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Estudios e Investigaciones Recientes

Symbolic sensitivity analysis of the new second-order IIR structure

Análisis simbólico sensitivo de la nueva estructura tipo IIR de segundo orden

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Abstract

An high-order IIR (In fi nite im pulse re sponse) fil ter is usu ally re al ized in the form of a cas cade or in the form of a par al lel con nec tion of sec ond-order sec tions. Con se quently, it is of in ter est to study prop er ties of a sec ond-order dig i tal fil ter struc tures. To this end, in this pa per we pro pose a new sec ond-order IIR fil ter struc ture. The ef fect of mul ti plier co ef fi cient quantization of the pro posed struc ture is an a lyzed us ing the MATLAB-based sym bolic anal y sis. The sen si tiv ity ma trix of the struc ture is com puted in a sym bolic form and its pole sen si tiv i ties are com pared with that of other known structures.

Key words: *IIR fil ter, sec ond-order sec tion, quantization, sen si tiv ity anal y sis, sen si tiv ity ma trix, pole sen si tiv ity.*

Resumen

Los filtros digitales de alto orden con la respuesta al impulso infinita (IIR) se implementan normalmente en forma de cascada o en forma de una conexión paralela de segundo orden, respectivamente. Por tal motivo, hay un interés para investigar las características de las secciones de segundo orden. En este artículo se propone una nueva estructura IIR de segundo orden. Los efectos de la cuantizacion de los coeficientes de la estructura propuesta son hechos utilizando herramienta simbólica de MATLAB. La matriz de la sensitividad de la estructura se obtiene de una forma simbólica y sus polos sensitivos son comparados con otras estructuras conocidas.

Descriptores: Filtro IIR, sección del orden dos, cuantización, análisis de sensitividad, matriz de la sensitividad, sensitividad de los polos.

Introduction

The main effect of multiplier coefficient quantization on Infinite Impulse Response (IIR) digital filters is to movethe poles and zeros to different locations from their original locations. As a result, the actual frequency response is different from the desired frequency response and may not be acceptable to the user (Mitra, 2006). Since the poles of the transfer function are more critical in determining the frequency response of the filter, we restrict our attention only to the movement of poles caused by quantization. If a pole remains close to the original location after coefficient quantization, the structure exhibits low pole sensitivity. Otherwise, the structure is expected to exhibit high pole sensitivity.

Additionally, a higher-order IIR filter is usually realized in the form of a cascade of second-order sections or in the form of a parallel connection of second orders sections. To this end, a low pole sensitivity of a high order IIR filter can be obtained by combining second order sections with low pole sensitivities.

Consider a second-order IIR filter transfer function

$$H(z) = \frac{N(z)}{D(z)}, \tag{1}$$

where the denominator polynomial is given by

$$D(z) = z^2 - bz + c \tag{2}$$

and b and c are positive constants.

The poles of $H(z)$ of equation (1) are at

$$p_{1,2} = re^{\pm j\theta}, \tag{3}$$

where r is radius and θ is the angle. Consequently,

$$c = r^2;\ b = 2r\cos(\theta). \tag{4}$$

Small changes of the coefficients b and c, by the amounts Δb and Δc respectively, result in a new denominator polynomial

$$\hat{D}(z) = z^2 - (b + \Delta b) \cdot z + (c + \Delta c). \tag{5}$$

The corresponding poles are at

$$p_{1,2} = (r + \Delta r)e^{\pm j(\theta + \Delta\theta)}. \tag{6}$$

Using equations (4)-(6) we relate the changes of the pole radius and angle with the changes of the coefficients b and c as

$$\begin{bmatrix} \Delta r \\ \Delta\theta \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \Delta c \\ \Delta b \end{bmatrix}, \tag{7}$$

where (Mitra, 2006)

$$\boldsymbol{A} = \begin{bmatrix} 1/2r & 0 \\ 1/2^2 tg(\theta) & -1/2r\sin(\theta) \end{bmatrix}. \tag{8}$$

We consider here digital filter structures with two distinct multiplier coefficients α and β. The most general form of the functional dependence of the constants b and c on α and β is given by:

$$c = c_1\alpha^{m_1} + c_2\beta^{m_2} + c_3\alpha^{m_3}\beta^{m_4}. \tag{9}$$

$$b = b_1\alpha^{r_1} + b_2\beta^{r_2} + b_3\alpha^{r_3}\beta^{r_4}.$$

From equation (9) we have

$$\begin{bmatrix} \Delta c \\ \Delta b \end{bmatrix} = \boldsymbol{B} \cdot \begin{bmatrix} \Delta\alpha \\ \Delta\beta \end{bmatrix} \tag{10}$$

Using Equations (7) and(10) we arrive at

$$\begin{bmatrix} \Delta r \\ \Delta\theta \end{bmatrix} = \boldsymbol{A} \cdot \boldsymbol{B} \cdot \begin{bmatrix} \Delta\alpha \\ \Delta\beta \end{bmatrix} = \boldsymbol{C} \cdot \begin{bmatrix} \Delta\alpha \\ \Delta\beta \end{bmatrix} \tag{11}$$

where $\boldsymbol{C}$ is the sensitivity matrix.

In order to compare the sensitivities of different structures we need a quantitative sensitivity measure. To this end, we use here the upper limit of the variations of Δr and $\Delta\theta$ in the pole radius and angle, respectively, for a given change in the multiplier values, $\Delta\alpha$ and $\Delta\beta$. The upper limit of variation of a function ΔF can be estimated by the *worst-case* method as (Lutovac & Tosic, 2001)

$$\Delta F|_{worst\ case} = \sum_{i=1}^{n} |S_{x_i}^F \Delta x_i|, \tag{12}$$

where $S_{x_i}^F$ is the sensitivity of F to the parameter x_i. Applying equation (11) to equation (12) we have

$$\Delta F = \begin{bmatrix} \Delta r \\ \Delta\theta \end{bmatrix};\ S_{x_i}^F = [\boldsymbol{C}];\ \Delta x_i = \begin{bmatrix} \Delta\alpha \\ \Delta\beta \end{bmatrix}. \tag{13}$$

Finally, it follows

$$\begin{bmatrix}\Delta r\\ \Delta\theta\end{bmatrix}\Bigg|_{worst\ case} = \left|[\mathbf{C}]\begin{bmatrix}\Delta\alpha\\ \Delta\beta\end{bmatrix}\right|, \qquad (14)$$

or

$$\Delta r|_{worst case} = |C(1,1)\Delta\alpha| + |C(1,2)\Delta\beta|,$$
$$\qquad (15)$$
$$\Delta\theta|_{worst case} = |C(2,1)\Delta\alpha| + |C(2,2)\Delta\beta|.$$

The rest of the paper is organized in the following manner. The new structure is proposed in the next section, followed by the derivation of the symbolic sensitivity matrix of the proposed structure. Finally, the pole sensitivities of the proposed structure are compared with that of other known structures.

Proposed structure

We propose a new structure, containing 3 multipliers as shown in figure 1.

Its transfer function is given by

$$H(z) = \frac{\beta}{z^2 - (1-\alpha\beta)z + \beta^2} \qquad (16)$$

The pole distributions of the (16) using $N=5$ number of bits to represent the multiplier values, is given in figure 2. The figure shows the poles for each value of α and ß, and $-1 < a$, $ß < 1$, for which the coefficients fulfill the requirements of stability

$$|b| < 1 + c$$
$$|c| < 1 \qquad (17)$$

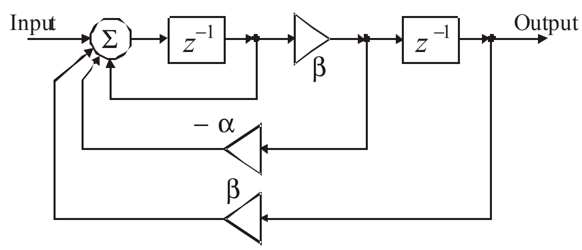


Figure 1. Proposed structure

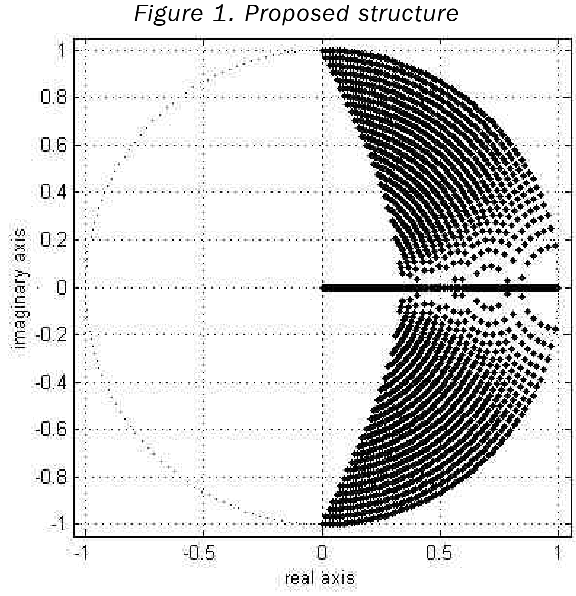


Figure 2. Pole distri bu tion of the proposed structure

Figure 2 indicates that there is a low sensitivity in the bandpass region. We use the sensitivity analysis described in Jovanovic Dolecek & Mitra (2006), to investigate in more details the sensitivity characteristics of the proposed structure.

General symbolic sensitivity matrix

In this section we use the result from Jovanovic Dolecek & Mitra (2006), to get the elements of the general sensitivity matrix **C** from equation (11) in the form

$$\boldsymbol{C}(1,1)=\frac{1}{2r}\,\frac{c_1\alpha^{m_1}m_1+c_3\alpha^{m_3}m_3\beta^{m_4}}{\alpha},$$

$$\boldsymbol{C}(1,2)=\frac{1}{2r}\,\frac{c_2\beta^{m_2}m_2+c_3\alpha^{m_3}m_4\beta^{m_4}}{\beta},$$

$$\boldsymbol{C}(2,1)=\frac{1}{2}\,\frac{(c_1\alpha^{m_1}m_1+c_3\alpha^{m_3}m_3\beta^{m_4})\cos(\theta)}{\alpha r^2\sin(\theta)}-$$

$$\frac{1}{2}\,\frac{(b_1\alpha^{n_1}n_1+b_3\alpha^{n_3}n_3\beta^{n_4})}{\alpha r\sin(\theta)}, \tag{18}$$

$$\boldsymbol{C}(2,2)=\frac{1}{2}\,\frac{(c_2\beta^{m_2}m_2+c_3\alpha^{m_3}m_4\beta^{m_4})\cos(\theta)}{\beta r^2\sin(\theta)}-$$

$$\frac{1}{2}\,\frac{(b_2\beta^{n_2}n_2+b_3\alpha^{n_3}n_4\beta^{n_4})}{\beta r\sin(\theta)}.$$

Next, the sensitivity matrix of the given structure is evaluated in two steps:

First, the symbolic coefficients in equations (18)

{b1, b2, b3, n1, n2, n3, n4, c1, c2, c3, m1, m2, m3, m4} (19)

are replaced with the actual values of these coefficients obtained from the transfer function of the proposed structure using the MATLAB command

$$C1=subs(C,\{symbolic\ coeffs\},\{actual\ values\}) \tag{20}$$

Next, the symbolic values of the multiplier coefficients α and β are replaced with their actual expressions as functions of the radius r and the angle θ of the poles using the MATLAB command

$$C2=subs(C1,\{\alpha,\beta\},\{actual\ expressions\}) \tag{21}$$

Comparing the denominator of equation (16) with the general form given by equation (9) we have the following actual values of the symbolic coefficients (19):

$$\begin{aligned}&b_1=1,b_2=0,b_3=-1,n_1=\\&n_2=0,n_3=n_4=1,\\&c_1=0,c_2=1,c_3=0,m_1=0,m_2=2,\\&m_3=0,m_4=0.\end{aligned} \tag{22}$$

Additionally it follows from equations (2), (9) and (16) :

$$\begin{aligned}\alpha&=\frac{1-2r\cos(\theta)}{r}\\ \beta&=r\end{aligned}. \tag{23}$$

In that way we get the symbolic sensitivity matrix of the proposed structure:

$$\boldsymbol{C}=\begin{bmatrix}0 & 1\\ \dfrac{1}{2\sin(\theta)} & \dfrac{1}{2r^2\sin(\theta)}\end{bmatrix}. \tag{24}$$

From equations (18) and (27) we have

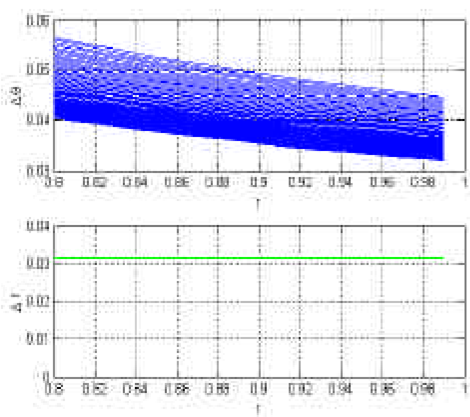
$$\Delta r|_{worst\ case} = |\Delta\beta|, \tag{25}$$

$$\Delta\theta|_{worst\ case} = \left|\frac{1}{2\sin(\theta)}\Delta\alpha\right| + \left|\frac{1}{2r^2\sin(\theta)}\Delta\beta\right|$$

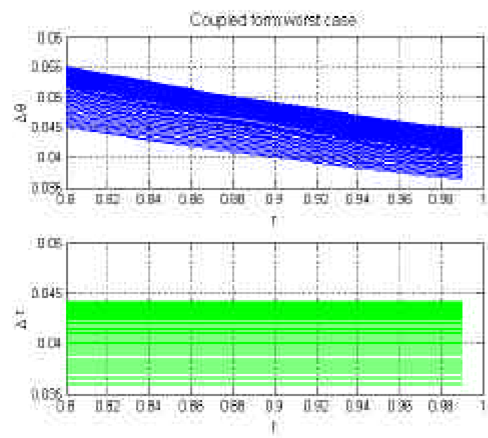
In a similar way we find the diagrams (25) for some known structures used in the next section.

Discussion

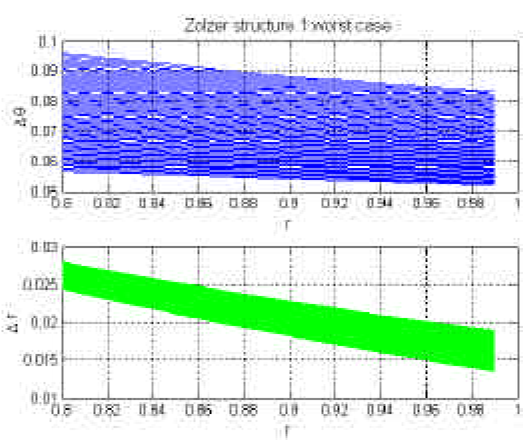
The sensitivity diagrams (25) for the proposed structure are plotted in figure 3 (a) for values of the pole radius in the range [0.8 – 0.99], and the phase values in the range [p/4 – p/2]. In order to compare the sensitivity of the proposed structure we find the sensitivities diagrams of some well known structures, using the same values of pole radius and the same phase values, and the method described in Jovanovic Dolecek & Mitra (2006). The corresponding sensitivity diagrams are shown in figure 3 (b)–(f). The results are summarized in the table 1.



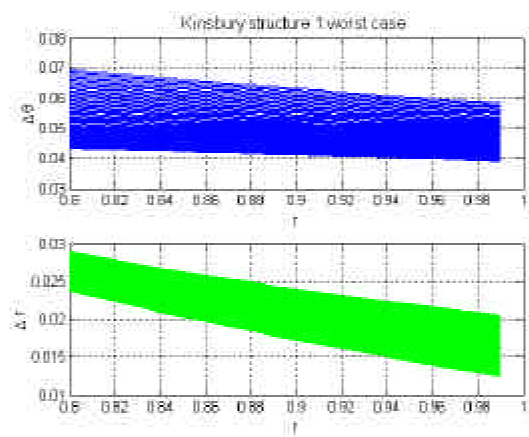
(a) Proposed structure



(b) Coupled struc ture (Gold & Reader, 1967)

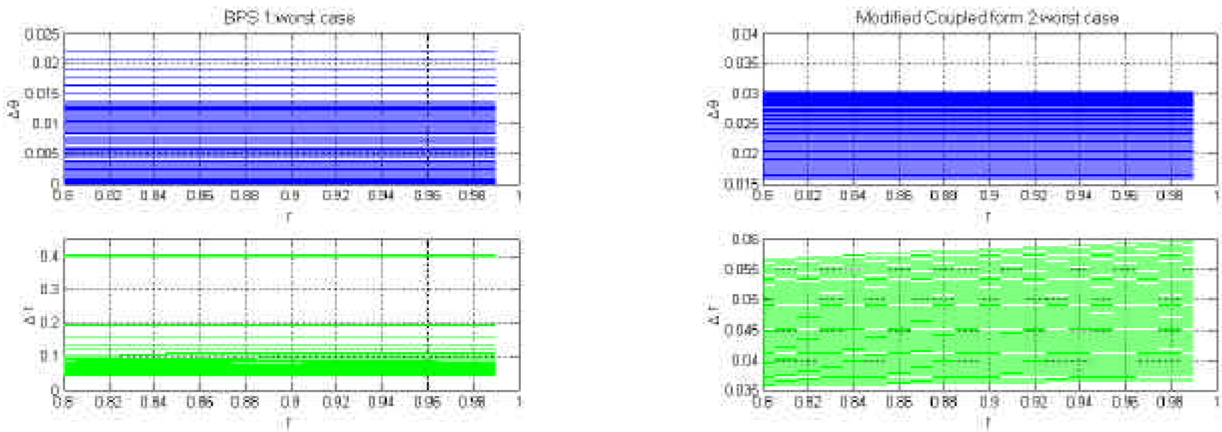


(c) Zolzer struc ture (Zolzer, 1990)



(d) Kingsbury struc ture (Kings bury, 1972)

Figure 3. Sensi tivity diagrams



(e) BPS 1, (Jovanovic & Mitra, 2002)

(f) Modified coupled struc ture (Yan & Mitra 1982).

Figure 3. Sensi tivity diagrams

Table 1. Comparison the proposed with some known structures

Structure	Max (Δθ)	Max (Δr)
Proposed	0.057	0.031
Kinsbury	0.07	0.029
Coupled	0.055	0.0448
BPS1	0.022	0.4
Zolzer	0.095	0.028
Modified coupled	0.03	0.06

Conclusions

The main effect of multiplier coefficient quantization on IIR digital filters is to move the poles and zeros to different locations from their original locations. As a result, the actual frequency response is different from the desired frequency response. Since the poles of the transfer function are more critical in determining the frequency response of the filter, we restricted our attention only to the movement of poles caused by quantization.

We proposed a new second order structure with three multipliers. The analysis of the effects of quantization on the structure is done using the MATLAB-based symbolic toolbox. We derived the sym- bolic sensitivity matrix of the proposed structure. Using the symbolic matrices we plotted the sensitivity diagrams to compare the sensitivity characteristics of the proposed and some known structures. The analysis demonstrates that the proposed structure has a low sensitivity for values of the pole radius in the range $[0.8 - 0.99]$, and the phase values in the range $[p/4 - p/2]$.

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