B(Y×B(Z) × ×B(Z))		sets of processes of individual designation of an image of an object by an individual unique signifying image of an object
Ax1.1.III		$\forall d \in D1.III, \exists d1 \in D1.I, \exists d2 \in D1.I ((pr1(d) = pr1(d1)) \& (pr2(d) = pr3(d1)) \& \&(pr1(d) = pr1(d2)) \& (pr3(d) = pr3(d2)) \& \&(pr2(d) \neq pr3(d)))$	A signifying and a signified images are subjectivied images of different material objects
Tr1.III	B(Y×B(Z) ×X)	$\{t \in Y \times B(Z) \times X \mid \exists t1 \in D1.I, ((pr1(t) = pr1(t1)) \& (pr3(t) = pr2(t1)) \& \&(pr1(t), pr2(t), pr2(t1)) \in D1.III)\}$	sets of reading processes of an individual unique signifying object, i.e. sets of triplets “a subject, a signified image, and an individual unique signifying object that stands for this image for the subject”.
D2.III	B(Y×B(Z) × ×B(Z))		sets of processes of individual designation of an abstract subjectivied image by an individual unique signifying image of an object
Ax2.1.III		$\forall d \in D2.III((pr1(d), pr2(d)) \in Tr11.I) \& \&((pr3(d), pr1(d)) \in D1.I) \& \&(\exists t1 \in Tr8.I(pr1(d) = pr1(t1)) \& \&(pr2(d) = pr3(t)) \& (pr3(d) \notin pr2(t1)))$	Signifying individual unique images are subjectivied images of objects that do not correspond to the signified abstract images.
Individual unique signs			
Tr2.III	B(Y×B(Z) ×X)	$\{t \in Y \times B(Z) \times X \mid ((pr1(t), pr2(t)) \in Tr11.I) \& \&(\exists d \in D1.I((pr1(t) = pr1(d)) \& \&(pr3(t) = pr2(d)))) \& \&((pr1(t), pr2(t), pr3(d)) \in D2.III))\}$	sets of reading processes of an individual signifying that signifies an abstract image, i.e. sets of triplets “a subject, a signified abstract image, and an individual unique signifying.
Tr5.III	B(B(Z) ×B(Z))	$\{t \in B(Z) \times B(Z) \mid (\exists d1 \in D1.III((pr1(t) = pr2(d1)) \& \&(pr2(t) = pr3(d1)))) \vee \vee(\exists d2 \in D2.III((pr1(t) = pr2(d1)) \& \&(pr2(t) = pr3(d1))))\}$	sets of pairs “an individual signified image and an individual unique signifying image”
Tr6.III	B(B(Z) ×X)	$\{t \in B(Z) \times X \mid \exists d \in D1.I((pr1(t) = pr3(d)) \& \&(pr2(t) = pr2(d)) \& \&(\exists t1 \in Tr4.III(pr2(t) \in pr2(t1))))\}$	sets of pairs “an individual unique signifying image and a corresponding individual unique signifying object”

Individual sign situations			
Individual sign situations involving individual unique signifying objects			
Name	Type	Expression	Interpretation
Tr7.III	B(B(Z) ×B(Z) × ×X)	$\{t \in B(Z) \times B(Z) \times X \mid \exists t1 \in Tr5.III, ((pr1(t) = pr1(t1) \& (pr2(t) = pr2(t1)) \& \&((pr2(t), pr3(t)) \in Tr6.III)))\}$	sets of individual unique signs, i.e. triplets "a signified image, an individual unique signifying image, and a corresponding individual unique signifying"
Tr10.III	B(Y×B(Z) ×B(Z))	Tr10.III=D1.III ∪ D2.III	sets of individual unique designation processes
Tr11.III	B(Y×B(Z) ×X)	Tr11.III=Tr1.III ∪ Tr2.III	sets of reading processes of a individual unique signifying
Individual sign situations with classes of signifying objects			
D3.III	B(Y×B(Z) × ×B(Z))		sets of processes of individual designation of an abstract image by an individual class-forming signifying image
Ax3.1.II I		$\forall d \in D3.III ((pr1(d), pr2(d)) \in Tr12.I) \& \&((pr1(d), pr3(d)) \in Tr30.I)$	A signified abstract image is a subjectivied abstract image. An individual signifying class-forming image is a kernel abstract image.
Tr13.III	B(Y×B(Z))	$\{t \in Y \times B(Z) \mid \exists t1 \in D3.III ((pr1(t) = pr1(t1)) \& (pr2(t) = pr3(t1)))\}$	sets of pairs "a subject and an individual signifying class-forming image"
Tr14.III	B(Y×B(Z) × ×B(X))	$\{t \in Y \times B(Z) \times B(X) \mid ((pr1(t), pr2(t)) \in Tr13.III) \& \&(\exists t1 \in Tr27.I, \exists t2 \in Tr29.I ((pr1(t) = pr1(t1)) \& (pr2(t) = pr2(t1)) \& \&((pr1(t) = pr1(t2)) \& (pr2(t) = pr2(t2)) \& \&(pr3(t) = pr3(t1) \cup pr3(t2))\}$	sets of triplets "a subject, an individual signifying class-forming image, and an individual class of signifying objects corresponding to a given individual signifying class-forming image"
Tr15.III	B(Y×B(Z) ×X)	$\{t \in Y \times B(Z) \times X \mid \exists t1 \in Tr14.III ((pr1(t) = pr1(t1)) \& (pr2(t) = pr2(t1)) \& \&(pr3(t) \in pr3(t1)))\}$	sets of triplets "a subject, an individual signifying class-forming image, and an instance of an individual class of signifying objects corresponding to a given individual signifying class-forming image"
Tr16.III	B(Y×B(Z) × ×B(X))	$\{t \in Y \times B(Z) \times B(X) \mid ((pr1(t), pr2(t)) \in Tr13.III) \& \&(t \in Tr28.I)\}$	sets of triplets "a subject, an individual signifying class-forming image, and a set of individual signifying objects corresponding to a given individual signifying class-forming image, whereas a subject does not possess their images" *(An auxiliary term for Tr18.III)*
Tr17.III	B(Y×B(Z) ×X)	$\{t \in Y \times B(Z) \times X \mid \exists t1 \in D3.III ((pr1(t) = pr1(t1)) \& (pr2(t) = pr2(t1)) \& \&((pr1(t), pr3(t1), pr3(t)) \in Tr15.III))\}$	sets of reading processes of an instance of an individual class of signifying objects, i.e. sets of triplets "a subject, a signified abstract image, and an instance of an individual class of signifying objects". *Input*: an instance of an

Individual sign situations			
Individual sign situations involving individual unique signifying objects			
Name	Type	Expression	Interpretation
			individual class of signifying objects. *Output: a* signified abstract image. A subject is a process's bearer.
Tr18.III	B(Y× (B(Z) × ×X))	$\{t \in Y \times (B(Z) \times X) \mid \exists t1 \in D3.III$ $((pr1(t) = pr1(t1))\,\&\,(pr2(t) = pr2(t1))\,\&$ $\&((pr1(t), pr3(t1), pr3(t)) \in Tr16.III))\}$	sets of reproducing processes of an instance of an individual class of signifying objects.
Tr19.III	B(B(Z) ×B(Z))	$\{t \in B(Z) \times B(Z) \mid \exists d \in D3.III$ $((pr1(t) = pr2(d))\,\&\,(pr2(t) = pr3(d)))\}$	sets of pairs "a signified abstract image and an individual signifying class-forming image"
Collective sign situations			
Collective sign situations with classes of signifying objects			
D4.III	B(B(Z) ×B(X))		Sets of pairs "a collective signified and a class of collective signifying objects"
Ax4.1.II I		Pr1(D4.III) $\subseteq$ Tr36.I	A collective signified is a collective class-forming set of values of properties (see Tr34.I)
Ax4.2.II I		Pr2(D4.III) $\subset$ Pr1(Tr32.I)	A class of collective signifying objects is a collective strict class (see Tr32.I).
Ax4.3.II I		$\forall d1 \in D4.III, \forall d2 \in D4.III$ $((pr2(d1) \neq pr2(d2)) \Rightarrow$ $\Rightarrow ((pr2(d1) \cap pr2(d2)) = \varnothing))$	Classes of collective signifying objects do not intersect.
Tr21.III	B(B(X))	Tr21.III=Pr2(D4.III)	Sets of classes of collective signifying objects
Tr25.III	B(Y×B(Z) ×B(Z))	$\{t \in Y \times B(Z) \times B(Z) \mid$ $\exists t1 \in Tr33.I, \exists t2 \in Tr35.I$ $((pr1(t) = pr1(t1))\,\&\,(pr3(t) = pr2(t1))\,\&$ $\&(pr1(t) = pr1(t2))\,\&\,(pr2(t) = pr2(t2))\,\&$ $\&((pr3(t2), pr4(t1)) \in D4.III))\}$	sets of triplets "a subject, a signified abstract image, and a signifying abstract image"
Tr26.III	B(B(Z) ×B(Z))	$\{t \in B(Z) \times B(Z) \mid \exists t1 \in Tr25.III$ $((pr1(t) = pr2(t1))\,\&\,(pr2(t) = pr3(t1)))\}$	sets of pairs "a signified abstract image and a signifying abstract image"
Collective sign situations with unique signifying objects			
D5.III	B(B(Z) ×X)		sets of pairs "a collective signified and a unique collective signifying"
Ax5.1.II I		Pr1(D4.III) $\subseteq$ Tr36.I	A collective signified is a collective class-forming set of values of properties.
Ax5.2.II I		$\forall y \in Y, \forall x \in Pr2(D5.III)$ $(y, x) \in Tr5.I$	Collective unique signifyings are material objects, each subject of a communicative group possessing their subjectivied images.

Individual sign situations			
Individual sign situations involving individual unique signifying objects			
Name	Type	Expression	Interpretation
Tr28.III	B(Y×B(Z) ×X)	$\{t \in Y \times B(Z) \times X \mid \exists d \in D5.III$ $((pr3(t) = pr2(d))\ \&$ $\&(\exists t1 \in Tr35.I((pr1(t) = pr1(t1))\ \&$ $\&(pr2(t) = pr2(t1))\ \&\ (pr1(d) = pr3(t))))\}$	sets of reading processes of a unique collective signifier, i.e. sets of triplets “a subject, a signified subjectivied abstract image, and a corresponding collective unique signifying”
Tr29.III	B(B(Z) ×B(Z))	$\{t \in B(Z) \times B(Z) \mid \exists t1 \in Tr28.III, \exists d \in D1.I$ $(pr1(t) = pr2(t))\ \&$ $\&(pr1(t1) = pr1(d))\ \&\ (pr3(t1) = pr2(d))\ \&$ $\&(pr2(t) = pr3(d))\}$	sets of pairs “a signified abstract image and a corresponding signifying abstract image”
Tr30.III	B(B(Z) ×B(Z) ×X)	$\{t \in B(Z) \times B(Z) \times X \mid \exists t1 \in Tr28.III, \exists d \in D1.I$ $(pr1(t) = pr2(t1))\ \&\ (pr3(t) = pr3(t1))\ \&$ $\&(pr1(t1) = pr1(d))\ \&\ (pr3(t) = pr2(d))\ \&$ $\&(pr2(t) = pr3(d))\}$	sets of unique collective signs, i.e. sets of triplets “a signified abstract image, a corresponding signifying image, and a corresponding unique collective signifying object”
Tr31.III	B(B(Z) ×B(Z))	Tr31.III=Tr26.III ∪ Tr29.III	sets of pair “a signified abstract image and a corresponding signifying image”
A union of sign situations			
Tr33.III	B(B(Z)×B(Z))	Tr33.III=Tr5.III ∪ Tr19.III ∪ Tr31.III	sets of pairs “a signified image and a corresponding signifying image” *This term corresponds to the set of signs according to Saussure.*