

# Evaluation of Vertical Handoff Decision Algorithms Based on MADM Methods for Heterogeneous Wireless Networks

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## ABSTRACT

In the forthcoming heterogeneous wireless environment, the mobility management of users roaming between different wireless access technologies is a challenging and important technical issue. New mobile devices such as netbooks, smartphones and tablets allow users to perform vertical handoffs among different wireless networks. The multiple attribute decision making (MADM) methods are suitable tools to model and study the vertical handoff process. Hence, recently several MADM methods such as SAW, MEW, TOPSIS, GRA, ELECTRE, VIKOR and WMC have been proposed for vertical handoff. In this paper, we present an extensive performance evaluation and comparative study of the seven MADM methods by means of numerical simulations in MATLAB. We evaluate the performance of each vertical handoff method under different applications such as voice, data, and cost-constrained connections. We also perform a sensitivity analysis and evaluate the computational complexity of each method in terms of number of floating point operations.

Keywords: vertical handoff, multiple attribute decision making, heterogeneous networks

## RESUMEN

En los próximos ambientes inalámbricos heterogéneos, la gestión de movilidad de los usuarios alternando entre diferentes tecnologías de acceso inalámbrico es un tema técnico muy importante y desafiante. Los nuevos dispositivos móviles como son las computadoras portátiles, teléfonos inteligentes y tabletas permiten a los usuarios móviles realizar traspasos verticales entre diferentes redes inalámbricas. Los métodos de decisión por atributos múltiples (MADM) son una herramienta adecuada para modelar y estudiar el proceso del traspaso vertical. Por lo tanto, recientemente se han propuesto algunos métodos MADM como SAW, MEW, TOPSIS, GRA, ELECTRE, VIKOR y WMC para el traspaso vertical. En este artículo, se presenta una evaluación de desempeño exhaustiva así como un estudio comparativo de los siete métodos MADM por medio de simulaciones numéricas en MATLAB. Se evalúa el desempeño de cada método de traspaso vertical bajo diferentes tipos de conexiones como voz, datos y costo restringido. Se realiza también un análisis de sensibilidad y se evalúa la complejidad computacional de cada método en términos del número de operaciones de punto flotante.

## 1. INTRODUCTION

A heterogeneous wireless network is an envisioned wireless system also called beyond third generation (B3G) or fourth generation (4G) system which is expected to integrate multiple wireless access networks over a common IP platform [1]. Networks such as wireless local area networks (WLAN/Wi-Fi), 2G/3G cellular networks (GSM/UMTS), wireless metropolitan area networks (WMAN/WiMAX), etc., tend to be integrated in the coming years allowing the best connectivity to users anywhere at anytime. Furthermore, new mobile devices such as netbooks, smartphones,

tablets, etc., allow users to switch between different wireless access technologies. This important mobility process is known as *vertical handoff* [2], [3]. Over the last few years, plenty of research efforts have been focused on this challenging mobility process in heterogeneous wireless systems.

The field of multiple attribute decision making (MADM) [4] has proved to be a suitable mathematical tool to study and to model the vertical handoff process. Several MADM methods

have been proposed in the literature for vertical handoff, methods such as SAW (simple additive weighting) [5], TOPSIS (technique for order preference by similarity to ideal solution) [5], MEW (multiplicative exponent weighting) [6], GRA (grey relational analysis) [7], ELECTRE (elimination and choice translating priority) [8], WMC (weighted Markov chain) [9], and VIKOR [10]. A considerable amount of research work to study, develop, and modify MADM methods for vertical handoff has been conducted in recent years [11], [12], [13], and [14]. However, it is necessary to widely evaluate and compare its performance under different scenarios in order to provide the best solution for a particular application.

In this paper, we present an extensive comparative study of seven MADM methods proposed in the literature for vertical handoff [5]-[10]. By means of numerical simulations in MATLAB, we evaluate the performance of each method under different applications such as voice, data, and cost-constrained connections. We also perform a sensitivity analysis and evaluate the computational complexity of each MADM method in terms of number of floating point operations (flops).

The rest of the paper is organized as follows. In Section 2, we introduce the vertical handoff decision problem and why it can be formulated as a MADM problem. In Section 3, we present the steps and procedures of the seven MADM methods that are considered in our performance evaluation. In Section 4, we present the numerical results. Finally, Section 5 concludes the paper.

## 2. Background

### 2.1 Vertical Handoff

The *vertical handoff process* can be divided into three main steps [2], [3], namely: *system*

*discovery*, *handoff decision*, and *handoff execution*. During the system discovery step, the mobile devices equipped with multiple interfaces have to determine which wireless networks can be used and the communication services available in each network. These wireless networks may also advertise the supported data rates for different services as well as other relevant information. During the handoff decision step, the mobile device determines which network it should connect to. The decision may depend on various parameters including the available bandwidth, packet delay, packet jitter, access cost, transmit power, battery status, and even the user's preferences. Finally, during the handoff execution step, the connection needs to be re-routed from the existing network in use to the new network in a seamless manner. This step also includes the authentication and authorization of the user in the new network as well as the transfer of user's context information.

There are several parameters that can be considered for the vertical handoff decision [2], [3]. The quality of service (QoS) parameters of a particular connection such as bandwidth or delay bounds are usually specified by the applications. The level of security and amount of cost may be directly specified by the user itself. All these parameters are gathered by the mobile device during the system discovery step. Figure 1 shows several of those parameters categorized under different groups (e.g., bandwidth, delay, power, cost, security, reliability). Thus, the vertical handoff decision becomes a complex decision problem which requires considering multiple parameters.

### 2.2 MADM Methods

Multiple attribute decision making (MADM) methods consider problems where making

Quality of Service				Signal Quality		Reliability		Battery energy		Access cost		Security				
Bandwidth		Delay														
Available bandwidth	Total bandwidth	Minimum bandwidth	Maximum bandwidth	Received signal strength	Signal to noise ratio	SINR, ISI, ICI, etc...	Packet loss	Frame loss	Bit error rate	Power consumption rate	Remaining energy	Cost per byte	Cost per time	Authentication and authorization	Encryption type	Identity access management

Figure 1. Decision parameters for vertical handoff.

preference decisions over available alternatives that are characterized by multiple and usually conflicting attributes are required [4]. MADM is a branch of the field multiple criteria decision making (MCDM). MADM problems are diverse in disciplines, but all share the following common characteristics: alternatives to select, multiple attributes describing the alternatives in different units of measurement, and a set of weights representing the relative importance among attributes. For notation, let  $M$  be the set of alternatives and  $N$  be the set of parameters or attributes. A MADM problem can be concisely expressed in a matrix format, where columns indicate attributes and rows list alternatives. A typical element of matrix  $x_{ij}$  indicates the performance rating of the  $i$ th alternative with respect to the  $j$ th attribute. Thus, a MADM problem with  $|M|$  alternatives and each with  $|N|$  parameters is given by

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{i|N|} & \dots & x_{i|N|} \\ x_{21} & x_{22} & \dots & x_{ij} & \dots & x_{2|N|} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_{|M|1} & x_{|M|2} & \dots & x_{|M|j} & \dots & x_{|M||N|} \end{bmatrix} \quad (1)$$

When the vertical handoff decision or network selection problem is formulated as an MADM problem, the alternatives to select are the candidate wireless networks and the attributes are the parameters describing the network which several authors call handoff metrics.

The MADM methods use scoring techniques to rank alternatives. An index or score is calculated by taking into account the contributions from each parameter. Before the calculation of the index, normalization of the parameters is required to deal with different units (e.g., bps, sec, price, etc.)

Additionally, a set of importance weights have to be defined for the calculation of the ranking. The value of such weights should represent the different levels of importance of a parameter for the decision maker. The set of importance weights has to satisfy the constraint:

$$\sum_{j \in N} w_j = 1 \quad (2)$$

In the case of the vertical handoff decision, the importance weights should represent the QoS requirements of the connection as well as the user's preferences. For example, in a voice connection, the importance weights of packet delay and jitter must have higher value than the bandwidth of the network. It is also important to note the difference among a *benefit* parameter and a *cost* parameter. For a benefit parameter, the larger the better, while for a cost parameter the smaller the better.

### 3. MADM Methods Applied for Vertical Handoff

In general, all the following MADM decision methods first calculate an index or score based on their specific procedures, then, the network selected for vertical handoff is the one with the best score value or the one in the first place in the ranking of candidate networks.

#### 3.1 SAW

As mentioned before, the MADM decision problem can be expressed in an  $M \times N$  decision matrix, where the  $j$ -th attribute of the  $i$ -th network is represented as  $x_{ij}$ . Simple additive weighting (SAW) is one of the best known and most widely used scoring methods because of its simplicity [4]. but it has also been considered in other recent work such as [6], [15], [16] and [17]. For vertical handoff decisions, the parameters usually have different measuring units, thus the values of the parameters require to be normalized first.

In SAW, the score of each candidate network  $i$  is obtained by adding the contributions from each attribute  $r_{ij}$  multiplied by the importance weight  $w_j$

Then, the selected network  $A_{SAW}^*$  is:

$$A_{SAW}^* = \arg \max_{i \in M} \sum_{j \in N} w_j r_{ij}, \quad (3)$$

where  $r_{ij} = x_{ij} / x_j^+$  for benefit parameters, and  $r_{ij} = x_j^- / x_{ij}$  for cost parameters, moreover,

$$x_j^+ = \max_{i \in M} x_{ij} \quad \text{and} \quad x_j^- = \min_{i \in M} x_{ij}, \quad \text{and the}$$

$$\sum_{j=1}^N w_j = I$$

importance weight vector must satisfy .

### 3.2 MEW

Multiplicative exponent weighting (MEW) is another MADM scoring method and is very similar to SAW. The main difference is that, in this method, instead of addition, there is multiplication [4]. It was initially proposed for vertical handoff in [6]. In MEW, the scores of the networks are determined by the weighted product of the attributes. The score  $S_i$  of network  $i$  is determined by the weighted product of the following attributes:

$$S_i = \prod_{j \in N} x_{ij}^{w_j}, \quad (4)$$

where  $x_{ij}$  denotes attribute  $j$  of candidate network  $i$ ,  $w_j$  denotes the weight of attributed  $j$ , and  $\sum_{j=1}^N w_j = I$ . Note that in (4),  $w_j$  is a positive

power for benefit metrics  $x_{ij}^{w_j}$ , and a negative power for cost metrics  $x_{ij}^{-w_j}$ . Since parameter normalization is not required (i.e., it is optional), the score of a network obtained by MEW does not have an upper bound [4], thus it is convenient to compare the score of each network with the score of the positive ideal network  $A^{**}$ .

This network is defined as the network with the best values in each metric. For a benefit metric, the best value is the largest. For a cost metric, the best value is the lowest. The value ratio  $R_i$  between network  $i$  and the positive ideal is calculated by

$$R_i = \frac{\prod_{j \in N} x_{ij}^{w_j}}{\prod_{j \in N} (x_{ij}^{**})^{w_j}}, \quad (5)$$

where  $0 \leq R_i \leq I$ . The selected network  $A_{MEW}^*$  is

$$A_{MEW}^* = \arg \max_{i \in M} R_i. \quad (6)$$

### 3.3 TOPSIS

In the algorithm based on the technique for order preference by similarity to ideal solution (TOPSIS) with  $|M|$  alternatives that are evaluated by  $|N|$ , the decision criteria is viewed as a geometric system with  $|M|$  points in the  $|N|$  dimensional space [4]. Here, the chosen candidate network is the one which has the shortest distance to the ideal solution and the longest distance to the worst case solution. The ideal solution is a "hypothetical solution" with the best values in each parameter while the negative ideal solution is the opposite. It was initially proposed for vertical handoff decision in [5] but it has also been considered in other recent work such as [6], [10], and [15]. In general, to compute the final ranking list, TOPSIS requires the following steps:

**Step 1:** Construct the normalized decision matrix, which allows comparison across the attributes, this matrix is given by

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i \in M} x_{ij}^2}}. \quad (7)$$

**Step 2:** Construct the weighted normalized decision matrix as  $v_{ij} = w_j * r_{ij}$ .

**Step 3:** Determine ideal and negative-ideal solutions by

$$A^+ = \{(\max_{i \in M} v_{ij} \mid j \in J), (\min_{i \in M} v_{ij} \mid j \in J')\}, \quad (8)$$

and

$$A^- = \{(\min_{i \in M} v_{ij} \mid j \in J), (\max_{i \in M} v_{ij} \mid j \in J')\}, \quad (9)$$

where  $J$  is the set of benefit parameters, and  $J'$  is the set of cost parameters.

**Step 4:** Calculate the separation measure between the networks and the positive and negative ideal networks by

$$s_i^+ = \sqrt{\sum_{j \in N} (v_{ij} - v_j^+)^2}, \quad s_i^- = \sqrt{\sum_{j \in N} (v_{ij} - v_j^-)^2}. \quad (10)$$

**Step 5:** Calculate the relative closeness to the ideal solution.

$$c_i^* = \frac{s_i^-}{(s_i^+ + s_i^-)}. \quad (11)$$

A set of alternatives can now be preference-ranked according to the descending order of  $c_i^*$ . Then the selected network  $A_{TOP}^*$  is

$$A_{TOP}^* = \arg \max_{i \in M} c_i^*. \quad (12)$$

### 3.4 ELECTRE

Elimination and choice translating priority (ELECTRE) is a MADM method which performs pair-wise comparisons among the alternatives using each of the attributes separately to establish outranking relationships between the alternatives [4]. It was initially proposed for vertical handoff decision in [8]. In general, for ELECTRE method, a reference attribute vector is used to adjust the raw attribute values for the alternative networks before they are compared. The value of each of the attributes in the decision matrix is compared with a corresponding reference attribute value  $x_j^{ref}$ . An absolute difference between the two values is taken to calculate a new matrix as follows:

$$r_{ij} = |x_{ij} - x_j^{ref}|. \quad (13)$$

Now, in this matrix all the attribute values can be considered to have a monotonically decreasing utility. Since a lower value for an adjusted attribute is considered an indication of a better network in the selection process, each attribute can be normalized as follows:

$$\hat{r}_{ij} = \frac{\max_{i \in M} \{r_{ij}\} - r_{ij}}{\max_{i \in M} \{r_{ij}\} - \min_{i \in M} \{r_{ij}\}}. \quad (14)$$

Now, it is necessary to take into consideration the relative importance of each of the attributes involved in the decision about network selection.

For the  $j$ -th attribute is assigned a weight  $w_j$ ,

such that  $\sum_{j=1}^N w_j = 1$ . Using the weights, an updated matrix is calculated by

$$\tilde{r}_{ij} = w_j \hat{r}_{ij}. \quad (15)$$

In order to compare the network alternatives, the concept of concordance and discordance has been introduced in ELECTRE, which are measures of satisfaction and dissatisfaction of the decision maker when one alternative is compared with another. It firstly uses pair-wise comparisons of networks to obtain the concordance set  $CSet(k, l)$  indicating the attribute of network  $k$  is better than network  $l$  and the discordance set  $DSet(k, l)$  indicating the attribute of network  $k$  is worse than network  $l$ . The concordance and discordance sets are formed as follows:

$$\begin{aligned} CSet_{kl} &= \{j \mid \tilde{r}_{kj} \geq \tilde{r}_{lj}\} \\ DSet_{kl} &= \{j \mid \tilde{r}_{kj} < \tilde{r}_{lj}\} \end{aligned} \quad (16)$$

Using the concordance and discordance sets, corresponding matrices are constructed. The elements of concordance matrix  $C$  can be represented as

$$C_{kl} = \sum_{j \in CSet_{kl}} w_j. \quad (17)$$

The entries for the concordance matrix are not defined for the diagonal. ELECTRE defines the elements of the discordance matrix as follows:

$$d_{kl} = \frac{\sum_{j \in DSet_{kl}} |\tilde{r}_{kj} - \tilde{r}_{lj}|}{\sum_{j \in N} |\tilde{r}_{kj} - \tilde{r}_{lj}|}. \quad (18)$$

Similarly, the entries for the discordance matrix are also not defined for the diagonal. A new parameter  $\tilde{C}_i$ , called the net concordance index is calculated.  $\tilde{C}_i$  is a measure of dominance of an alternative  $i$  over other alternatives. It can be calculated as follows:

$$\tilde{C}_i = \sum_{j \in N, j \neq i} C_{ij} - \sum_{j \in N, j \neq i} C_{ji}. \quad (19)$$

Similarly, the term net discordance index  $D_i$  is defined as a measure of relative weakness of alternative  $i$  over other alternatives and can be calculated as

$$\tilde{D}_i = \sum_{j \in N, j \neq i} D_{ij} - \sum_{j \in N, j \neq i} D_{ji}. \quad (20)$$

An alternative with the highest value of net concordance index  $\tilde{C}$  and the lowest value of net discordance index  $\tilde{D}$  would be preferred. However, if it is not the case, the alternatives are ranked based on the concordance and discordance indexes where each alternative is ranked by taking the average of these two rankings. The alternative with the highest average ranking is considered to be the best alternative. Alternatives with the same average ranking would be considered equally suited.

### 3.5 VIKOR

MADM method VIKOR is a compromise ranking method for MADM problems presented in [18]. It works for decision making on the idea of an aggregating function representing closeness to the ideal solution. For VIKOR, a compromise solution is a feasible solution that is the closest to the ideal solution. It was initially proposed for vertical handoff decision in [15] but it has also been studied later in [10]. For VIKOR method the following steps are required:

**Step 1:** For each parameter  $j = 1, 2, 3, \dots, N$ , determine the best and worst values given by

$$F_j^+ = \{(\max_{i \in M} x_{ij} | j \in N_b), (\min_{i \in M} x_{ij} | j \in N_c)\}, \quad (21)$$

and

$$F_j^- = \{(\min_{i \in M} x_{ij} | j \in N_b), (\max_{i \in M} x_{ij} | j \in N_c)\}, \quad (22)$$

where  $N_b \subset N$  is the set of benefit parameters, and  $N_c \subset N$  is the set of cost parameters.

**Step 2:** Compute the values of  $S_i$  and  $R_i$  for  $i = 1, 2, 3, \dots, M$  given by

$$S_i = \sum_{j \in N} w_j \frac{(F_j^+ - x_{ij})}{(F_j^+ - F_j^-)}, \quad (23)$$

and

$$R_i = \max_{j \in N} \left[ w_j \frac{(F_j^+ - x_{ij})}{(F_j^+ - F_j^-)} \right], \quad (24)$$

where  $w_j$  is the importance weight of parameter  $j$ .

**Step 3:** Compute the values of  $Q_i$  for  $i = 1, 2, 3, \dots, M$  given by

$$Q_i = \gamma \left( \frac{S_i - S^+}{S^- - S^+} \right) + (1 - \gamma) \left( \frac{R_i - R^+}{R^- - R^+} \right), \quad (25)$$

where

$$\begin{aligned} S^+ &= \min_{i \in M} S_i, & S^- &= \max_{i \in M} S_i, \\ R^+ &= \min_{i \in M} R_i, & R^- &= \max_{i \in M} R_i, \end{aligned} \quad (26)$$

and parameter  $\gamma$  with  $0 \leq \gamma \leq 1$  is the weight of the strategy. It also represents the majority of criteria. Here, we set  $\gamma = 0.5$ .

**Step 4:** Given the values for  $Q$ , for all  $i \in M$ , rank the candidate networks in an increasing order. The selected network  $A_{VIK}^*$  is

$$A_{VIK}^* = \arg \min_{i \in M} Q_i^*. \quad (27)$$

### 3.6 GRA

In the algorithm based on the Grey Relational Analysis (GRA), the grey relational coefficient

(GRC) is used as the coefficient to describe the similarity between each candidate network and the best reference network (i.e., an ideal network formed by choosing the best value of each attribute). It was initially proposed for vertical handoff decision in [7] but it has also been studied later in [6]. In this method, the normalization of the sequence data is performed according to the two situations (larger-the-better and smaller-the-better) as follows:

$$r_{ij} = \frac{x_{ij} - \min_{i \in M} x_{ij}}{\max_{i \in M} x_{ij} - \min_{i \in M} x_{ij}}, \quad (28)$$

and

$$r_{ij} = \frac{\max_{i \in M} x_{ij} - x_{ij}}{\max_{i \in M} x_{ij} - \min_{i \in M} x_{ij}}, \quad (29)$$

The next step is to find the ideal reference sequence  $x_0$  to contain the upper bound or lower bound, respectively, in larger-the-better or smaller-the-better situations, and then calculate the correlated coefficient:

$$GRC_i = \frac{I}{N} \sum_{j \in N} \frac{\Delta_{\min} + \Delta_{\max}}{\Delta_i + \Delta_{\max}}, \quad (30)$$

where  $\Delta_i = |x_{0j} - r_{ij}|$ , is the grey correlated distance, and it is:

$$\Delta_{\max} = \max_{i \in M, j \in N} \Delta_i, \quad \Delta_{\min} = \min_{i \in M, j \in N} \Delta_i, \quad (31)$$

The larger the GRC, the more preferable the network will be. The selected network  $A_{GRA}^*$  is

$$A_{GRA}^* = \arg \max_{i \in M} GRC_i. \quad (32)$$

### 3.7 WMC

The algorithm based on the weighted Markov chain (**WMC**) is a rank aggregation method initially developed for web searching [9]. The WMC method begins by constructing a Markov chain transition matrix based on ranking lists. Later, it is updated based on experience and importance weights. Stationary probability is then derived and used to sort candidates. WMC was initially proposed for vertical handoff decision in [9] but it has also been considered later in [10]. It includes the following steps:

**Step 1:** Construction of weighted Markov chain transition matrix  $MC$ . Initialize an  $M \times M$  matrix  $MC = \{mc_{ij}\}$  with all element values equal to 0, in which  $mc_{ij}$  represents the transition probability from alternative  $p_i$  to the network  $p_j$ .

**Step 2:** For each decision factor  $q$ , a ranking list is obtained as

$$\tau_q = [p_1 \geq p_2 \geq \dots \geq p_M], \quad (33)$$

where " $\geq$ " represents some ordering relation, and  $\tau_q(p)$  denotes the ranking of the alternative  $p$  with regard to factor  $q$ .

**Step 3:** For each  $mc_{ij}$  in  $MC$ , update

$$mc_{ij} = mc_{ij} + \frac{w_q}{\tau_q(p_i)}, \quad \text{if } \tau_q(p_i) \geq \tau_q(p_j). \quad (34)$$

**Step 4:** Computation of stationary probabilities:

$$\pi_j = \sum_{i \in M} \pi_i mc_{ij}, \quad \sum_{j \in N} \pi_j = 1. \quad (35)$$

The selected network  $A_{WMC}^*$  is

$$A_{WMC}^* = \arg \max_{j \in M} (\pi_j). \quad (36)$$

## 4. Numerical Results

In this section, we present a performance comparison and evaluation of the MADM decision methods for vertical handoff described in Section 3.

### 4.1 Scenario and Simulation Set Up

We consider a scenario of an heterogeneous wireless environment integrated by three different types of wireless networks such as [19]: WLAN, UMTS and WiMAX. We also consider that there are two networks of each type for a total of six candidate networks. The networks are labeled as follows: networks 1 and 2 correspond to networks UMTS1 and UMTS2 respectively, networks 3 and 4 correspond to networks WLAN1 and WLAN 2 respectively, and networks 5 and 6 correspond to

networks WiMAX1 and WiMAX2 respectively. Six decision parameters from the ones shown in Figure 1 are considered in this study. The decision parameters are the following: available bandwidth in Mbps, total bandwidth in Mbps, packet delay in ms, packet jitter in ms, packet loss per each  $10^6$  packets, and cost per byte. The ranges of the values of the parameters in each network are shown in Table 1. The values of each network are selected from typical operational and standardized ranges, and assuming that the 3G cellular UMTS networks are of type 3GPP Release 4 [20], the WLAN1 is of type IEEE 802.11b [21], the WLAN2 is of type IEEE 802.11g [22], and the WiMAX networks are of type IEEE 802.16e [23]. Finally, the ranges are also selected to satisfy QoS requirements as defined in the 3GPP technical specification for services and services capabilities [24].

The values of the importance weights in (1) for different services or traffic classes considered in this study are the following: case 1, all parameters have the same importance weight, this is the baseline case; case 2, the packet delay and packet jitter have 70% of importance and the rest is equally distributed among the other parameters, this case is suitable for voice connections; case 3, the available bandwidth and total bandwidth have 70% of importance, this case is suitable for data connections; and finally case 4, cost per byte (price) has 50% of importance, this case is suitable for cost-constrained connections.

In each vertical handoff decision point (i.e., the point of the specific handoff decision step which is before the system discovery step and after the handover execution step), the attribute values may be the same, increase or decrease within the range shown in Table 1. In order to vary the values of the decision criteria, a discrete event simulation is conducted where a discrete-time Markov chain is used to model the evolution on time of each

decision parameter [25]. The discrete-event simulation is implemented in MATLAB [26]. The transition probabilities for an increment or decrement are  $p_c$ , while the probability of being in the same value is  $p_s$ . The transition probabilities for an increment or decrement are  $p_c=0.4$ , while the probability of being in the same value is  $p_s=0.2$ . For each application, we consider 50 vertical handoff points in the simulation. Each vertical handoff decision point is the average of 10 simulations with different and random initial values.

#### 4.2 Results Case 1 vs. Case 2 (For Voice Connections)

Figure 2 shows the packet delay in ms achieved by the seven vertical handoff algorithms. The first 50 vertical handoff decision points (i.e., points 1 to 50 in the x-axis and left-hand side of Figure 2) correspond to case 1 and the final 50 points (i.e., points 51 to 100 and right-hand side of Figure 2) corresponds to case 2. Note that in the first part of the simulation, with the exception of GRA and MEW, the rest of the decision algorithms tend to select the same networks and hence achieving very similar performance in terms of a packet delay of around 80 ms. This imply that internally, GRA and MEW based on their own specific procedures select, score and evaluate the networks differently even with a uniform assignment of weights in (1). On the other hand, in the second part of the simulation, when the assignment of weights in (1) is according to case 2 (i.e., aimed for voice connections), the performance of the decision algorithms differs. First note that VIKOR, SAW and TOPSIS in all their decisions always achieve a packet delay lower than 68 ms but, on average, the lowest value of packet delay is achieved by VIKOR. Second, note that MEW, ELECTRE and WMC are able to achieve values of packet delay bounded between 68 to 78 ms. Finally, note that the values of packet delay for GRA are the highest (around 125 ms) and remain very close to the ones

Parameter	UMTS1	UMTS2	WLAN1	WLAN2	WiMAX1	WiMAX2
Available Bandwidth (Mbps)	0.1-2	0.1-2	1-11	1-54	1-60	1-60
Total Bandwidth (Mbps)	2	2	11	54	60	60
Packet Delay (ms)	25-50	25-50	100-150	60-150	60-100	60-100
Packet Jitter (ms)	5-10	5-10	10-20	10-20	3-10	3-10
Packet Loss (per $10^6$ )	20-80	20-80	20-80	20-80	20-80	20-80
Cost per Byte (price)	0.6	0.8	0.1	0.05	0.5	0.4

Table 1. Range of values of the wireless network parameters.



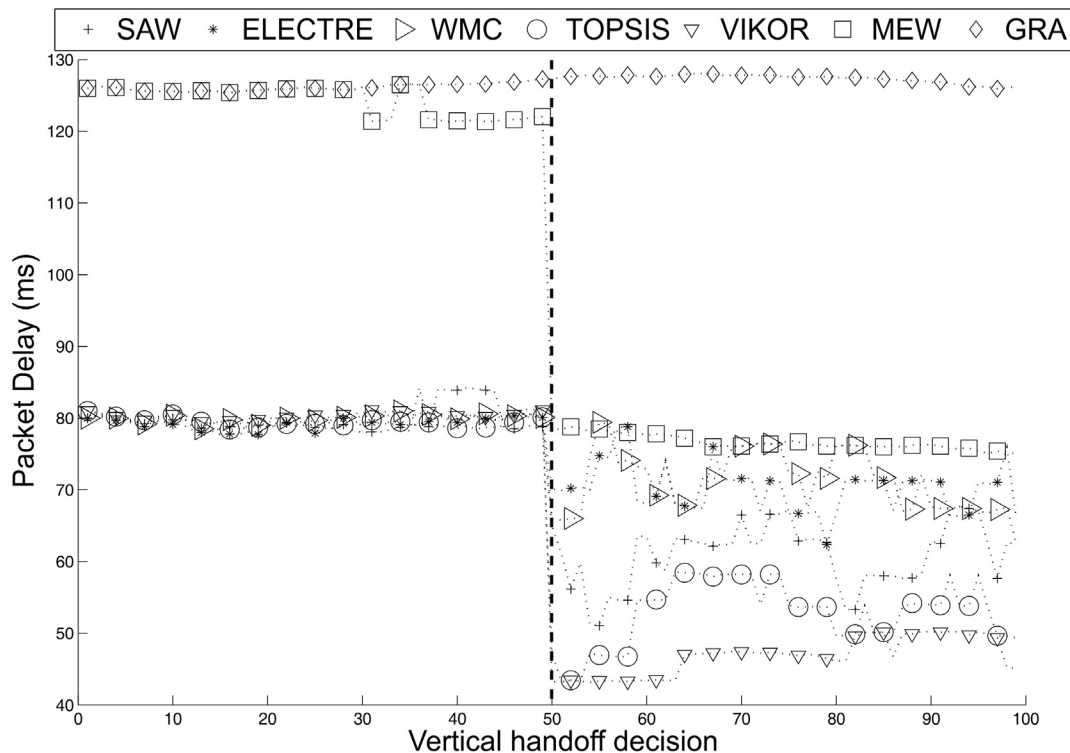


Figure 2. Packet delay achieved in case 1 (baseline) vs. case 2 (voice).

in case 1. Thus, for voice connections, the best option for the vertical handoff decision algorithm is VIKOR since it is able to achieve, on average, values of packet delay -16% lower than TOPSIS, -25% lower than SAW, -37% lower than WMC, -39% lower than ELECTRE, -42% lower than MEW, and -64% lower than GRA.

#### 4.3 Results Case 1 Vs Case 3 (For Data Connections)

Figure 3 shows the available bandwidth in Mbps achieved by the seven vertical handoff algorithms. The first 50 vertical handoff decision points (i.e., points 1 to 50 in the x-axis and left-hand side of Figure 3) correspond to case 1, and the final 50 points (i.e., points 51 to 100 and right-hand side of Figure 3) correspond to case 3. Note again that in the first part of the simulation, with the exception of GRA and MEW, the rest of the decision algorithms

tend to often select the same networks and hence achieve a close performance in terms of an available bandwidth between 29 to 32 Mbps. On the other hand, in the second part of the simulation, when the assignment of weights in (2) is according to case 3 (i.e., aimed for data connections), the performance of the decision algorithms is improved specially for GRA and MEW. First, note that the values of the available bandwidth for WMC are the lowest (on average around 29.4 Mbps) and remain very close to the ones in case 1. Second, note that the rest of the decision algorithms increase their values of available bandwidth with respect to case 1 remaining bounded in this case between 31 and up to 33.5 Mbps. Finally, note that MEW is the decision algorithm with the slightly highest values of available bandwidth but certainly the one with the best improvement of +26% with respect to case 1.

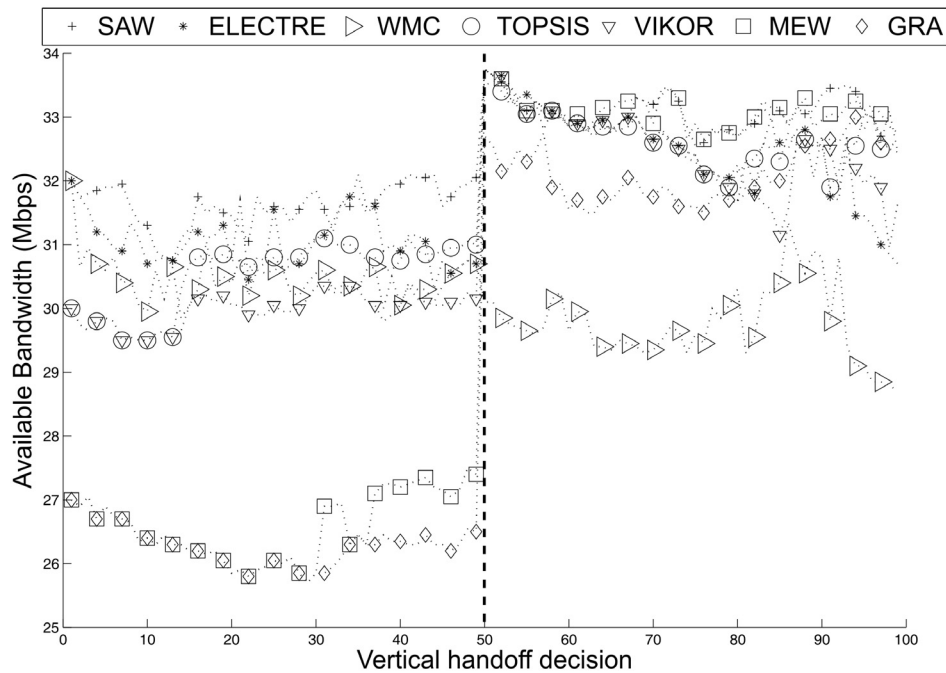


Figure 3. Available bandwidth achieved in case 1 (baseline) vs. case 3 (data).

#### 4.4 Results Case 1 vs. Case 4 (For Cost-Constrained Connections)

Figure 4 shows the cost per byte achieved by the seven vertical handoff algorithms. The first 50 vertical handoff decision points (i.e., points 1 to 50 in the x-axis and left-hand side of Figure 4) correspond to case 1 and the final 50 points (i.e., points 51 to 100 and right-hand side of Figure 4) correspond to case 4. As expected, note again that in the first part of the simulation, with the exception of GRA and MEW, the rest of the decision algorithms tend to often select the same networks and hence achieve a close performance in terms of cost per byte between 0.4 to 0.5. In the second part of the simulation, when the assignment of weights in (1) is according to case 4 (i.e., aimed for cost-constrained connections), the performance of the decision algorithms, with the exception of GRA, is improved. First, note that the values of cost per byte for GRA are the highest (on average around

0.6) in case 4 and also quite higher compared to the ones of GRA in case 1. Second, note that the rest of the decision algorithms reduce their values of cost per byte with respect to case 1 remaining fixed in 0.05. Finally, note that ELECTRE is the only decision algorithm with slightly higher values but certainly with a reduction of -66% with respect to case 1.

#### 4.5 Sensitivity Analysis

Besides the performance results shown in the previous sub-sections, it is also relevant to perform a sensitivity analysis on the importance weights. Table 2 shows the specific numerical example considered for the sensitivity analysis. The values in Table 2 are selected from a snapshot from all the possible simulation scenarios defined by Table 1. Figure 5 shows the selected network for vertical handoff for each of the seven algorithms when the weight of the available bandwidth is

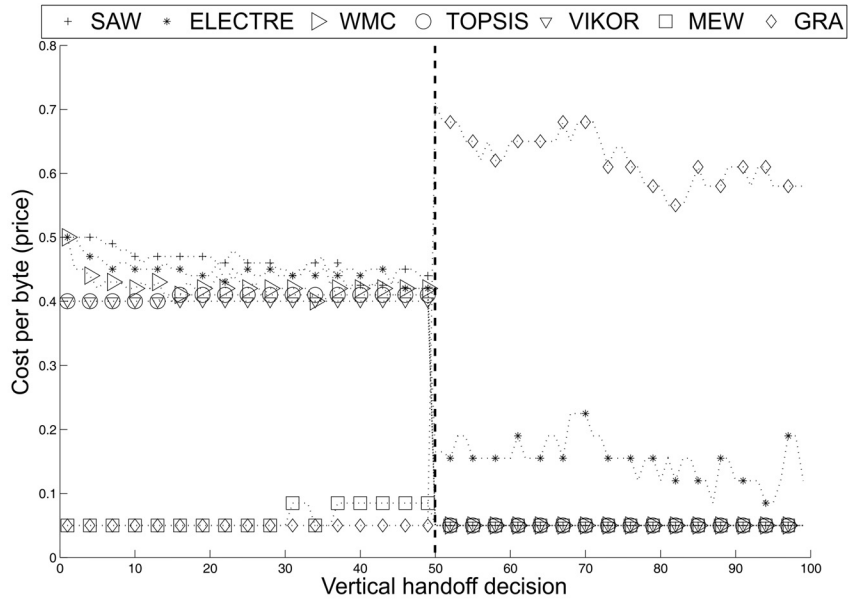


Figure 4. Cost achieved in case 1 (baseline) vs. case 4 (cost-constrained).

varied from 0 to 1. Note that after the value of 0.5, all the algorithms converge to network 5. Note in Table 2 that network 5 corresponds to network WiMAX1 which is the one with the largest value of available bandwidth. It is important to note in Figure 5 that GRA and MEW have a different initial network selection for vertical handoff (i.e., network 4) and also that they are the decision methods that are more sensible to the change on the values assigned to the weights in (1). This feature explain why GRA and MEW usually have, in several cases, a different performance compared to the rest of the decision algorithms, as shown in the simulation results in Figures 2, 3 and 4.

#### 4.6 Results for Computational Complexity

Finally, in order to complete the performance comparison among the seven vertical handoff algorithms, it is also important to show the computational complexity of each algorithm in terms of the number of floating point operations (flops). To measure the number of flops, the Performance Application Programming Interface (PAPI) [27] MATLAB extension is installed on a computer with a 1Ghz Pentium III processor and 512 MB in RAM. PAPI [28] is a set of tools that define a standardized interface and methodology to measure computational performance. The PAPI

Parameter	UMTS1	UMTS2	WLAN1	WLAN2	WiMAX1	WiMAX2
Available Bandwidth (Mbps)	1	1.2	6	27	32	30
Total Bandwidth (Mbps)	2	2	11	54	60	60
Packet Delay (ms)	37	38	125	126	80	82
Packet Jitter (ms)	7	8	15	16	6	8
Packet Loss (per $10^6$ )	50	51	50	49	48	45
Cost per byte (price)	0.6	0.8	0.1	0.05	0.5	0.4

Table 2. Values for sensitivity analysis.

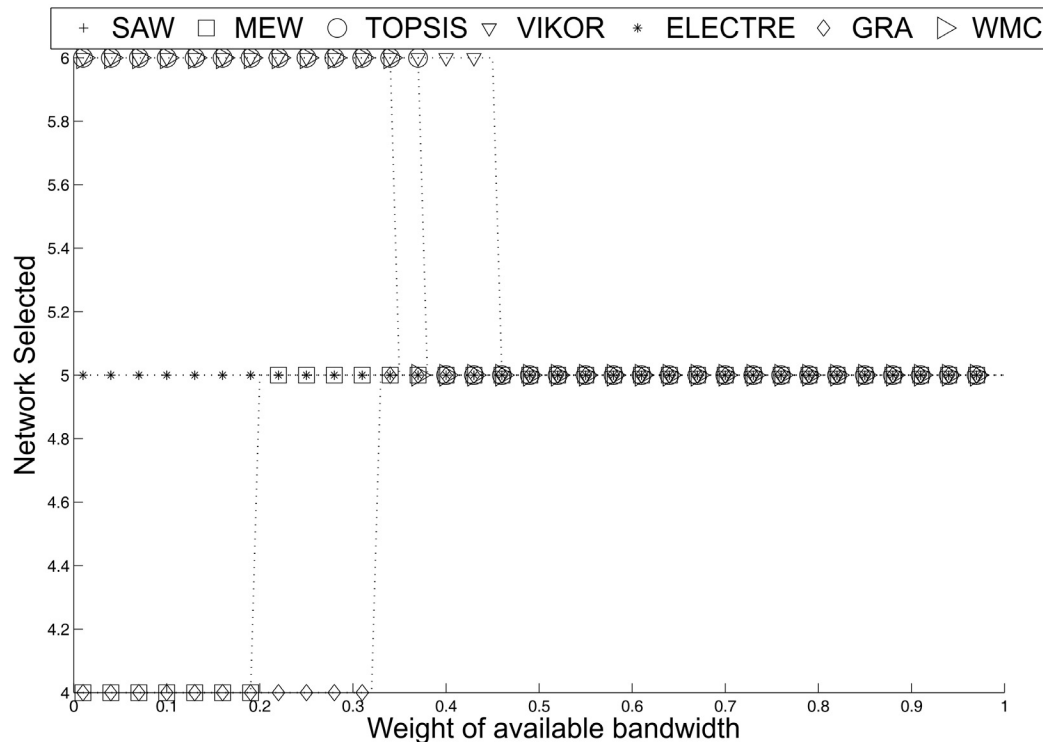


Figure 5. Sensitivity analysis for available bandwidth in case 3 (data).

extension incorporates the flops function to a custom MATLAB installation. Then, to test the algorithms, 100 vertical handoff decision points are simulated for case 1. Figure 6 shows the number of flops achieved by the seven vertical handoff algorithms for decision matrixes from size 3X3 to 6X6 (i.e., three to six, respectively, networks and decision parameters). Note that SAW and GRA are the algorithms requiring fewer operations. MEW, TOPSIS and VIKOR are around the same number of operations. Finally, note that WMC and ELECTRE are the algorithms requiring the larger number of floating point operations.

#### 4.7 Summary of Results

After evaluating the performance of the seven vertical handoff decision algorithms under different applications (i.e., voice, data, and cost-constrained) as well as measuring the computational complexity in terms of the number of floating point (flops) operations, we can summarize our performance evaluation as follows:

1) In case of voice connections, the *best* option for the vertical handoff decision algorithm is VIKOR. TOPSIS and SAW provide a *satisfactory* level of performance and the rest of the decision algorithms provide a *low* level of performance in terms of packet delay.

2) In case of data connections, the best option for the vertical handoff decision algorithm is MEW. SAW, TOPSIS, VIKOR and ELECTRE provide a *satisfactory* level of performance and the rest of the decision algorithms provide a *low* level of performance in terms of available bandwidth.

3) In case of cost-constrained connections, the *best* options for the vertical handoff algorithm are SAW, MEW, TOPSIS, VIKOR and WMC. ELECTRE provides a *satisfactory* level of performance and GRA provides a *low* level of performance in terms of cost per byte.

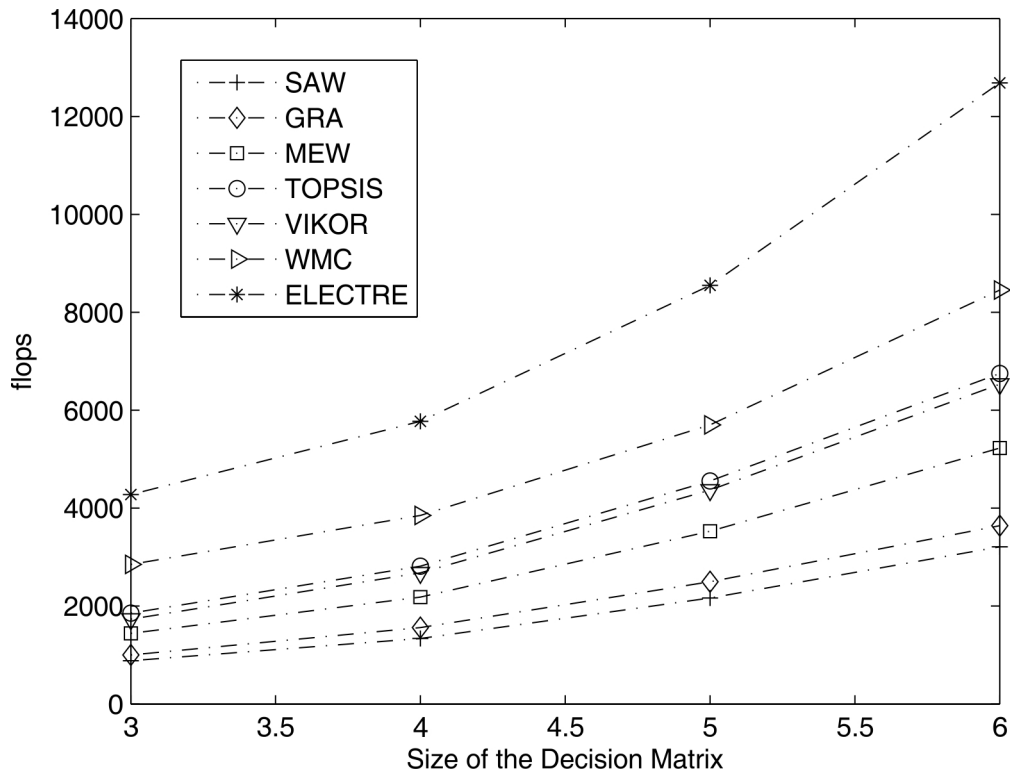


Figure 6. Number of floating point (flops) operations vs. size of decision matrix.

4) Based on the sensitivity analysis, GRA and MEW are decision methods that are very sensible to the assignment of weights in (1); hence, it impacts on their overall performance.

5) Based on the computational complexity evaluation, SAW and GRA are the *best* options for the vertical handoff decision algorithm. MEW, TOPSIS and VIKOR provide a *satisfactory* level of performance and WMC and ELECTRE provide a *low* level of performance in terms of floating point operations (flops). Please refer to Table 3.

## 5. Conclusions

In this paper, we presented an extensive performance evaluation and a comparative study of seven MADM methods proposed in the literature for vertical handoff named SAW, MEW, TOPSIS, GRA, ELECTRE, VIKOR and WMC. By means of numerical simulations in MATLAB, we evaluated the performance of each decision method under different applications such as voice, data, and cost-constrained connections. Hence, we found that VIKOR and MEW are the best decision

Vertical handoff algorithm	Voice	Data	Cost	Flops
SAW [5]	Satisfactory	Satisfactory	Best	Best
MEW [6]	Low	Best	Best	Satisfactory
TOPSIS [5]	Satisfactory	Satisfactory	Best	Satisfactory
GRA [7]	Low	Low	Low	Best
VIKOR [10]	Best	Satisfactory	Best	Satisfactory
ELECTRE [8]	Low	Satisfactory	Satisfactory	Low
WMC [9]	Low	Low	Best	Low

Table 3. Summary of results.

methods for vertical handoff when voice connections and data connections are considered, respectively. We also performed a sensitivity analysis and evaluated the computational complexity of each MADM method in terms of number of floating point operations. The results also indicate that SAW and GRA are the best decision methods for vertical handoff due to their low computational complexity. We expect that the results and findings presented in this paper can be helpful to mobile devices manufacturers for planning and electronic design as well as to application programmers aiming to explore the benefits of a heterogeneous wireless environment.

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