

# Tracking Control for an Underactuated Two-Dimensional Overhead Crane

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## ABSTRACT

In this paper, the tracking control problem is considered for the payload transportation with an underactuated two-dimensional overhead crane. Two sliding mode controllers are designed to perform the trajectory tracking. One is proposed to control hoisting and lowering the suspended payload, and the other one is proposed to control both trolley positioning and payload swaying. Considering the second sliding mode controller is used to control two degrees of freedom (DOFs), a fuzzy inference algorithm is proposed to dynamically adjust the coupling factor between the two DOFs. The two controllers make the payload track a predefined trajectory and be safely transported as fast and accurately as possible with a small swing angle, and then place the payload at the desired position. Simulations are performed with the proposed controllers and the results show their effectiveness.

Keywords: Sliding mode, fuzzy inference, trajectory tracking, underactuated system, crane.

## 1. Introduction

Overhead cranes are widely used in workshops and harbors to transport all kinds of massive goods. It is desired for the overhead crane to transport payloads to the required position as fast and as accurately as possible without collision with equipment while placing payloads at an appropriate position. In addition to these requirements, the payload swing angle should be kept as small as possible and it is desired that the payload can track a predefined trajectory.

Much work has been done in controlling the overhead crane. Garrido [1] adopted the input shaping control method; however, input shaping must be precalculated accurately according to the system model and the approach lacks robustness to external disturbances. Lee [2] used fuzzy logic only for anti-swing control and position servo control for position and swing damping. Michael [3] adopted fuzzy logic to control both positioning and swing damping, but it was difficult to set both the fuzzy rules and the parameters of the controller only according to experiences. Moreover, most of the above methods do not take payload hoisting and lowering into account.

Anti-sway trajectory tracking of overhead cranes has been studied in the following literatures. To

improve the performance of swing suppression, Lee [4] proposed a nonlinear switching control law for 2-D overhead cranes with the feedback linearization method and the Lyapunov stability theorem. Lee [5] proposed an anti-swing trajectory tracking control law with a coupling control by defining a coupled variable of the trolley traveling motion and the load sway motion. However, the drawback of that work lies in that they have considered less practical issues such as robustness against parameter uncertainty, unmodeled dynamics, and actuator nonlinearity [6]. Sliding mode control (SMC) is a robust design methodology using a systematic scheme based on a sliding mode surface and the Lyapunov stability theorem [7]. The main advantage of SMC is that the system uncertainties and external disturbances can be handled under the invariance characteristics of the system's sliding mode state with guaranteed system stability [7]. The SMC was used by Er [8] and Mansour [9] for positioning control and hoisting control, but another state feedback control scheme was needed to control the payload swing. A reference model was defined by Hasanul [10] to track through a linearized system model. All those methods have difficulties in automatically tuning the relationship between position control and sway control. Park [6] proposed adaptive fuzzy sliding-

mode anti-sway tracking control and Yang [11] proposed adaptive tracking control for overhead crane, but the payload hoisting and lowering were not considered.

This paper presents a solution to control the overhead crane, where the payload swing, crane motion, and payload hoisting (and lowering) are considered altogether. It is well known that SMC is capable of tackling non-linear systems with parameter uncertainties and external disturbances hence the external disturbances and system parameters varying are not taken into consideration in modeling. This paper is organized as follows: Section 2 gives the dynamical model of the overhead crane. In Section 3, two sliding mode controllers are designed for the overhead crane and the fuzzy tuning algorithm is designed to adjust the control of two DOFs. In Section 4, simulations are performed and the results are given. In Section 5, the conclusions are presented.

## 2. Dynamical model of the two-dimensional overhead crane

Figure 1 shows the overhead crane system, where  $m_1$ ,  $m_2$ ,  $x$ ,  $l$ ,  $\theta$ ,  $f_1$  and  $f_2$ , respectively, are the trolley mass, payload mass, trolley position, rope length, swing angle, driving force of the trolley and driving force of hoisting (lowering) along the rope. There are three DOFs (i.e.,  $x$ ,  $l$  and  $\theta$ ) to be controlled but only two control actions (i.e.,  $f_1$  and  $f_2$ ) are available thus the overhead crane is an under-actuated system.

According to the Lagrangian equation:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = T_i \quad (i=1,2,3) \quad (1)$$

where  $L = K - U$  is the Lagrangian function,  $K$  is the system's kinetic energy,  $U$  is the system's potential energy,  $q_i$  is the generalized coordinate (here is  $x$ ,  $l$  or  $\theta$ ), and  $T_i$  is the external force (here  $f_1$  or  $f_2$ ). Without considering friction, the motion equation

of the overhead crane system can be obtained:

$$\begin{aligned} (m_1 + m_2)\ddot{x} + 2m_2\dot{l}\dot{\theta}\cos\theta + m_2l\ddot{\theta}\cos\theta - m_2l\dot{\theta}^2\sin\theta + m_2\ddot{l}\sin\theta &= f_1 \\ m_2\ddot{l} + m_2\ddot{x}\sin\theta - m_2\dot{\theta}^2l - m_2g\cos\theta &= f_2 \\ m_2l^2\ddot{\theta} + m_2\dot{x}\dot{\theta}\cos\theta + 2m_2l\dot{\theta} + m_2gl\sin\theta &= 0 \end{aligned} \quad (2)$$

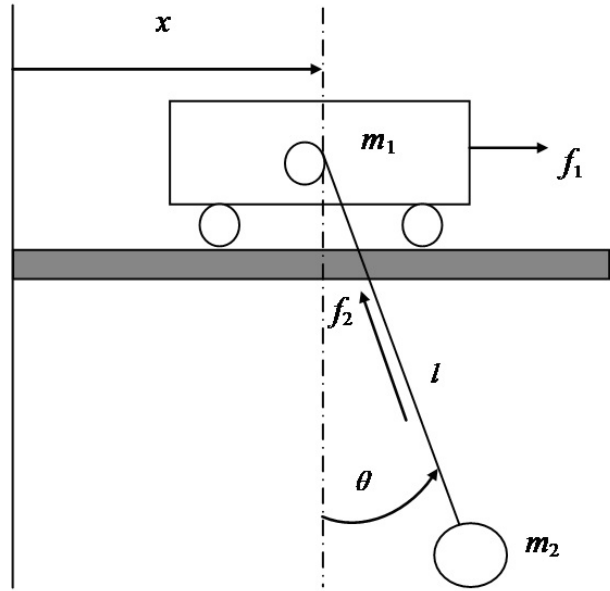


Figure 1. Two-dimensional overhead crane.

From the system motion equation (2), the following equation can be obtained:

$$\begin{aligned} \ddot{x} &= (f_1 - f_2 \sin\theta) / m_1 \\ \ddot{l} &= f_2 / m_2 + \dot{\theta}^2 l + g \cos\theta - (f_1 - f_2 \sin\theta) \sin\theta / m_1 \\ \ddot{\theta} &= -((f_1 - f_2 \sin\theta) \cos\theta / m_1 + 2\dot{l}\dot{\theta} + g \sin\theta) / l \end{aligned} \quad (3)$$

Assume the trolley position is  $x_1 = x$ , the rope length is  $x_2 = l$ , and the swing angle is  $x_3 = \theta$ ; moreover,  $x_4 = \dot{x}$ ,  $x_5 = \dot{l}$ ,  $x_6 = \dot{\theta}$ . The state equation of the overhead crane system can be rewritten as

$$\begin{aligned}
 \dot{x}_1 &= x_4 \\
 \dot{x}_2 &= x_5 \\
 \dot{x}_3 &= x_6 \\
 \dot{x}_4 &= (f_1 - f_2 \sin x_3) / m_1 \\
 \dot{x}_5 &= f_2 / m_2 - (f_1 - f_2 \sin x_3) \sin x_3 / m_1 + x_6^2 x_2 + g \cos x_3 \\
 \dot{x}_6 &= -((f_1 - f_2 \sin x_3) \cos x_3 / m_1 + 2x_5 x_6 + g \sin x_3) / x_2
 \end{aligned} \tag{4}$$

### 3. Design of the tracking controller for the crane

The control objective of the overhead crane is to transport the payload along a predefined trajectory from the initial position to the final position. In the control process, the payload should be transported as fast as possible and the payload swaying should be as small as possible. The control problems are complex because the overhead crane is under-actuated.

The tracking error vector is defined as

$$e_i = x_i - x_i^d \quad (i = 1, 2, \dots, 6) \tag{5}$$

where superscript 'd' indicates the corresponding desired value. To apply the SMC strategy, three time-varying sliding mode functions are defined for each DOF as follows:

$$s_1 = e_4 + \lambda_1 e_1 \tag{6}$$

$$s_2 = e_5 + \lambda_2 e_2 \tag{7}$$

$$s_3 = e_6 + \lambda_3 e_3 \tag{8}$$

where  $\lambda_i$  ( $i=1,2,3$ ) are suitable positive constants.

The predefined trajectory is planned according to the transportation distance and the obstacle standing in the transporting environment. Without considering the swing angle, the predefined trajectory can be expressed as

$$x_2^d = f(x_1) \tag{9}$$

During the transportation process, the payload should be transported along the predefined trajectory; moreover, the payload is often massive. Therefore, the swing angle should not be controlled by system input  $f_2$  in order to prevent situations such as lowering in the hoisting process and hoisting in the lowering process, but it can be controlled by system input  $f_1$ . Consequently,  $f_1$  controls two DOFs, namely, the crane position and the payload swing angle. Another sliding mode function  $s_4$  is defined by

$$s_4 = s_1 + ks_3 \tag{10}$$

where  $k$  is set as a real negative variable to be designed below and it can be regarded as a coupling factor between sliding mode functions  $s_1$  and  $s_3$ . The coupling factor can not only connect sliding mode surfaces  $s_1 = 0$  and  $s_3 = 0$  but also adjust their importance in the system control; therefore, the coupling factor will play an important role in the interactive control between sliding mode surfaces  $s_1 = 0$  and  $s_3 = 0$ .

In SMC, if the following discontinuous dynamics are imposed on the corresponding  $s_i$  ( $i=2,4$ ):

$$\dot{s}_2 = -\eta_2 \operatorname{sgn}(s_2) \tag{11}$$

$$\dot{s}_4 = -\eta_4 \operatorname{sgn}(s_4) \tag{12}$$

the sliding mode conditions ( $s_i \dot{s}_i = -\eta_i |s_i|$ ) are satisfied, in which  $\eta_2$  and  $\eta_4$  are suitable positive constants. That is to say, system states will achieve the corresponding sliding mode surface  $s_i = 0$  in a finite amount of time. Where  $\operatorname{sgn}()$  is a sign function:

$$\operatorname{sgn}(s) = \begin{cases} 1 & \text{for } s > 0 \\ 0 & \text{for } s = 0 \\ -1 & \text{for } s < 0 \end{cases}$$

The satisfied sliding mode conditions and stable sliding mode surfaces can ensure the stability of

the overhead crane control system. As is well known, using the sign function in SMC often causes system chattering in practice that is not expected. One solution is to introduce a boundary layer around the sliding mode surface [7]; therefore, saturation functions are adopted to replace the sign function in (11) and (12):

$$\dot{s}_2 = -\eta_2 \text{sat}(s_2 / \Phi_2) \quad (13)$$

$$\dot{s}_4 = -\eta_4 \text{sat}(s_4 / \Phi_4) \quad (14)$$

where positive constant factors  $\Phi_i$  ( $i=2,4$ ) are the width of the boundary layers. Saturation functions  $\text{sat}(s_i / \Phi_i)$  are defined as

$$\text{sat}(s_i / \Phi_i) = \begin{cases} s_i / \Phi_i & \text{for } |s_i / \Phi_i| \leq 1 \\ \text{sgn}(s_i / \Phi_i) & \text{for } |s_i / \Phi_i| > 1 \end{cases}$$

From (6), (7), (8) and (10), we have

$$\dot{x}_5 - \dot{x}_5^d + \lambda_2(\dot{x}_2 - \dot{x}_2^d) + \eta_2 \text{sat}(s_2 / \Phi_2) = 0 \quad (15)$$

$$\dot{x}_4 - \dot{x}_4^d + \lambda_1(\dot{x}_1 - \dot{x}_1^d) + k(\dot{x}_6 - \dot{x}_6^d) + k\lambda_3(\dot{x}_3 - \dot{x}_3^d) + \eta_4 \text{sat}(s_4 / \Phi_4) = 0 \quad (16)$$

Generally, the desired swing angle is zero thus we have  $\dot{x}_6^d = 0$  and  $\dot{x}_3^d = 0$ . From (4), (15) and (16), we can obtain the following control law:

$$f_1 = m_2 \sin x_3 (b + a \sin x_3 - x_6^2 x_2 - g \cos x_3) + m_1 a \quad (17)$$

$$f_2 = m_2 (b + a \sin x_3 - x_6^2 x_2 - g \cos x_3) \quad (18)$$

where

$$a = (k(2x_5x_6 + g \sin x_3 - \lambda_3x_2x_6) - \lambda_1x_2x_4 + x_2\dot{x}_4^d + \lambda_1x_2x_4^d - \eta_4x_2 \text{sat}(s_4 / \Phi_4)) / (x_2 - k \cos x_3) \quad (19)$$

$$b = \dot{x}_5^d - \lambda_2(x_5 - \dot{x}_2^d) + \eta_2 \text{sat}(s_2 / \Phi_2) \quad (20)$$

Considering that the coupling factor plays an important role in the coordinated control between the crane position and the payload swing,  $k$  should be appropriately chosen to achieve satisfactory decoupling performance for different system states. Here, an adaptive fuzzy tuning method is proposed to generate the coupling factor. As is well known, if the swing angle is large, the payload swaying control should be reinforced in order to damp the payload swing, and if the swing angle is small, the crane position control should be strengthened. A base value  $k^b$  is given to ensure a basic performance, a dynamic variable  $\Delta k$  is dynamically tuned in real time, and a breadth  $K$  is used to limit the range of the coupling factor. Then the dynamic coupling factor is defined by

$$k = k^b + K\Delta k \quad (21)$$

Therefore, the coupling factor can be automatically adjusted within  $[k^b, k^b + K]$  through tuning the dynamic variable according to the system states.

The fuzzy rule sets can be expressed as

$$R_i: \text{IF } |x_3| \text{ IS } F_i \text{ THEN } \Delta k \text{ IS } \Delta K_i$$

where  $R_i$  is the  $i$ th rule of  $n$  rules,  $F_i$  and  $\Delta K_i$  are the corresponding membership functions of  $|x_3|$  and  $\Delta k$  in the  $i$ th rule. The simplified inference method is used to derive the inference result as

$$\Delta k = \left( \sum_{i=1}^n \mu_{F_i}(|x_3|) \times \Delta K_i \right) / \sum_{i=1}^n \mu_{F_i}(|x_3|) \quad (22)$$

where  $\mu_{F_i}(|x_3|)$  is the firing degree of the  $i$ th rule. By the way, the motor speed is usually limited in practice, and the trolley's rated speed  $v_e$  transformed from the motor's rated speed is the optimal transportation speed. For long-distance transportation, in order to prevent the acceleration

from becoming too great during the acceleration process and to make the trolley transport with  $v_e$  after the acceleration process, position error  $e_1$  can be limited to

$$e_1 = \eta_1 \text{sat}((x_1 - x_1^d) / \eta_1) \quad (23)$$

where coefficient  $\eta_1$  can be calculated by

$$\eta_1 = v_e / \lambda_1 \quad (24)$$

Using (23) and (24) in (6), we get

$$s_1 = e_4 + v_e \text{sat}(e_1 / \eta_1)$$

When  $x_4^d = 0$ , transport velocity  $x_4$  will approach trolley's rated speed  $v_e$  and keep it until the deceleration process takes place because sliding mode function  $s_1$  approaches zero under the control action of driving force  $f_1$  in (17). The deceleration process begins when position error  $e_1$  become less than  $\eta_1$ .

**Remark 1:** The stability of the closed-loop system consists of the accessibility of the sliding mode surface and the stability on the sliding mode surface. The first one is apparently guaranteed through the design process of the sliding mode controller, and the second one is decided by coupling factor  $k$  in Equation (10) and the definitions of the swing angle and driving force  $f_1$ . Assume a positive angle and positive  $f_1$  directions are defined as Figure 1 shows. For sliding mode function  $s_1 > 0$ , a negative driving force is required such that sliding function  $s_1$  approaches sliding mode surface  $s_1 = 0$ ; however, for sliding mode function  $s_3 > 0$ , a positive driving force is required such that sliding function  $s_3$  approaches sliding mode surface  $s_3 = 0$ . That is to say, the roles of the two sliding mode functions in control are inconsistent. Therefore, coupling factor  $k$

should be negative for the stability on the sliding mode surface.

**Remark 2:** In practice, rope length  $x_2$  is always larger than zero. Swing angle  $x_3$  is within  $(-\pi/2, \pi/2)$  to ensure safe operation; moreover, variable  $k$  is negative. Hence, the term  $(x_2 - k \cos x_3)$  in Equation (19) is greatly larger than zero. Therefore, there are no singularities of the controllers of Equations (17) and (18).

**Remark 3:** With the proposed control scheme, the overhead crane can be stabilized to a fixed position or a trajectory. The saturation function in Equation (23) does not affect the accessibility of the sliding mode surface and the stability on the sliding mode surface. Therefore, the saturation function in Equation (23) will not affect the stability of the closed-loop system.

#### 4. Control Simulations

Simulations are performed with the system given by (4) and the controllers given by Equation (17) and (18) with (6), (7), (8), (10), (19), (20), (21) and (22). In the simulations, the parameters of the overhead crane system are chosen as follows: trolley mass  $m_1 = 20\text{kg}$ , payload mass  $m_2 = 10\text{kg}$ , gravitational acceleration  $g = 9.8\text{m/s}^2$ . The hoisting and lowering motion will track the reference trajectory, along which the suspended payload will not collide with other objects and the energy consumed in transportation is the lowest. Assuming that the obstacles in the transporting environment look like those in Figure 2, a parabolic reference trajectory is calculated offline for transporting.

Hence, Equation (9) can be expressed as

$$x_2^d = \begin{cases} l_0 + \rho(P_1 - x_1)^2 & \text{for } x_1 \leq P_1 \\ l_0 & \text{for } P_1 < x_1 < P_2 \\ l_0 + \rho(x_1 - P_2)^2 & \text{for } x_1 \geq P_2 \end{cases} \quad (25)$$

In simulations, the above parameters are chosen:  $l_0 = 5$ ,  $\rho = 0.05$ ,  $P_1 = 10$ ,  $P_2 = 30$ . The reference trajectory is also shown in Figure 2.

System performance is sensitive to the parameters of the sliding surface [12, 13]. For the sliding surface slope, if a large value is available, the system will be more stable but the tracking accuracy may be degraded because of a longer reaching time of the representative point to the surface. Conversely, if a small value is chosen, the convergence speed on the sliding surface itself will be slow, leading to longer tracking times. For the sliding mode switch control value, a large value will lead to a large chattering and a small value will lead to a long reaching time. Moreover, the switch value is limited by the system's maximum output. The introduction of a boundary layer reduces chattering at the cost of deteriorated the tracking precision, i.e., a large value will lead to a small chattering but a low tracking precision. According to the above principles, the parameters of the sliding mode surfaces are chosen as  $\lambda_1 = 0.25$ ,  $\lambda_2 = 0.8$ ,  $\lambda_3 = 8$ ,  $\eta_1 = 8$ ,  $\eta_2 = 10$ ,  $\eta_4 = 10$ ,

$\Phi_1 = 0.5$ ,  $\Phi_2 = 0.5$ ; the parameters of coupling factor are  $k^b = -1$ ,  $K = -0.4$ ; the trolley's rated speed is  $v_e = 2\text{m/s}$ ; and the maximal acceleration (or deceleration) of the trolley is  $1\text{m/s}^2$ .

The self-tuning fuzzy inference controller takes the absolute value of the swing angle as antecedent variable whose fuzzy subsets are constructed as small (S), medium (M) and big (B) and are shown in Figure 3.

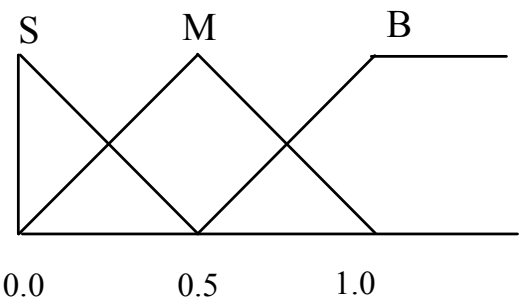


Figure 3. Membership functions of dynamic variable.

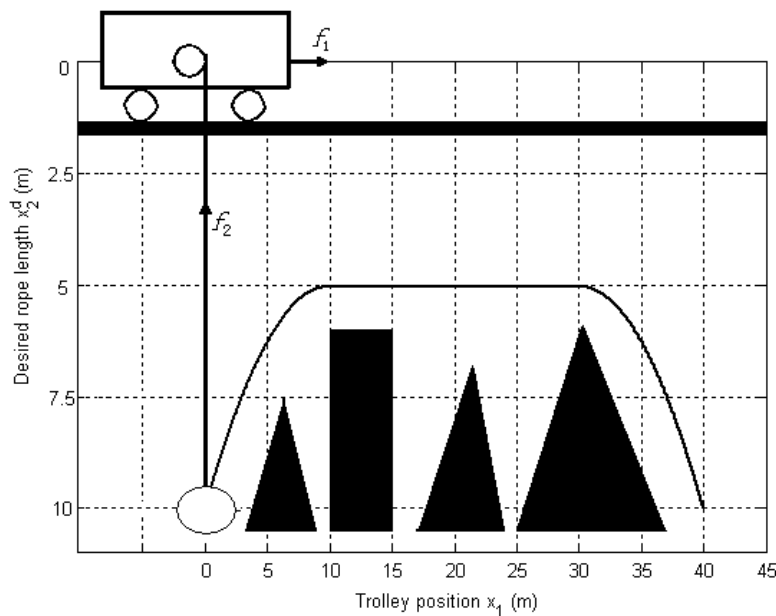


Figure 2. Reference trajectory.

The dynamic variable of the coupling factor can be inferred by the fuzzy rules in Table 1.

Antecedent variable	Consequent variable
$ x_3 $	$\Delta k$
S	0.0
M	0.5
B	1.0

Table 1. Rules for dynamic variable.

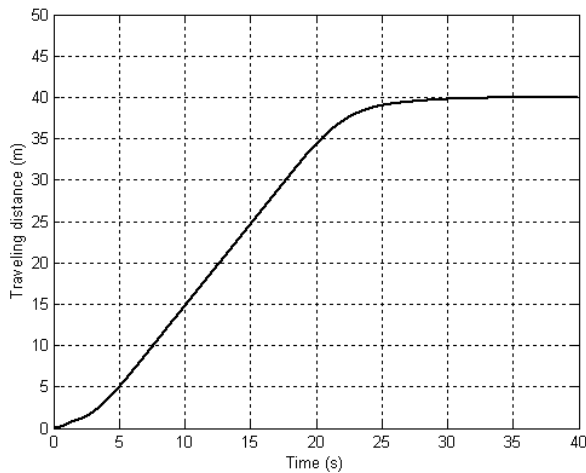
Using the proposed control law with the above parameters and zero initial condition, the first simulation is performed with MATLAB. The transportation task is to move the payload that is located at (0, 10) along the predefined trajectory to (40, 10). The simulation results are shown in Figure 4.

Then, the simulation results showed the following: position overshoot is little, and collision with other objectives is prevented. The velocity of the trolley quickly approaches the rated speed hence the transporting time is sharply shortened. The swing angle can be damped in all the transportation process, and the payload can strictly track the predefined trajectory.

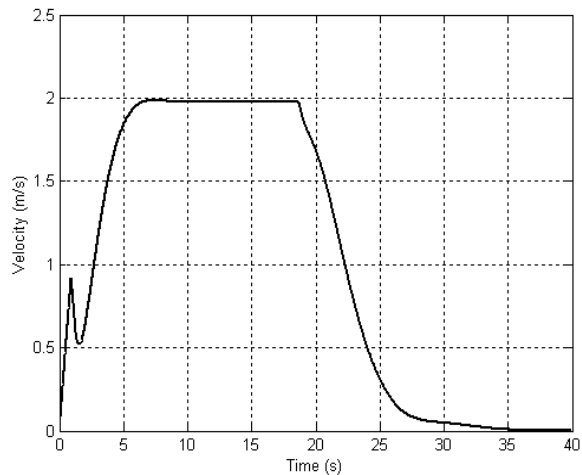
In order to validate the robustness of the proposed control algorithm with the same transportation task, the second simulation with nonzero initial condition, external disturbance and system parameter varying is performed. The payload initial swing angle is  $-0.05\text{rad}$ , a man-made swing angle ( $0.05\text{rad}$ ) at 10s is added to the overhead crane system, and

system parameter  $m_2$  is changed from 10kg to 8kg. The second simulation's results are shown in Figure 5. The results show the proposed control algorithm can effectively resist external disturbances and system parameters' variation, and the initial swing angle can be damped quickly.

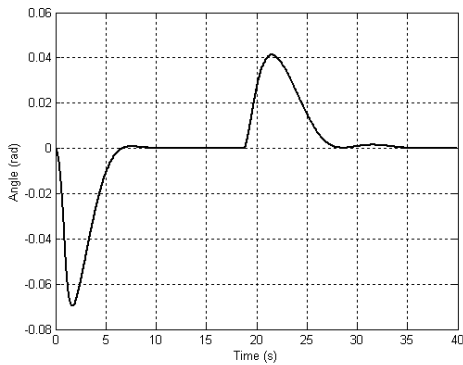
As expected, in the constant velocity zones, the trolley transports with the rated speed and the payload swing angle is almost zero. Moreover, the proposed self-tuning fuzzy inference algorithm can automatically improve the control performance through adjusting the importance of those two sliding mode surfaces in control.



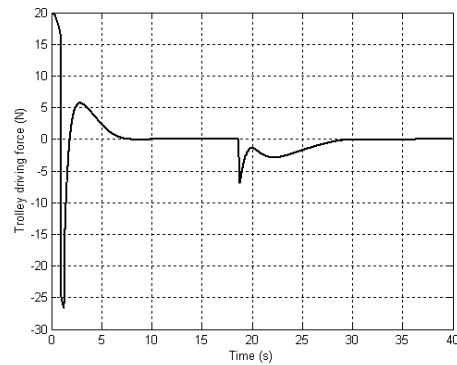
(a) Position response



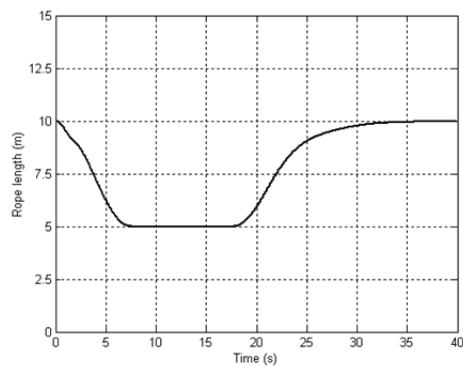
(b) Velocity response



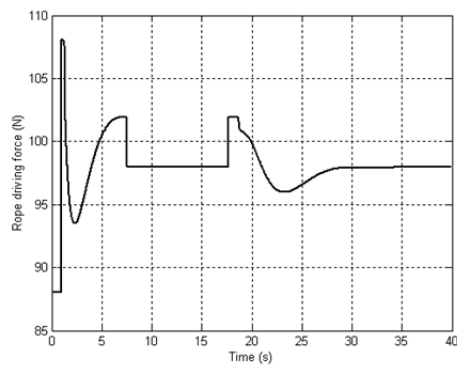
(c) Sway angle response



(d) Trolley driving force response



(e) Rope length response



(f) Rope driving force response

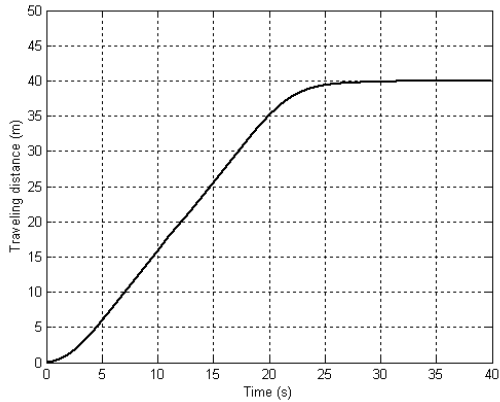
Figure 4. Simulation results without nonzero initial condition, extern disturbance and system parameter varying.

## 5. Conclusions

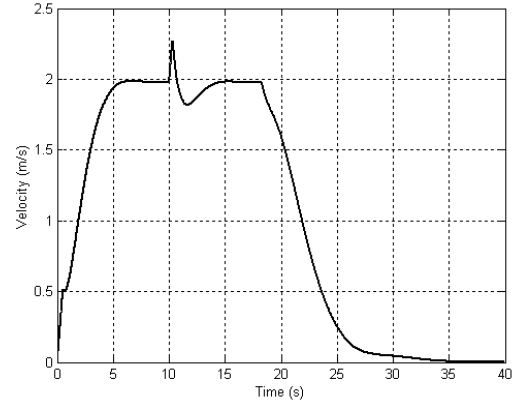
The dynamic model of overhead crane is obtained according to the Lagrangian equation, and two sliding mode controllers are proposed for an overhead crane to transport the payload along a predefined trajectory. One sliding mode controller is designed to hoist and lower the payload to track the predefined trajectory, and another one is used to control the trolley position and payload swing angle. Because the overhead crane is an underactuated system, a self-tuning fuzzy inference algorithm has been proposed to adjust the relationship between trolley positioning control and payload swaying control. The simulation results have supported the theoretical results and

have shown the effectiveness of the proposed control law. The performance of the proposed algorithm is less affected by the initial condition, external disturbance and parameters uncertainty. Consequently, the proposed algorithm has a great potential for high-performance control of overhead crane.

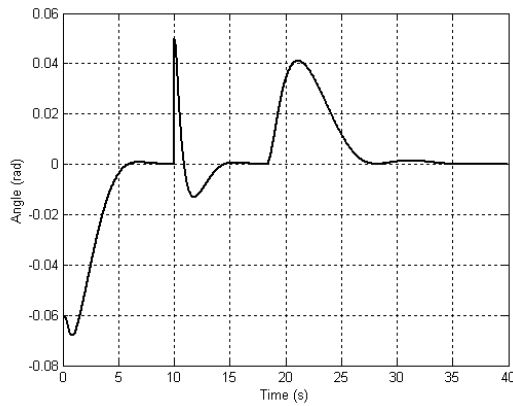




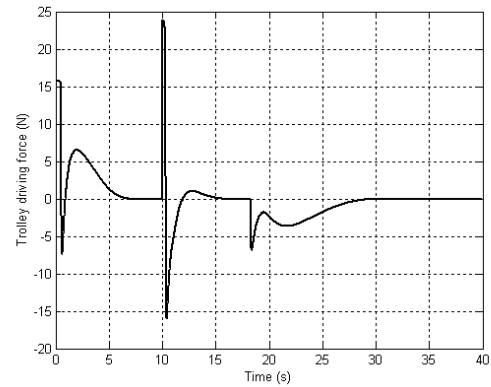
(a) Position response



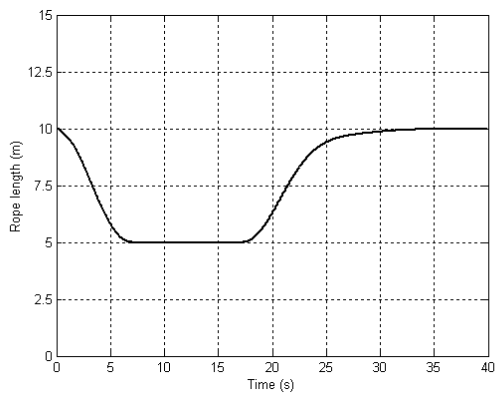
(b) Velocity response



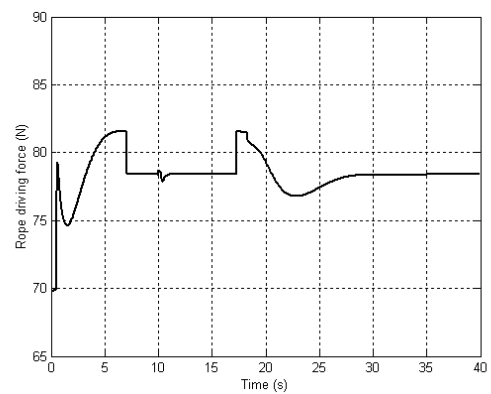
(c) Sway angle response



(d) Trolley driving force response



(e) Rope length response



(f) Rope driving force response

Figure 5. Simulation results with nonzero initial condition, extern disturbance and system parameter varying.

### Acknowledgements

This work is supported by the National Natural Science Foundation of China (No.61175086) and the Research Award Fund for Outstanding Middle-aged and Young Scientist of Shandong Province (No.BS2009DX021).

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