
A NALYSIS OF A TRANSPORT NETWORK BASED ON GEOREFERENCE SYSTEM AND MULTICOMMODITY NETWORK FLOW.

Nava-Fonseca, C. A.

Laboratorio de Transporte, Departamento de Sistemas, Facultad de Ingeniería, UNAM

Received: September 1st, 2005. Accepted: March 16th, 2006

ABSTRACT

The methodology used for the prediction of trips through the allocation of these according to the origin-destination (OD) survey, constitutes a traditional methodology to cover the demand of existing transport between the zones of analysis. When the transportation system fulfills the characteristics of a network (topology, routed arcs, nodes with commodity supply and demand, etc.) in addition, fleet restrictions are modeled as constraints on this network (capacity, transfer, flow balance, etc.); therefore the transportation system can be worked like a Multicommodity Network Flow Problem (or MCNF). If these data are contained in georeferenced databases, it offers the possibility of spatial analysis through specialized software, allowing the manipulation of such so it can be formulated as a minimum cost multicommodity network flow. The results offer better policies for the analyzed network than the results of the traditional methodology. Finally, the variations applied in the sensitivity analysis, purpose options such as: design for the future expansion, recommendations to the Master Plan of Electric Transport Systems, as well as of operating options to the present topology of the network.

RESUMEN

La metodología usada para la predicción de viajes por medio de la asignación de viajes según la encuesta origen-destino (OD), constituye una metodología tradicional para cubrir la demanda de transporte existente entre las zonas de análisis. Cuando el sistema de transporte cumple con características de una red (topología, arcos disecionados, nodos con entradas y salidas, etc.) y además se cuentan con las restricciones aplicadas a ésta red (restricciones de flotilla, capacidad, transbordo, balance de flujo, etc.) el sistema de transporte se puede trabajar como un modelo de red de flujo multiproducto (Multicommodity Network Flow o MCNF). Estos datos contenidos en bases de datos georreferenciadas ofrecen la capacidad de análisis a través de un software especializado en análisis espacial, permite la manipulación de tal forma que se pueda formular el problema de flujo en redes multiproducto de mínimo costo, y cuyos resultados ofrecen mejores datos de la operación de la red analizada. Finalmente, las variaciones a las restricciones en el análisis de sensibilidad, realizadas en el problema principal y el modelo planteado, resultan en opciones: tanto de diseño para la expansión futura, realizando recomendaciones al Plan Maestro, así como de opciones de operación a la topología actual de la red..

KEYWORDS: *Multicommodity Network Flow, Network Design, Transportation Network.*

1. INTRODUCTION

The intent of this work is to use the operation research tools in the field of network flow in specific the use of multicommodity network flow to design an urban transport network through the creation of a new methodology of solution. Also while employing these tools, it was also required to understand and use Geographical Information Systems (GIS) for the formulation of this methodology. This methodology was applied in the urban rail transportation network of Mexico City. The governing document in which this network has planned its expansion is the Master Plan

of Metropolitan and Suburban Electric Trains. The design considers only the trips attracted and generated by the zone of analysis. With only these data the future routes were designed. In contrast, the proposed methodology needs more data as well as other specialized models, in order to generate more results than the methodology with which the Master Plan was designed. The data required for the analysis and formulation of the MCNF model was: the Origin-Destination survey (made by the Mexican National Institute of Statistical Geography and Computer Science), the operative data of the Metropolitan Urban Transport System and its network cost, routes and topology. From these data, the manipulation of databases, application of transport and optimization models of the final model construction was made and later its resolution and sensitivity analysis. The model is the minimum cost multicommodity network flow and it represents the shorter trips the user would make within the network; therefore the system must offer the best service that it can grant while covering the trip demand. While the sensitivity analysis of the model offered results like: projected passenger routing before the construction of the expansion is made, impacts when the fleet is increased or the itinerary frequency is increased or decreased compared with the present network.

2. MCNF PROBLEM FORMULATION

There are mainly two forms of formulating the multicommodity problem, depending on the type of problem or in the form in which the problem is being solved, since the computational order varies when the commodity is the OD pair and the notation is the node-arc notation. Let N denote the set of all nodes in G , A the set of all arcs, and K the set of all commodities.

For commodity k with an origin s_k and destination t_k , c_{ij}^k represents its per unit flow cost (i, j) and x_{ij}^k the flow on arc (i, j) . Let b_i^k the supply demand at node i , and B^k the total demand units of commodity k . Let u_{ij} be the arc capacity on arc (i, j) . We assume each unit of each commodity consume one unit of capacity from each arc on which it flows.

2.1 NODE-ARC FORM

The node-arc form of the MCNF problem is a direct extension of the conventional single commodity network formulation. It can be formulated as follows:

$$\min \sum_{k \in K} \sum_{(i, j) \in A} c_{ij}^k x_{ij}^k = Z^*(x)$$

s.a.

$$\sum_{(i, j) \in A} x_{ij}^k - \sum_{(j, i) \in A} x_{ji}^k = b_i^k \quad \forall i \in N, \forall k \in K \quad (1)$$

$$\sum_{k \in K} x_{ij}^k \leq u_{ij} \quad \forall (i, j) \in A \quad (2)$$

$$x_{ij}^k \geq 0 \quad \forall (i, j) \in A, \forall k \in K \quad (3)$$

Where $b_i^k = B^k$ if $i = s_k$, $b_i^k = -B^k$ if $i = t_k$, and $b_i^k = 0$ if $i \in N \setminus \{s_k, t_k\}$.

The set of constraints (1) represent the demand and supply satisfaction in the node of study. The set of constraints (2), are the mutual capacity for a group of variables in an arc, also known as packing constraints. The set of constraints (3) are the non-negative conditions for the decision variables. Additionally it is possible to add to the model the side constraints, which limit the flow between the OD pairs, in real world problems these constraints describe the limitations in: inventory, load platforms, trucks or train load.

2.2 MCNF PROBLEM DECOMPOSITION

The MCNF has a block-angular structure, because of this; specific methods and algorithms for its solution exist. Kennington, J.L. and R.V. Helgason [1], they describe the methodologies to decompose the problem by pricing, resources and primal partition. In the case of the problem modeled with the node-arc form, the decomposition of the problem generates the minimum cost network flow problem. The main characteristic of the generated subproblem through this partition is that it has many constraints. In contrast, when the problem is formulated with the node-path form, the decomposition results in the short path subproblem, being less number of constraints and the short path problem is easier to solve. I-Ling Wan [2] proposes algorithms that optimize the resolution of the problem of shorter route to solve the MCNF.

2.3 SOLUTION METHODS TO THE MCNF PROBLEM

We mentioned that the MCNF problem has a block-angular structure, to exploit this characteristic, specific method, algorithms and solutions strategies exist; the objective is to invert the base matrix and make more efficient its resolution. Hartman and Lasdon [3] propose the technique to create superior limits within the base in a specialized simplex method. Because the number of active restrictions in the optimal solution is less than the number of arcs with capacity, it is possible to divide the constraints in two groups: active and the secondary ones; like Kennington proposes [4] in the primal partition .or McBride [5][6][7] that who developed EMNET, which bases its solution on factorization methods. EMNET is designed to solve linear problems that count with a network structure and network problems with lateral restrictions. Using a heuristic decomposition, heuristic it provides a feasible solution to the primal base computed by EMNET and provides a smaller compute time for the resolution of the problem. Murtagh and Saunders [8], proposed a strategy where the variables could be classified in 3 types: basic, notnon-basic and superbasic. From this strategy Castro and Nabona [9] implemented to the MCNF a library called PPRN to solve the subproblem of minimum cost of the MCNF, which can have an linear or nonlinear objective function linear or nonlinear. For this work, the primal partitioning method was used and including side constraints to the model.

3. CONSTRUCTION OF THE MODEL

The Origin-Destination survey made by the Mexican National Institute of Statistical Geography and Computer Science define geographic areas where users generate trips by type (automobile, bus, urban rail, taxi and other). From this survey we can catalog the pattern of trips to model the demand and supply of the transportation.

3.1 DECISION VARIABLES

The decision variable is set to tell the quantity of commodities from an origin node to a terminal node traveling by the arc considered in the network. The meaning is the amount of passenger who start a trip in a determined station withand end it at another station using the urban rail transport system, under the assumption of short path. The problem has 52668 decision variables.

3.2 SUPPLY AND DEMAND OD MATRIX

With the information of the georeferenced survey data, the databases where filtered with the purpose to eliminate non-relevant data. Therefore, each Transport Analysis Zone (or ZAT) only has the attributes for generated and attracted trip by the transport system analyzed: urban rail network. Having a new theme map generated by the filtering, it was possible to work on other theme map which included the urban plan of Mexico City and its suburban zones; also, another theme map was used: the georeferenced urban rail network and its stations. With this layers combined, the working map was developed, where the geometric centroid of each ZAT established the origin of the trip and was assigned to the nearest rail station (as seen in the Figure 1). The same procedure was used to the attracted trips of

each ZAT. Each station was a node, for that reason now the new map had new database entries: total of supply and demand trips for each node. For these new generated databases, bounded to spatial information, the trips must be balanced in order to assure no accumulation within the network. After the balance in the supply and demand trips, using the double restricted growth model, the trip distribution OD matrix was constructed. The follow

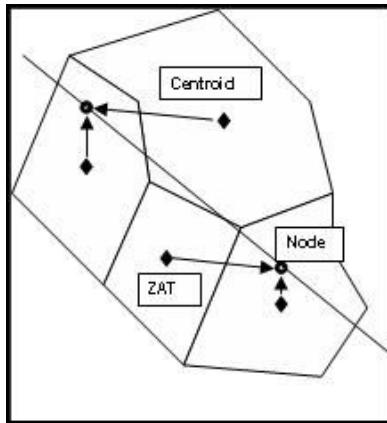


Figure 1

$$T_{ij} = t_{ij} * a_i * b_j$$

s.t.

$$\sum_j T_{ij} = P_i$$

$$\sum_i T_{ij} = A_j$$

Where:

T_{ij} = is the forecasted flow produced by the zone i and attracted to zone j

t_{ij} = is the base production of zone i attracted to zone j

a_i = is the balance factor for line i

b_j = is the balance factor for line j

P_i = is the number of trips generated by zone i

A_j = is the number of trips generated by zone j

The solution of the problem converge handling it iteratively, with the generated matrix, the amount of products in form of OD pairs determine the amount of passengers for each node of the problem.

3.2 COST MATRIX

It was useful to use the georeference data of the rail system, because among the characteristics of this stations and lines (spatial position, travel length distance, operation speed) the generation of the cost matrix was based on time travel between stations. So this study is time cost-dependant, so it is possible to assign actual or future time cost to the traveling time, making easier the economic evaluation of the proposed alternatives. Therefore the results are analyzed taking the travel-hour as the base unit in the cost matrix.

3.3 NETWORK MATRIX

The transport network considered in this study is divided in lines; actually it is structured by 10 rail lines and one light rail line. Altogether these 11 lines are composed by 157 nodes and 342 directed arcs. By using the identifiers from the GIS databases as a label, the arcs are composed as an OD pair, denoting where is the trip generated and in which node is its arrival. Finally, according to its topology, the network matrix was generated with the OD pairs.

3.4 MUTUAL AND ARC CAPACITY VECTOR

For the set of 11 lines that compose the network, each line has its own fleet of trains (which make certain number of runs per day) and its rush hour capacity. To the working database, new entries were inserted to each of the OD pairs forming an arc and the network; and then identifying to which line it belongs. Lastly, the fleet capacity was added to the database per line identified as a new entry to the database. This new table contained both mutual and arc capacity, since the fleet capacity is equal to the mutual capacity for the OD pairs during the trip along the same line.

4 RESULTS AND SENSITIVITY ANALYSIS

The results were worked in such a way that their interpretation could suggest how to operate more efficiently the network, and also what to expect in case of an expansion. This was worked with the value of the objective function, de value of the decision variables and the shadow price for mutual capacity constraints.

4.1 MODEL VALIDATION

The solution to the problem must reflect the present operation of the urban rail transport system. The value of the mutual capacity constraints, besides the slack and shadow were used to compare the actual operation of the transport system. Each year the Collective Rail Transport System emits an annual report containing the statistical use of each line, like: the total of passengers, day with the minimum and maximum influx, average of train usage per day, etc. The data is classified according to the operation and then verify that the lines presenting saturation are the ones with the lowest or none slack. Also, some arcs are known to have saturation, this can be corroborated if the mutual capacity constraint is active. In addition, there are well known ZAT where the contribution of passengers is very low (for example, when the land-use is industrial and the traffic is by other transport system), but their OD pairs describe the trips search to reach modal hubs. Following this, analysis, it was determined that there were only slight variations from the multicommodity model, and it reflected the behavior of the passenger and the operation of the whole system.

4.2 SENSITIVITY ANALYSIS

The following variations were made to the model:

1. Modify the mutual capacity vector; thus, the general model was granted more capability to distribute the passengers through the network.
2. Propose new arcs between nodes, this would expand the cost matrix and modify the topology of the network (increasing the network matrix).
3. Finally, propose new arcs between the nodes with the most highest passenger traffic.

4.3 RESULTS AND INTERPRETATION

4.3.1 ITINERARY OR FLEET MODIFICATION

This scene describes when the first modification of the sensitivity analysis was made. The increase in the mutual capacity vector represents the increase of trains in the fleet or more passage frequency of the trains through the stations. As it is seen in the figure 2, the value of the objective function decreases until the capacity does not allow more trips being routed despite a greater capacity is offered. Also we noticed that the increase in the fleet capacity by any of these modifications (itinerary frequency or more trains) offer a global diminish in the objective function value by reducing the passenger-hour-trip. While examining the operative measures, it is not possible to increase the fleet more than certain value; this is because due to security measures, each train needs an specific distance to make an emergency maneuver.

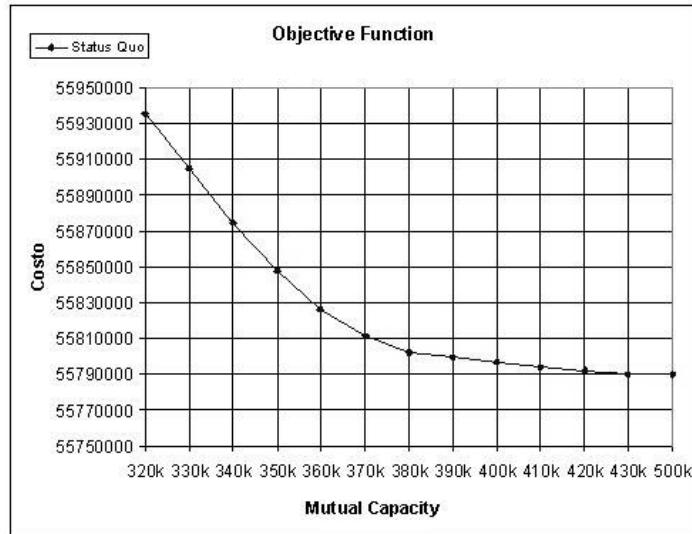


Figure 2

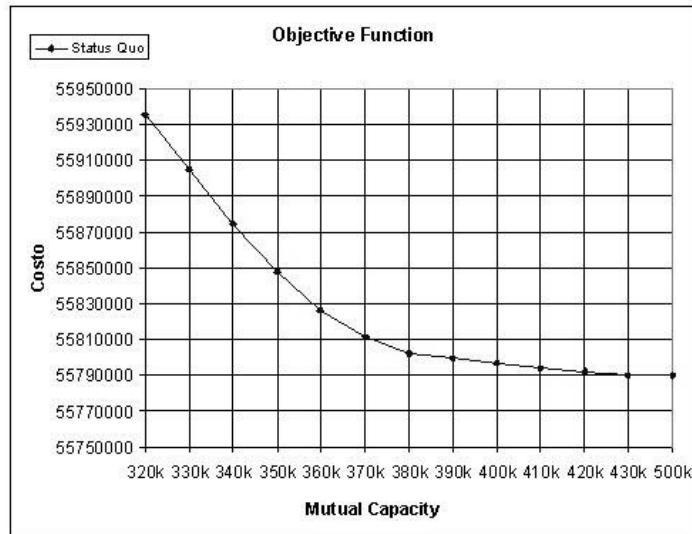


Figure 3

4.3.2 NETWORK EXPANSION

The Master Plan for Urban Electric Trains contains where and what expansions will be made to existing lines and the creation of new lines. This document stipulates that line 8 must connect with line 3 allowing a transfer with line 5; and, line 6 will expand to make a transfer with line B. These modifications constitute the second scene in the sensitivity

analysis. In the following years, according to the expansion plan, these lines and stations will be constructed and put in operation. To forecast the use of the planned lines, the original problem was modified, also its fleet is modified (as the previous scene in the sensitivity analysis). As it is seen in the figure 3 in the same graphic it is compared the expansion scene with the previous scene; with an easier route for passenger to reach their destinations, the behavior is to save time in their trips; and, while increasing the fleet management the value of the objective function also decreases. The report of the shadow price and mutual capacity of the expansion arcs allows noticing the routed trips along the planned arcs; therefore it is possible to determine the size of the new frequency itinerary or the new fleet size and plan the acquisition of new trains if necessary.

4.3.3 EXPRESS TRAINS

The third modification to the model is based on an uninterrupted operation between transshipment stations. These stations allow transfers from one line to another, and some of them have the capacity for modal transfer, creating more demand to reach these stations. The stations selected for this setting operate with saturation or near it; also the stations were on the lines with the most trips. Under the scope of better service, it is possible to suggest that most of the passengers must be routed with greater agility. After analyzing the value of the decision variables the lines that present saturation have most of its arcs occupied because most of the passengers make long trips. For this type of operation and expansion, as it is observed in the figure 4, the value of the objective function is smaller than the previous scenes, but contrary to the other two, increasing the fleet capacity does not decrease to much the value of the objective function. As it is observed in the figure 4, the diminution of the objective function value decrease notably compared with the actual operation. Consequently, the use of bays in the stations to promote the use of express lines might actually help to save time among the passengers.

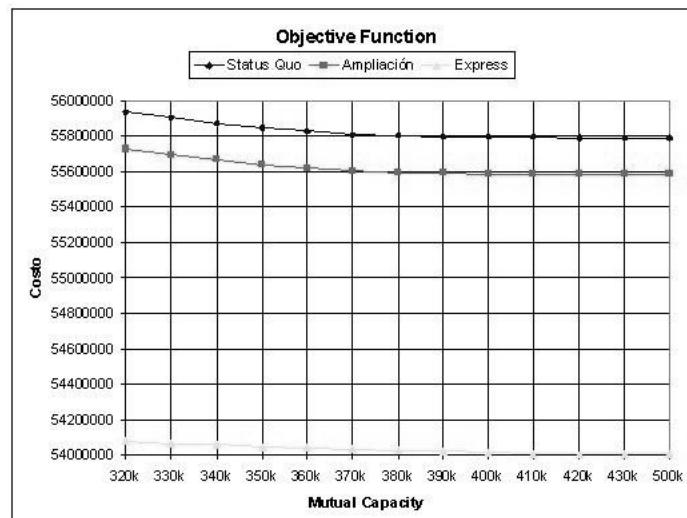


Figure 4

5. CONCLUSIONS

In the global behavior, the cost of the number of trips is reduced in a range of 0.5% up to 3%, depending on the chosen alternative. This means that the solution searched for better level of service must account the use and resource of each line. Also this saving of travel-passenger-hour allows setting the starting point and analyzing what type of expansion will be pursued. The methodology used for the prediction of trips by the model uses the multicommodity network flow approach, and its solution provide other data for evaluation in addition to the methods used to design the Master Plan of Electric Rails. The Plan states that its content is subject of to changes and it is possible to add new techniques and technology to provide a better design and service. Since this methodology consider the methods of allocation, generation and distribution of trips, but also the conditions and restrictions of the transport network, it is possible that the sensitivity analysis give results as: estimation of trips through the fictitious arcs, fleet increase by expansion or level of service, etc.

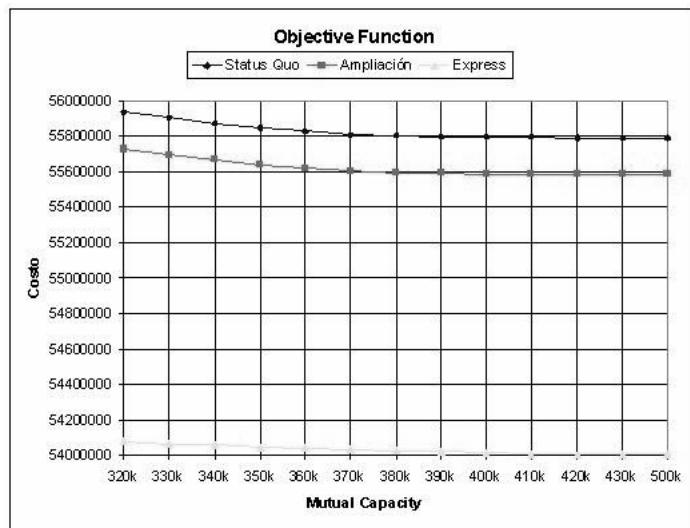


Figure 4.3

The planning of expansion and design of the transport network now consider aspects such as spatial position (and land use if necessary), better trip movement and estimation and ways to improve the actual operation of the network (detecting long trips and bottlenecks).

6. FUTURE WORK

After this research, it is possible to find other ways of improving the actual operations by making other changes in the sensitivity analysis, since this work only modified the mutual capacity vector. Also the future work will try to incorporate this methodology to other transportation network like buses, and logistic supply chains.

Finally, all the results of this analysis were obtained in worksheet or text tables, it will be better to change these tables to databases and incorporate them to the original GIS databases, so new thematic maps could be created.

7. REFERENCES

- [1] Kennington, J.L. and R.V. Helgason, "Algorithms for network programming", John Wiley & Sons, New York. 1980.
- [2] I-Ling Wan, "Shortest Paths and Multicommodity Network Flows", Georgia Institute of Technology. 2003.
- [3] Hartman, J. and Lasdon, L., "A generalized upper bounding algorithm for multicommodity network flow problems", Networks, vol. 1, no. 4, pp 333-354, 1972.
- [4] Kennington, J., "Solving multicommodity transportation problems using a primal partitioning simplex technique", Naval Research Logistics Quarterly, vol. 24, pp 309-325, 1977.
- [5] McBride, R., "Solving embedded generalized network problems" European Journal of Operational Research, vol. 21, no. 1, pp 82-92, 1985
- [6] McBride, R., "Advances in solving the multicommodity flow problem", Interfaces, vol. 28, no. 2, pp 32-41, 1998
- [7] McBride, R., "Progress made in solving the multicommodity flow problem", SIAM Journal on Optimization, vol. 8, pp. 947-955, 1988
- [8] Murtagh, B. and Saunders, M., "Large-scale linearly constrained optimization", Mathematical Programming, vol. 14, no. 1, pp 41-72, 1978

[9] Castro, J. and Nabona, N., "An implementation of linear and nonlinear multicommodity network flows", European Journal of Operational Research, vol. 92, pp 37-53, 1996.



Carlos Alberto Nava Fonseca.

2002 Civil Engineer, Facultad de Ingeniería, UNAM, 2005 Master in Engineering, Facultad de Ingeniería, UNAM, Worked at the Transportation Laboratory in the Systems Department for the Graduate School of Engineering since 2000 until 2005. During this period he cooperated with the Social Development Secretariat of Mexico in courses and seminars involving Urban Planning and Transportation; also worked as a second lecturer for the undergraduate course in Traffic Engineering. He worked in various joint projects including logistics and urban development. Moved to the consultancy sector for chain supply management during 2005. Currently, since April 2006, he is in a Doctoral Course at the Tohoku University in Sendai, Japan. He is conducting research in Urban Transportation and Planning. His research topics include: Traffic Simulation, Network flows and Urban Transportation.