An Unambiguous Tracking Scheme Using Partial-Pulses for BOC Signals

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ABSTRACT
The recent researches on the tracking of binary offset carrier (BOC) modulated signals have been studied focusing on resolving the ambiguity problem caused by the multiple side-peaks in BOC autocorrelation. In this paper, we propose a novel unambiguous BOC tracking scheme with an improved tracking performance by using partial-pulses of BOC signals. Firstly, we observe that a sub-carrier consists of two partial rectangular pulses referred to as partial-pulses, and then, generate multiple partial-correlations composing the BOC autocorrelation. Finally, a correlation function with no side-peak is constructed by combining the partial-correlations, and then, a delay lock loop employing the proposed unambiguous correlation function adjusts the phase of the local BOC signals. From numerical results, it is confirmed that the proposed scheme provides a better tracking performance than the conventional schemes in terms of the tracking error standard deviation.

Keywords: Binary offset carrier (BOC), global navigation satellite systems (GNSSs), tracking, side-peaks, ambiguity problem.

1. Introduction

Driven by the increasing demand for more precise location-based services [1], [2], new GNSSs including European Galileo and modernized global positioning system (GPS) employ the binary offset carrier (BOC) modulation to provide a higher positioning accuracy than the phase shift keying (PSK) modulation used in the conventional GPS [3]-[6]. The BOC modulated signal is generated by multiplying a pseudorandom noise (PRN) code with a sub-carrier of sine- or cosine-phased square wave, denoted as $\mathrm{BOC}_{\sin}(kn,n)$ or $\mathrm{BOC}_{\cos}(kn,n)$, respectively: Here, k represents the ratio of the chip period T_c of the PRN code to the period of the sub-carrier, and n denotes the ratio of T_c^{-1} to 1.023 MHz [7], [8].

Due to a sharp main-peak of BOC autocorrelation, the BOC signals offer a better tracking performance than the PSK signals. However, the BOC autocorrelation has multiple side-peaks around its main-peak, and thus, the tracking loop might be locked on one of the multiple side-peaks, causing a biased tracking result, which is referred to as the ambiguity problem [9]. Thus, several unambiguous tracking schemes [10]-[12] have been proposed to provide unambiguous correlation functions by removing side-peaks. A correlation function with no side-peak was firstly proposed for $\mathrm{BOC_{sin}}(n,n)$ in [10] by subtracting the correlation between the received BOC and local PRN signals from the BOC autocorrelation. Then, in [11], specially designed local signals with a tunable parameter κ are employed to provide unambiguous correlation functions for $\mathrm{BOC_{sin}}(kn,n)$. In [12], an interesting scheme applicable to both $\mathrm{BOC_{sin}}(kn,n)$ and $\mathrm{BOC_{cos}}(kn,n)$ was proposed based on a combination of sub-correlations of the BOC autocorrelation, which exhibits a better tracking performance than the schemes of [10] and [11]; However, the scheme of [12] has focused only on cancellation of correlation side-peaks without considering the tracking performance improvement.

The motivation of the paper comes from the fact that the effect of a tracking error can be reduced by sharpening the correlation main-peak [3]. Thus, in this paper, we address an unambiguous tracking scheme remove the side-peaks and improve the tracking performance by generating an unambiguous correlation function with a main-peak sharper than those of the conventional unambiguous correlation functions. We interpret a sub-carrier pulse as a sum of two partial pulses, and then, generate $4k$ partial-correlations composing BOC autocorrelation. In the first stage, we generate an unambiguous correlation function with a low and narrow main-peak by combining four partial-correlations out of $4k$ partial-correlations. In the second stage, we generate multiple unambiguous correlation functions with the same main-peak width by combining the correlation function obtained at the first stage with partial-correlations, and then, construct an unambiguous correlation function with a high and sharp main-peak by increasing the height of the main-peak. Moreover, the proposed scheme not only provides an improved tracking performance but also is applicable to both $\mathrm{BOC_{sin}}(kn,n)$ and $\mathrm{BOC_{cos}}(kn,n)$.

The rest of this paper is organized as follows. In Section 2, we describe the BOC signal model and its partial-correlations. Section 3 proposes an unambiguous correlation function with a higher and sharper main-peak. In Section 4, tracking performances of unambiguous tracking schemes are compared in terms of the tracking error standard deviation (TESD). Finally conclusion is presented in Section 5.

2. Signal model

Interpreting that a sub-carrier pulse is the sum of two rectangular pulses (referred to as partial-pulses), we can express the baseband equivalent of BOC signal as

$$b(t) = \sqrt{S}\sum_{i=-\infty}^{\infty} c_i p_{T_c}(t - iT_c)c_{sc}(t), \tag{1}$$

where S is the signal power, $c_i \in \{-1,1\}$ is the ith chip of a PRN code with period T, $p_\alpha(t)$ is the unit rectangular pulse over $[0,\alpha)$, and T_c is the chip period of the PRN code. In addition, $c_{sc}(t) = \sum_{l=0}^{4k-1} h_l p_{T_s}(t - iT_c - lT_s)$ is the square wave sub-carrier, $T_s = T_c / 4k = \frac{1}{4kn\times 1.023\,\mathrm{MHz}}$ is the duration of a partial-pulse, and $h_l \in \{-1,1\}$ is the sign of the lth partial-pulse: For $\mathrm{BOC_{sin}}(kn,n)$ and $\mathrm{BOC_{cos}}(kn,n)$, h_l is specified as $(-1)^{2ki+\lceil \frac{l}{2} \rceil}$ and $(-1)^{2ki+\lfloor \frac{l}{2} \rfloor}$, respectively, where $\lceil x \rceil$ is the smallest integer not smaller than x and $\lfloor x \rfloor$ is the largest integer not larger than x. We can also see that there exist $4k$ partial-pulses in a chip period T_c. We assume that every chip of the PRN code is an independent random variable taking on +1 and -1 with equal probability and the code period T is sufficiently large compared with the chip period T_c. The normalized BOC autocorrelation can be expressed as

$$\begin{aligned} R(\tau) &= \frac{1}{ST}\int_0^T b(t)b(t+\tau)dt \\ &= \sum_{l=0}^{4k-1}\{\frac{1}{4k}\sum_{m=0}^{4k-1} h_l h_m \Lambda_{T_s}(\tau + (l-m)T_s)\} \\ &= \sum_{l=0}^{4k-1} P_l(\tau), \end{aligned} \tag{2}$$

where

$$\Lambda_{\varepsilon}(\tau)=\begin{cases}1-\dfrac{|\tau|}{\varepsilon}, & |\tau|\le\varepsilon,\\ 0, & |\tau|>\varepsilon\end{cases} \quad (3)$$

is a triangular function and

$$P_l(\tau)=\frac{1}{4k}\sum_{m=0}^{4k-1}h_l h_m \Lambda_{T_s}(\tau+(l-m)T_s) \quad (4)$$

is the lth partial-correlation function. Figure 1 shows partial-correlations $\{P_l(\tau)\}_{l=0}^{4k-1}$ and autocorrelations for $\mathrm{BOC_{sin}}(kn,n)$ and $\mathrm{BOC_{cos}}(kn,n)$. From the figure, we confirm that the BOC autocorrelation is the sum of the partial-correlations.

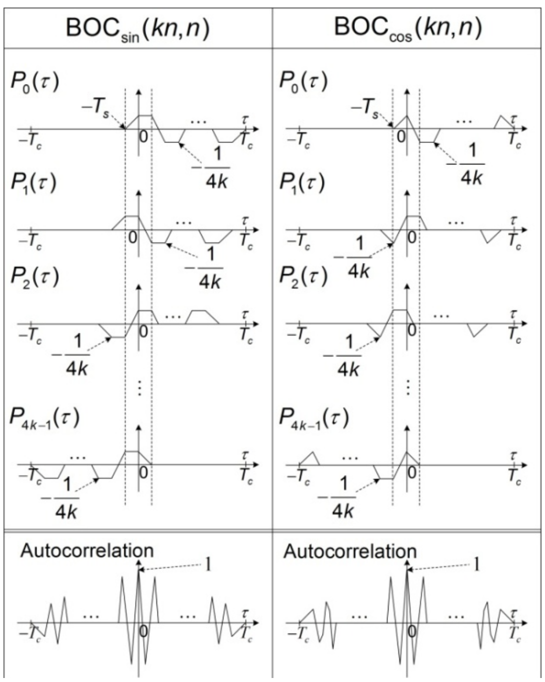


Figure 1. Partial-correlations and autocorrelations for $\mathrm{BOC_{sin}}(kn,n)$ and $\mathrm{BOC_{cos}}(kn,n)$.

3. Proposed correlation function

We generate an unambiguous correlation function with a high and sharp main-peak in two stages. The first and second stages are described in Figures 2 and 3, respectively, for $\mathrm{BOC_{sin}}(kn,n)$.

In the first stage, we combine $P_0(\tau)$ and $P_{4k-1}(\tau)$ as

$$\begin{aligned}R_0(\tau)&=P_0(\tau)\oplus P_{4k-1}(\tau)\\ &\triangleq |P_0(\tau)|+|P_{4k-1}(\tau)|-|P_0(\tau)-P_{4k-1}(\tau)|\end{aligned} \quad (5)$$

to generate an unambiguous correlation function $R_0(\tau)$ with no side-peak, where $a\oplus b\triangleq|a|+|b|-|a-b|$. From Figure 2, we can observe that the main-peak width of $R_0(\tau)$ is determined by the location of a zero-crossing point nearest to $\tau=0$ in $P_0(\tau)$ and $P_{4k-1}(\tau)$. Thus, to generate an unambiguous correlation function with a narrow main-peak, we combine $R_0(\tau)$ with $P_1(\tau)-P_{4k-2}(\tau)$ and $P_{4k-2}(\tau)-P_1(\tau)$ as

$$\begin{cases}T_1(\tau)=(P_1(\tau)-P_{4k-2}(\tau))\boxplus R_0(\tau)\\ \qquad\triangleq |P_1(\tau)-P_{4k-2}(\tau)+R_0(\tau)|\\ \qquad\quad -|P_1(\tau)-P_{4k-2}(\tau)|\\ T_2(\tau)=(P_{4k-2}(\tau)-P_1(\tau))\boxplus R_0(\tau),\end{cases} \quad (6)$$

where $a\boxplus b\triangleq|a+b|-|a|$. For example, the zero-crossing points nearest to $\tau=0$ for $T_1(\tau)$ and $T_2(\tau)$ are $0.34T_s$ for $\mathrm{BOC_{sin}}(kn,n)$, which is obviously much narrower than the main-peak width T_s of the scheme of [12]. Then, we combine $T_1(\tau)$ and $T_2(\tau)$ as

$$R_1(\tau)=T_1(\tau)\oplus T_2(\tau) \quad (7)$$

yielding an unambiguous correlation function with a low and narrow main-peak; however, $R_1(\tau)$ does not contain sufficient signal energy required for reliable tracking since $R_1(\tau)$ is generated by using only four partial-correlations out of $4k$ partial-correlations , which causes a significant energy loss in the tracking process.

Thus, in the second stage we construct multiple unambiguous correlation functions by using additional combination of $\{P_l(\tau)\oplus R_1(\tau)\}_{l=1}^{4k-2}$, and then, increase the height of a low and narrow main-peak as

$$R_{\text{proposed}}(\tau) = R_1(\tau) + \sum_{l=1}^{4k-2} P_l(\tau) \oplus R_1(\tau) \tag{8}$$

yielding an unambiguous correlation function with a high and sharp main-peak as shown in Figure 3. The value of the main-peak is always two since its value can be calculated as $4/(4k)+(4k-2)\times 2/(4k)$, where $4/(4k)$ comes from $R_1(\tau)$ and $(4k-2)\times 2/(4k)$ comes from $\sum_{l=1}^{4k-2} P_l(\tau)\oplus R_1(\tau)$. It should be noted that the proposed two-stage method is also directly applicable to although the proposed method is described for $\text{BOC}_{\text{sin}}(kn,n)$ only.

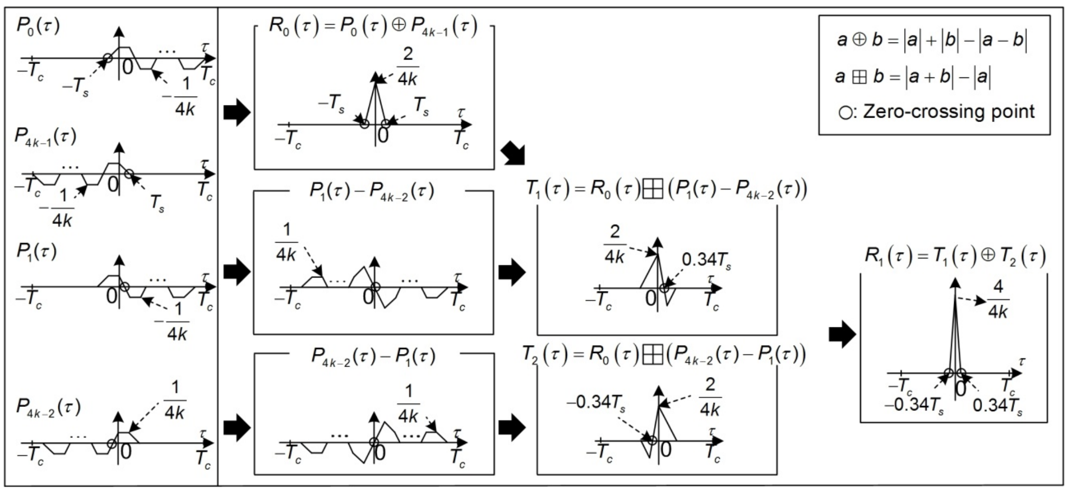


Figure 2. The first stage of the proposed scheme for $\text{BOC}_{\text{sin}}(kn,n)$.

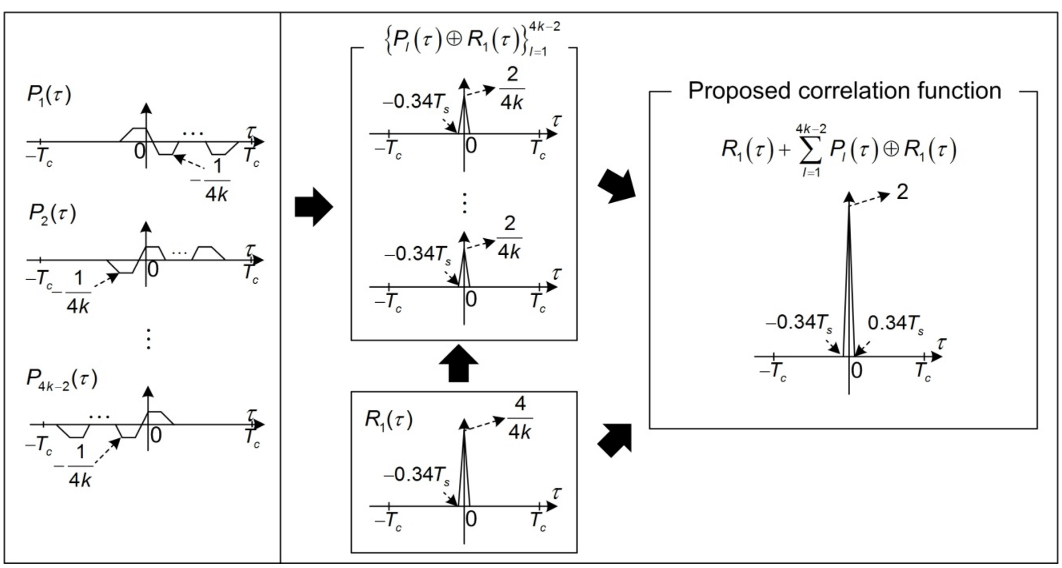
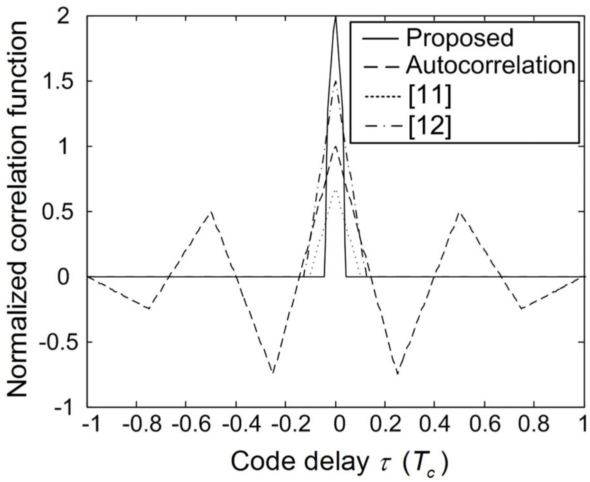
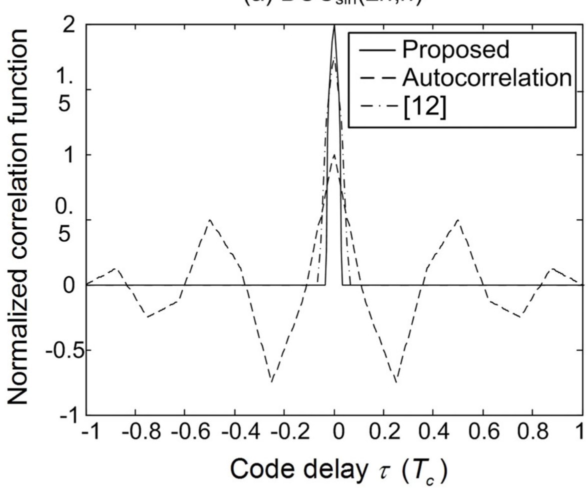


Figure 3. The second stage of the proposed scheme for $\text{BOC}_{\text{sin}}(kn,n)$.

Figure 4 shows the normalized correlation function of the proposed and conventional schemes for $\mathrm{BOC}_{\sin}(2n,n)$ and $\mathrm{BOC}_{\cos}(2n,n)$: Here, it should be note that the schemes of [10] and [11] are dedicated to $\mathrm{BOC}_{\sin}(n,n)$ and $\mathrm{BOC}_{\sin}(kn,n)$, respectively, and the tunable parameter κ is set to 0.3 which guarantees the best performance [11]. From the figure, we can see that the proposed correlation function has a main-peak sharper than those of the conventional correlation functions. Specifically, the normalized height of the main-peak of the proposed correlation function is two regardless of the value of k. The main-peak widths are $0.68T_s$ and $0.5T_s$ for $\mathrm{BOC}_{\sin}(kn,n)$ and $\mathrm{BOC}_{\cos}(kn,n)$, respectively. It is straightforward to show that the absolute slope of the main-peak of the proposed correlation function for $\mathrm{BOC}_{\sin}(kn,n)$ is $23.52k$, and is larger than the absolute slope $8k-4$ of the main-peak of the correlation function in [12]. Moreover, the proposed correlation function for $\mathrm{BOC}_{\cos}(kn,n)$ has an absolute slope $32k$ of the main-peak, which is larger than the absolute slope $16k-4$ of the main-peak of the correlation function of [12].



(a) $\mathrm{BOC}_{\sin}(2n,n)$



(b) $\mathrm{BOC}_{\cos}(2n,n)$

Figure 4. The normalized correlation functions of the proposed and conventional schemes for (a) $\mathrm{BOC}_{\sin}(2n,n)$ and (b) $\mathrm{BOC}_{\cos}(2n,n)$.

As the main-peak width of the proposed correlation function is narrower than those of the conventional correlation functions, the available code tracking range of the proposed correlation function is smaller than those of the conventional correlation functions in [10] and [11].; however, the absolute slope of the main-peak of the proposed correlation function is much larger than those of the conventional correlation functions. Thus, we can anticipate that the proposed correlation function will offer a performance improvement over the conventional correlation functions.

Figure 5 illustrates a delay lock loop (DLL) structure of the proposed noncoherent BOC signal tracking scheme, where τ represents the phase difference between the received and locally generated BOC signals and Δ is the early-late spacing. In the correlator, the received BOC signal $b(t)$ is first multiplied with the early version of the locally generated BOC signal $b\left(t+\tau+\frac{\Delta}{2}\right)$ (late version $b\left(t+\tau-\frac{\Delta}{2}\right)$), and then, correlated every T_s seconds, yielding partial-correlations of early version $\{P_l(\tau+\frac{\Delta}{2})\}_{l=0}^{4k-1}$ (late version $\{P_l(\tau-\frac{\Delta}{2})\}_{l=0}^{4k-1}$). Then, in the combiner, partial-correlations of early version $\{P_l(\tau+\frac{\Delta}{2})\}_{l=0}^{4k-1}$ (late version $\{P_l(\tau-\frac{\Delta}{2})\}_{l=0}^{4k-1}$) are combined as in Eq. 8 to produce the proposed correlation function of early version $R_{\text{proposed}}\left(\tau+\frac{\Delta}{2}\right)$ (late version $R_{\text{proposed}}\left(\tau-\frac{\Delta}{2}\right)$). Finally, in the tracking process, the discriminator output

$$D(\tau) = R^2_{\text{proposed}}\left(\tau+\frac{\Delta}{2}\right) - R^2_{\text{proposed}}\left(\tau-\frac{\Delta}{2}\right) \tag{9}$$

is applied to the loop filter to drive the numerically controlled oscillator (NCO), which advances or delays the clock of the local signal generator until τ becomes zero.

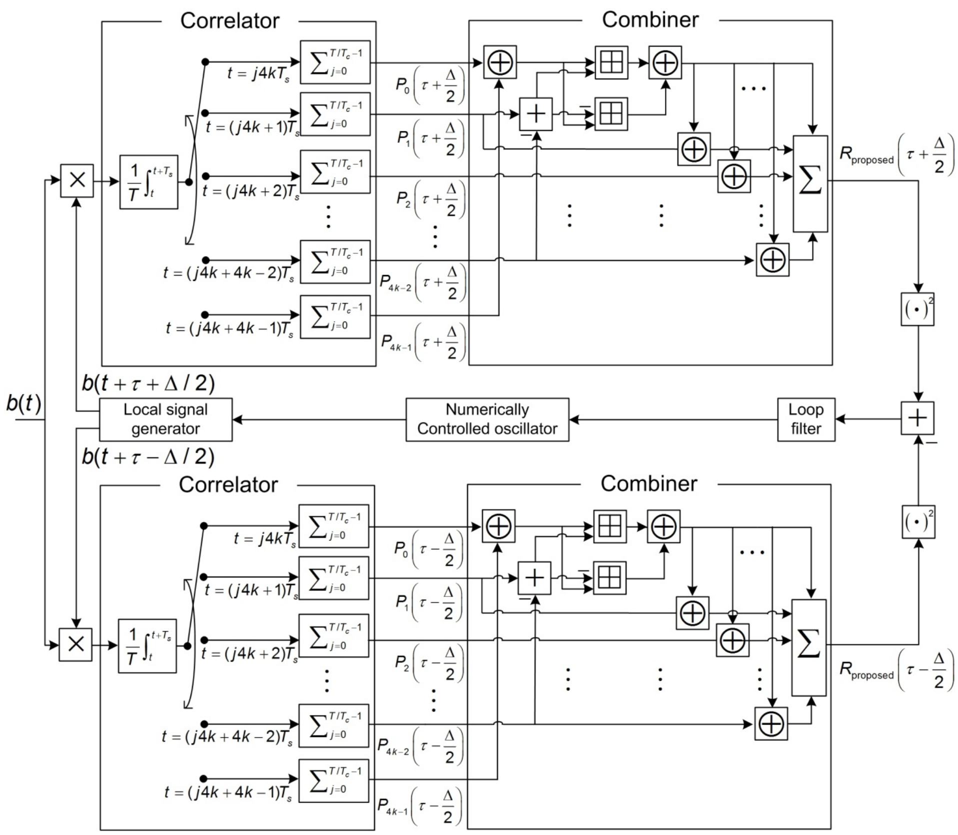
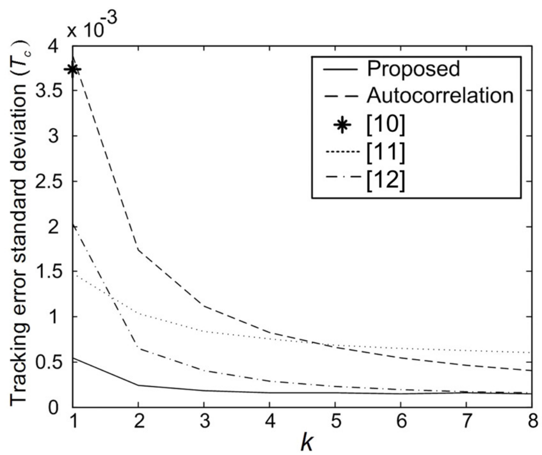


Figure 5. A delay lock loop structure of the noncoherent BOC signal tracking scheme.

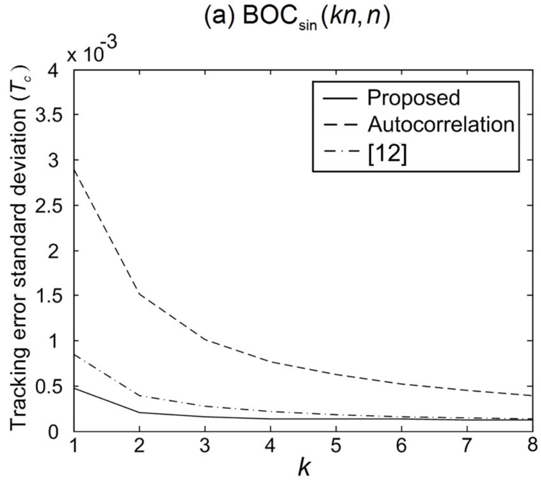
4. Numerical results

Tracking performances of the proposed and conventional schemes are compared in terms of TESD: Here, the TESD is defined as $\frac{\sigma}{G}\sqrt{2B_L T_I}$, where σ is the standard deviation of $D(0)$, B_L is the bandwidth of the loop filter, T_I is integration time, and $G = \frac{dD(\tau)}{d\tau}|_{\tau=0}$ is the discriminator gain [14]. We assume the following parameters for simulations: Galileo E1-B PRN code with period $T = 4$ ms, $T_I = T$, $\Delta = T_s / 4$, $B_L = 1$ Hz, $\kappa = 0.3$ for the scheme of [11], and $T_c^{-1} = 1.023$ MHz .

Figures 6 and 7 show the TESD of the tracking schemes with the proposed and conventional correlation functions as a function of k when the carrier-to-noise ratio (CNR) is 30 and 35 dB-Hz, respectively: Here, the CNR is defined as S / N_0 where N_0 is the noise power spectral density. From the figures, it is clearly observed that the proposed correlation function provides a better TESD than the conventional correlation functions. Although the TESD of the conventional correlation functions becomes smaller as the value of k increases, the proposed correlation function always provides the best performance due to its sharp main-peak.



(a) $\mathrm{BOC}_{\sin}(kn, n)$



(b) $\mathrm{BOC}_{\cos}(kn, n)$

Figure 6. Tracking error standard deviation of the proposed and conventional schemes as a function of k when CNR = 30 dB-Hz.

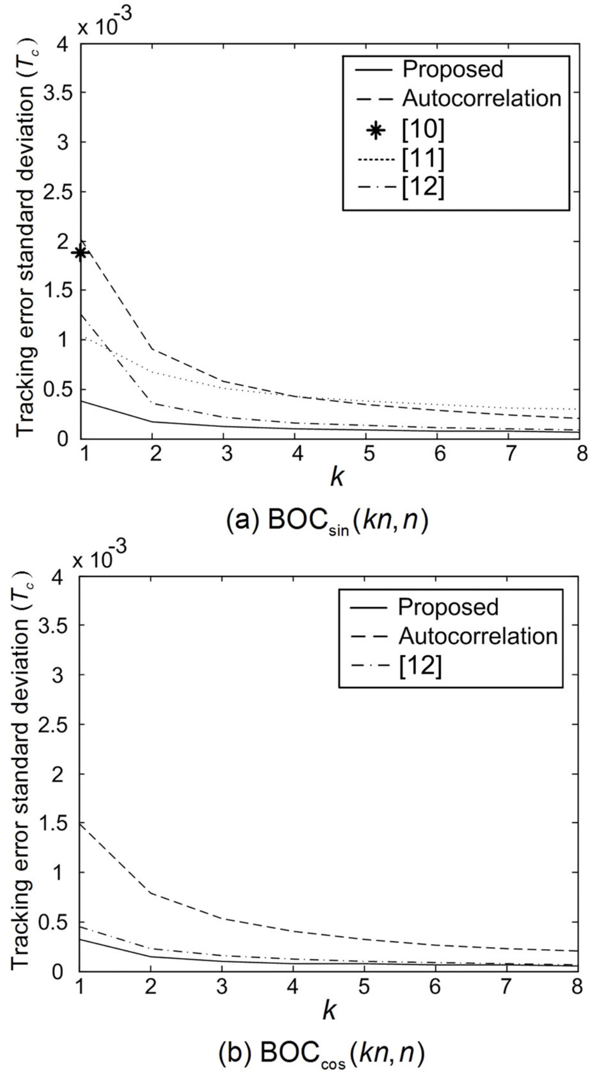
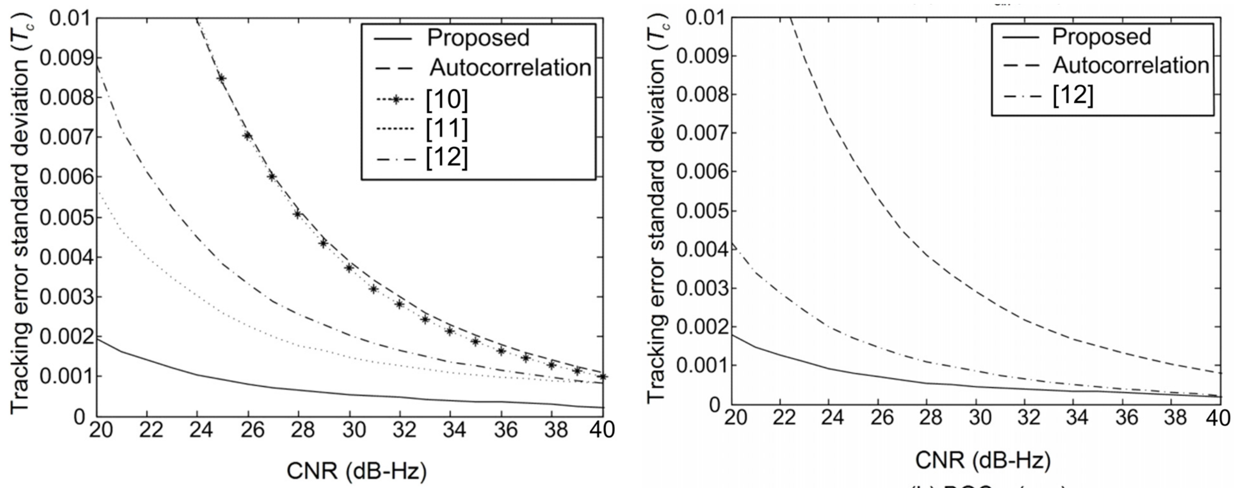


Figure 7. Tracking error standard deviation of the proposed and conventional schemes as a function of k when CNR = 30 dB-Hz.

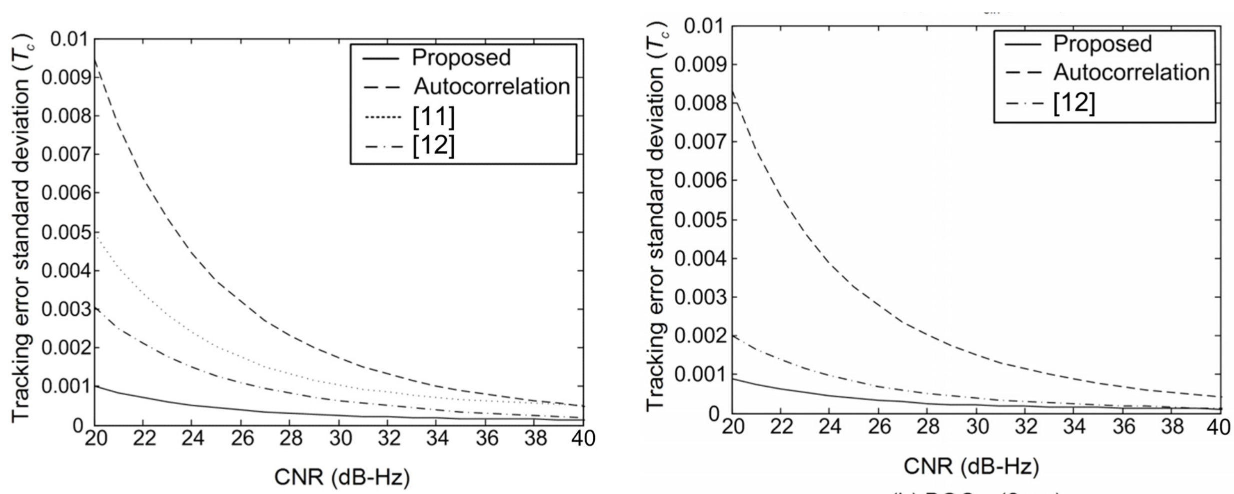
Figures 8-10 show the TESD of the tracking schemes with the proposed and conventional correlation functions as a function of CNR when $k = 1$, 2, and 4, respectively. From the figures, we can confirm that the proposed correlation function also provides a significant performance improvement over the conventional correlation functions in the CNR range of 20 ~ 40 dB-Hz of practical interest.

We would like to stress that the improved performance of the proposed scheme comes from the fact that the discriminator output of the proposed correlation function has a much higher slope (i.e., the discriminator gain) than that of the conventional one as shown in Figure 11. It should be also observed that the proposed scheme has the same tracking range, which is defined as the range of the linear part around $\tau = 0$, as that of the conventional one of [12].



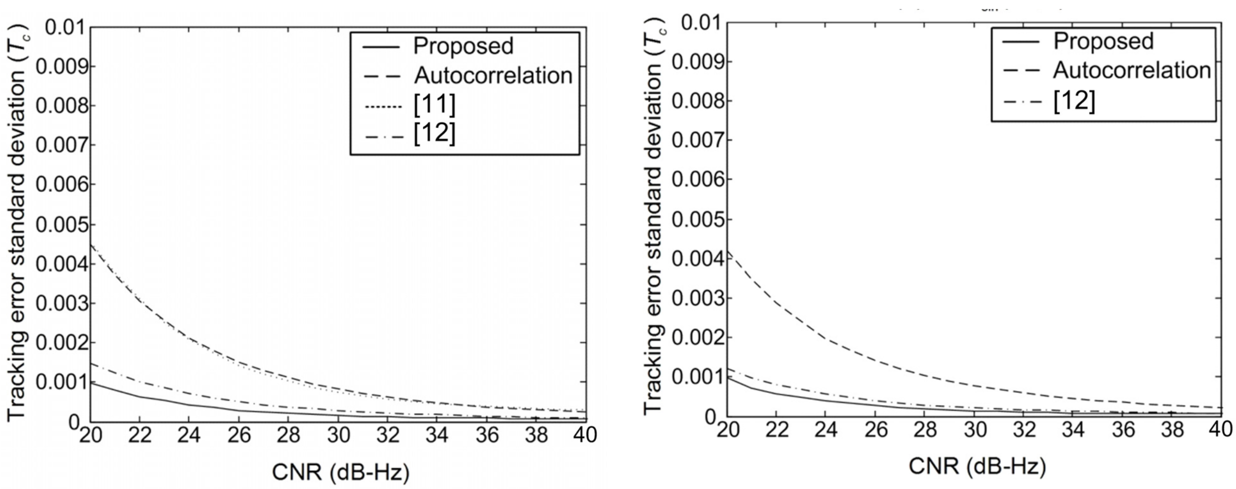
(a) $BOC_{sin}(n,n)$ (b) $BOC_{cos}(n,n)$

Figure 8. Tracking error standard deviation of the proposed and conventional schemes as a function of CNR when $k=1$.



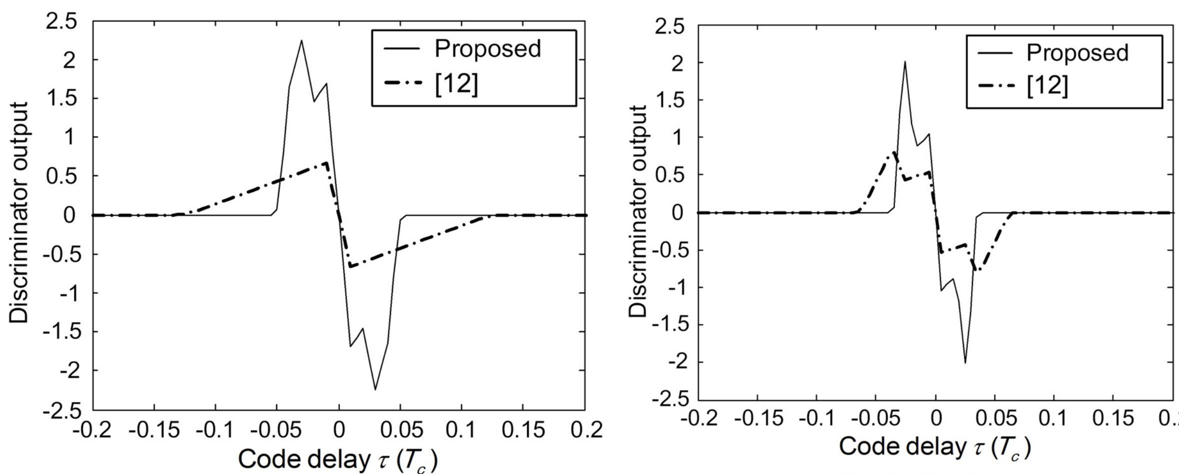
(a) $BOC_{sin}(2n,n)$ (b) $BOC_{cos}(2n,n)$

Figure 9. Tracking error standard deviation of the proposed and conventional schemes as a function of CNR when $k=2$.



(a) $\mathrm{BOC_{sin}}(4n,n)$ (b) $\mathrm{BOC_{cos}}(4n,n)$

Figure 10 . Tracking error standard deviation of the proposed and conventional schemes as a function of CNR when $k=4$.



(a) $\mathrm{BOC_{sin}}(2n,n)$ (b) $\mathrm{BOC_{cos}}(2n,n)$

Figure 11. Discriminator outputs of the proposed and conventional scheme of [12] for $\mathrm{BOC_{sin}}(2n,n)$ and $\mathrm{BOC_{cos}}(2n,n)$.

5. Conclusion

In this paper, we have proposed an unambiguous tracking scheme using partial-pulses by generating an unambiguous correlation function with a high and sharp main-peak. The proposed scheme not only removes the side-peaks completely but also provides an improved tracking performance compared with the conventional schemes. Specifically, we have generated 4k partial correlations using partial-pulses, and then, produced an unambiguous correlation function with a low and narrow main-peak based on the combination of four partial-correlations in first stage. In second stage, we have generated an unambiguous correlation function with a high and sharp main-peak using the correlation function obtained at the first stage and partial-correlations. The numerical results have demonstrated that the proposed scheme provides a better tracking accuracy than the conventional schemes in terms of the TESD.

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