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A Mamdani Type-Fuzzy Inference - Alignment Matrix Method for Evaluation of Competencies Acquired by Students Enrolling at the Mexican Higher Middle Education System I: Formulation and Explanation Based on Simulation, and a Real but Incomplete Data Set

Cecilia Leal-Ramírez[1], Héctor Alonso Echavarría-Heras[1]
Hajasya Maraay Romero-Escobar[2]

[1] Centro de Investigación Científica y de Estudios Superiores de Ensenada, Departamento de Ecología, Mexico

[2] Universidad Tecnológica de Tijuana, Facultad de Energías Renovables y Ambiental, Mexico

cleal@cicese.mx, heheras@icloud.com, hajasya@hotmail.com

Abstract. The last reforms at the Mexican educational system attempt to modify graduates' traditional profiles to enable them to investigate, develop and propose solutions to various problems, think more assertively, form opinions, interact with multidisciplinary groups of people, and ultimately assume a proactive role in the community. The implementation of curricula promoting the development of competencies becomes essential in achieving envisioned goals. The alignment of competencies to study plans and the evaluation of the development of related skills becomes critical at the scheme's implementation. Currently, relating duties mainly depends on the teacher's will, but the lack of a systematic protocol to achieve involved endeavors promotes designs based on subjectivity. On top of this, concurring student's performance grading scheme relies on a thorough alphabetic character rating approach, thereby masking objective scoring. Assessment at the Mexican educational system realm conceives upon criteria designed abroad. Thereby linking particularities are left behind at performing evaluation tasks. This contribution proposes a method based on a fuzzy inference approach that aims to provide a formal structure that allows teachers to achieve the alignment of competencies and evaluate their development by students relying on a quantitative paradigm. We offer a comprehensive display of the formalities of the proposed fuzzy method. Explanation of its functionality relies on both simulated and real data. The aim here is on addressing the student-teacher aspect of alignment and evaluation procedures. Extension to other precincts within the whole Mexican educational system also contemplates and intends to appear in further contributions.

Keywords. Alignment matrix, competencies, mathematical relation.

1 Introduction

Reforms the Mexican educational system that initiated in the last decade in Mexico address all levels and have in common a curriculum design promoting the development of competencies [1]. The ensuing strategy aims to transform education so that graduates investigate, develop and propose solutions to various problems, think more assertively, form opinions, interact with multidisciplinary teams and ultimately assume a proactive role in their communities [2]. In what follows the composite Mexican educational system could be also referred simply as educational system for short. So far, the educational system's protocol for the evaluation of student's performance has mainly mimicked international criteria. Subsequent assessments have so far revealed unfavorable results.

For example, the values reported by the International Program for the Evaluation of Students [3-6] demonstrate that since 2006 the output of the Mexican educational endeavors exhibited severe deficiencies, mainly in mathematics, science, and reading. Therefore, the procedures involved in the whole educational system should be revised, particularly those contributing to students' development of competencies and the concurring evaluation of their acquisition.

A competency describes as a composite of a coordinated and integrated knowledge set along with procedures and attitudes so that the individual can display the "know-how to do" and the "know-how to be" while performing professional practice [7]. From this, it follows that when individuals become competent, they can select, coordinate and efficiently mobilize a set of articulated and interrelated knowledge to solve problems in a specific context. Given this interpretation, competencies could conceive as complex skills that are difficult to conceptualize, especially concerning what it means to integrate them into practice because there is no unified theory for their implementation and evaluation. For example, the definition of attitudes and values, by their very nature, implies a noticeable subjective burden on the evaluating entities since they are intangible and dependent on human interpretation.

This aspect adds ambiguity or vagueness to the evaluation because it is a function of the experiences. There is an uncertainty load when a context conforms to vagueness, imprecision, and lack of data. Fortunately, techniques and instruments proposed by a Fuzzy Logic approach have efficiently contributed to modeling uncertain phenomena in educational sciences. Applications cover assessing student-centered learning [8]. Besides, fuzzy paradigms contributed classifying the student academic profile [9]. And fuzzy logic applications also appear for the cognitive diagnosis of the student [10], as well as, at evaluating student's attitude [11].

Currently, the Mexican education system structures a study program based on units, learning outcomes, and complementary activities, assigning each one of them a specific value relative to the whole program [12].

Ensuing percentage values express without specifying any mathematical relationship that allows evaluating the development of competencies in each activity and despite this, teachers must maintain indicators of skills acquired by each of the students. Besides, teachers do not have at hand a specified protocol that allows them to align the competencies with the unit's objectives, learning outcomes, and learning activities.

A lack of a systematized approach to achieve the alignment of competencies also makes it difficult to integrate their acquisition into the student's assessment protocol. In addition, the evaluation scheme implemented at the technological branch of the Mexican higher middle education system constitutes impairing the implementation and evaluation of competencies. For example, the use of alphabetic characters to label grades describes an effort to deter discrimination scenarios [12]. But such an approach could mask the student's objective evaluation.

Also, in the current evaluation approach, a student does not "approve" or "fail" in the literal interpretation of the words but is instead declared “Competent" or "Not competent" [12]. Such an evaluation scheme makes it difficult to compare results on a quantitative basis. For a student, it is enough to obtain "Competent," which achieves when 70% of academic activities evaluate at least with the letter "S" (Sufficient) [12]. Since students know how the system operates, so they efficiently manage to know when they have reached 70% of their evaluation. This way, many students resolve not to participate more actively in in their educational task. In addition, students assume that even though a lack of commitment at the end of the period, they will not get a “failed," This characteristic of the educational system undermines all the efforts of teachers. And the fact that there is still no methodology based on a mathematical formalism to establish an alignment of the student's competencies with the learning activities reduces the effectiveness of the teacher's competencies own skills.

In the literature, it is possible to find models on the alignment and evaluation of acquired competencies. For instance, Biggs in [13] points out that the way to carry out a constructive alignment is by elaborating learning objectives, which can be created by establishing the subject centered on the action. This identifies the task or product or, more generally, the accomplishment to be carried out. Vargas in [14] proposed a meta-model to assess the grade of acquisition of competencies. They established a relationship between competencies, a learning outcome, and an evaluation tool through a tree. The children's weight of the internal nodes has an arbitrary value between 0 and 1, and their sum is 1. Romero-Escobar in [12] proposed a model that establishes an alignment of the objectives of the learning activities with the competencies of the teacher and the student's competencies. The approach offers a procedure to construct a relevance matrix to indicate which competencies will be considered in the design of learning activities. However, the resulting alignment does not express the mathematical relationship between the competencies and the objectives of the learning activities mathematically. The most outstanding aspect of Romero-Escobar's work is conceiving the relevance matrix and the outline from which its formal structure can be arranged.

The approach that we offer in this work involves the development of a system composed of an alignment method, whereby means of a matrix on its mathematical conception, the teacher can make an alignment indicating the competencies and attributes formally integrated at the process of evaluation of learning activities.

The alignment is strictly a task consigned to the teachers; their criterion traces the student's path to achieve full development of competencies. Based on this integration, the system creates weight values rating the relative importance for the learning units, and activities contemplated in the study program. It is worth emphasizing that our approach undertakes using a mathematical formalism. A weight value will represent the teacher's criteria applied in said alignment, totally transparent and easy to accomplish. Proposed alignment formalization will also allow knowing the grade of development of the competencies acquired by the student or by the group.

In addition, our construct integrates a Mamdani type fuzzy inference model to evaluate the efficiency with which the student performs their activities through three elements, such as the knowledge acquired, the procedure developed and the attitude shown during the process, that is, extracted evidence when developing the learning activities. The teacher's mastery of the subject must create learning activities aligned with the acquisition of competencies [15-17] and provide the techniques to evaluate them.

Fuzzy systems based in fuzzy logic [18] are used in many scientific applications, they are classified into two types. Type 1 fuzzy systems can handle linguistic variables and expert reasoning and reproduce the knowledge of the systems to be controlled (e.g., [19]).

Type 2 fuzzy systems can model complex non-linear systems, achieving better performance from controllers designed under this approach (e.g., [18, 20, 21]).

The fuzzy logic systems of Type 2 by interval are a representation of reduced complexity of the fuzzy systems Type 2, it is the most used now (e.g., [22]). Uncertainty is inherent in the information regardless of the type of technique, treatment or any other factor relying in its use. Regarding the evaluation of competencies, fuzzy logic has been used to assess student responses (e.g., [23-26]) or to construct fuzzy rubrics (e.g., [27]).

Our Mamdani-typo-1 fuzzy inference system integrates three elements: knowledge, procedure and attitude, defined as linguistic variables to evaluate efficiency, which has not been done so far. To begin with, talking about attitude implies identifying the kind, we refer to [28]. It would be very difficult to integrate all kinds of attitudes in a single Type 1 fuzzy system, considering that elements such as knowledge and the procedure are to be also integrated in the evaluation of the efficiency. The system that we develop is also relevant because it attempts to adapt a first step by mathematically formalizing the alignment of competencies with learning activities in accordance with the objectives established by the study program.

Once the alignment is made, our approach provides a way to evaluate the grade of development of the competencies with transparency and with the certainty that the competences acquired by the student are those established in said alignment. In addition, the Mamdani type-1 fuzzy inference system evaluates the efficiency with which the competencies were developed in each of the learning activities, integrating three fundamental elements that define the set of articulated and interrelated knowledge to solve problems in a specific context.

What remains of the paper organizes as follows. In Section 2, we present basic concepts needed to comprehend addressed matters: included are outlines of the Mamdani Type 2 fuzzy inference system, the alignment method and mathematical formulism involved at evaluating acquired competences. In Section 3, we present the results of a simulated data study case and a counterpart based on real data. In Section 4, we present the discussion of the results linking a detailed description of the problem. Conclusions are presented in Section 5. Finally, in an Appendix we present some concepts and definitions of fuzzy set theory that are relevant to build the addressed fuzzy inference system.

2 Methods

2.1 Competencies

The paradigm of acquisition of competencies recently adopted by the Mexican Higher Middle Educational System aims to the development of generic, disciplinary and professional competencies by the students (Figure 1). Pertaining explanations of terms appear in the document entitled Integral Reform of the Higher Middle Education (RIEMS in Spanish) [2].

The development of generic competencies requires to all graduates and is pertinent to all the subjects taught. The disciplinary competencies comprise two categories. They could be basic disciplinary competencies that are intended to be achieved in every subject composing the curricular design at a whole system level. There could be also extended disciplinary competencies whose accomplishment is required to be achieved by the enrolment in the educational scheme pertaining to constituting subsystems. Professional competencies also split into basic and extended categories and with similar associations to the whole system and subsystem levels. Finally, generic competencies comprise 11 different dimensions.

In the present study, we center on generic competencies because they ought to be acquired by every student. Certainly, their development could entail a successful performance in any professional field, thereby increasing hiring chances. Generic competencies compose 11 dimensions and each one integrates a set of attributes that every student must develop (Table 1).

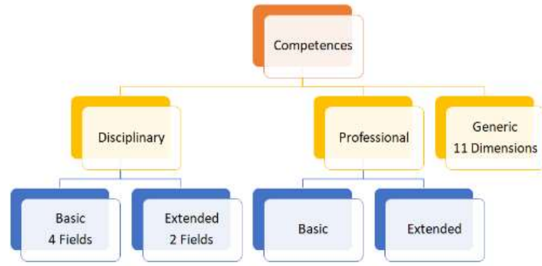

Fig 1. Competencies of the student of higher middle education level defined by the RIEMS [2]

The process of evaluating learning by competencies requires a comprehensive evaluation of the student's attainment of affective, cognitive and psychomotor characteristics. In the affective aspect, the know how to be at coexistence is promoted, the individual is encouraged to be a good citizen so that he assumes his rights and duties with responsibility and builds her (his) personal identity; in the cognitive aspect, the known to know is stimulated so the student could seek to develop the ability to use a set of tools that allow processing and interpreting information, and provide solutions to situations that arise; in the psychomotor aspect, the know how to do is heartened by focusing on the individual's performance in real life situations, it induces the use of knowledge in a systemic and reflective way to achieve goals; know-how implies an individual's performance in a proficient way in the completion of an activity or in the solution of a problem based on planning and understanding the domain of the problem [27].

An evaluation criterion should aim to assess the development of the student's acquisition of characteristics associating with the cognitive, affective, and psychomotor domains [29-30]. Using linguistic appraisals of performance conditions, assessment procedures must allow establishing the level of development of competences. Tobón

Table 1. Description of the generic competencies and number of attributes they compose. The letter C symbolizes the word "Competence" and the subscript indicates the competence framed in [2]. Also, a detailed description of the attributes is presented in [2]

Competence	Description	No. of attributes
	Self-determines and takes care of itself	
C_1	Knows and self-values, and addresses problems and challenges, considering the pursued objectives.	6
C_2	Sensitive to art and participates in the appreciation and interpretation of its expressions in different genres.	3
C_3	Chooses and practices healthy lifestyles.	3
	Expresses oneself and communicates with others	
C_4	Listens to, interprets and delivers relevant messages in different contexts using appropriate media, codes and tools.	5
	Thinks critically and reflectively	
C_5	Develops innovations and proposes solutions to problems based on established methods	6
C_6	Holds a personal stance on topics of general interest and relevance, considering other points of view critically and reflectively.	4
	Learns autonomously	
C_7	Learns by own initiative and self-interests throughout life.	3
	Works collaboratively	
C_8	Participates and collaborates effectively on diverse teams.	3
	Participates responsibly in society	
C_9	Participates with a civic and ethical conscience at community, region, Mexico and the world levels.	6
C_{10}	Maintains a respectful attitude towards interculturality and diversity of beliefs, values, ideas and social practices.	3
C_{11}	Contributes to sustainable development in a critical way, with responsible actions.	3

[30] proposed as evaluation elements those corresponding to the affective, cognitive and psychomotor characteristics (Figure 2).

Concisely, an evaluation of acquisition of competencies amounts to collecting evidence through the analysis of learning activities and application of techniques aimed to their assessment. Gonczi and Athanasoub [31] establish that the development of competencies must be evaluated integrally by selecting the most

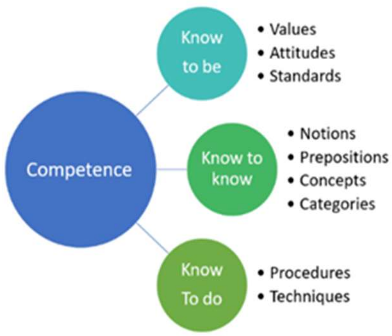

Fig. 2. Elements to evaluate the student's attainment of affective, cognitive and psychomotor characteristics as proposed by [30]

suitable methods, for example, written tests, observation, problem-solving, or a combination of techniques depending on the skill or competency to be evaluated, more examples are presented in Table 2.

Regardless of the evaluation technique used by the teacher, for present aims we have considered that evidence of success in a learning activity should base on the display of following elements: knowledge (notions, concepts, prepositions or/and categories), the procedure or technique carried out, and the attitude shown by the student during achieving a certain process. In other words, according to our approach, the evaluation of the effectiveness of each learning activity should consider the formation of listed elements.

The traditional procedures for evaluating learning, for the most part, centers on the proficiency attained by the student at a specific moment and addresses only the cognitive aspect as the basis of the assessment.

The reason underneath adopting a fuzzy inference system is that it offers a scheme that allows evaluation according to a comprehensive perspective, wherein competence assessment considers affective, cognitive, and psychomotor aspects.

All affective aspects are essential; however, we consider that the student's attitude turns out to be a key feature since it sets; the type of emotion displayed while carrying out a given activity, the way of interacting, or the acting manner in the face of a specific situation or stimulus [32-33]. Castillero-Mimenza [30] classifies different types of attitudes according to various criteria (Table 3).

We consider attitude as evidence according to its practical value, how the subject values the environment or situation, positive or negative. The effects of a positive attitude favor optimistic interpretation regardless of the difficulties that may arise, bringing the subject closer to stimulation or action and to pursue achieving goals through a healthy, confident, and generally disciplined way [30].

The effects of a negative attitude maximize the aversive experience, the subject could minimize the relevance of a circumstance, or directly fail to acknowledge the positive aspects of the situation.

A negative attitude can also induce them to withdraw from acting or acquire complaining behavior beyond what is rational, making it difficult to achieve goals [30].

To acquire competencies, the student must achieve a certain number of learning activities. These activities are guided by the teacher, who should also aim to obtain sufficient evidence to determine how efficiently a student develops them.

The teacher can choose any technique (Table 2) to evaluate a learning activity, as long as the criteria include the ranks that the teacher assigns to knowledge, procedure, and attitude, such that the way in which these elements combine, coordinate, and integrate into determining the student's performance can be appraised [7].

2.2 Fuzzy Inference System

We use the notation convention described in the Appendix to construct the fuzzy inference system. We begin by defining the membership functions that characterize the fuzzy sets inherent to the fuzzy system's antecedents and consequents.

Based in an expert system knowledge base, we consider three input variables, $\mathrm{Knowledge} = \mathrm{X}_1$, $\mathrm{Procedure} = \mathrm{X}_2$ and $\mathrm{Attitude} = \mathrm{X}_3$, to be the antecedents of the fuzzy system and an output variable, $\mathrm{Efficiency} = \mathrm{Y}$, entailing the consequent. To the input variable X_1 we associate the linguistic terms "Little", "Enough" and "Much", characterized by z-shaped (Equation (A6)), Gaussian (Equation (A4)) and sigmoidal (Equation (A5)) membership functions, respectively (see Figure 3).

Similarly, the input variable X_2 describes by the linguistic terms "Not attempted", "Incomplete" and "Achieved" and these characterize by means of the

Table 2. Assessment techniques and their characteristics: (1) conceptual content, (a) facts and data, (b) principles and concepts, (2) procedural content, (3) attitudes and values, (4) thinking skills and (5) auxiliary techniques [12]

Techniques	1		2	3	4	5
	a	b				
Mental maps	X	X	X		X	Checklist.
Problem solving	X	X	X	X	X	Interview, checklist, rubrics and ranks.
Case method	X	X	X	X	X	Interview, checklist, rubrics and ranks.
Projects	X	X	X	X	X	Interview, checklist, rubrics and ranks.
Daily	X	X	X	X	X	Interview
Debate	X	X	X	X	X	Checklist and rubrics.
Techniques based in questions	X	X	X	X	X	Interview and checklist.
Essays	X	X	X	X	X	Interview, checklist, rubrics and ranks.
Briefcase	X	X	X	X	X	Interview, checklist, rubrics and ranks.

Table 3. Classification of the types of student's attitude according with Castillero-Memenza in [30]

Classification	**Type**	**Attitude**
1	**According to an effective valence** How they allow to assess the environment and the situation.	• Positive • Negative • Neutral
2	**According to activity orientation** In what way do they generate a concrete approach or orientation towards the idea of carrying out a behavior or activity?	• Proactive • Reactive
3	**According to the motivation to act** What motivates them to carry out a behavior or activity?	• Interested • Selfless
4	**Depending on the relationship with others** How do they get along with others?	• Collaborator • Manipulative • Assertive • Permissive • Passive • Aggressive
5	**According to the type of elements used to assess the stimuli** How do you process reality?	• Emotional • Rational

z-shaped (Equation (A6)), Gaussian (Equation (A4)) and sigmoidal (Equation (A5)) membership functions, respectively (see Figure 3).

In turn, the input variable X_3 characterizes by a trapezoidal membership function defined in the Equation (A7) (see Figure 3).In the same way, the linguistic terms "Terrible", "Very Bad', "Bad", "Regular", "Good", "Very Good", "Excellent" and "Outstanding" associate to the output variable Y and characterize by a triangular membership function defined by Equation (A4) (see Figure 3).

The process by which the IF-THEN rules composing the addressed fuzzy inference system are built must consider the relationship among the variables knowledge, procedure, attitude and efficiency (Table 4).

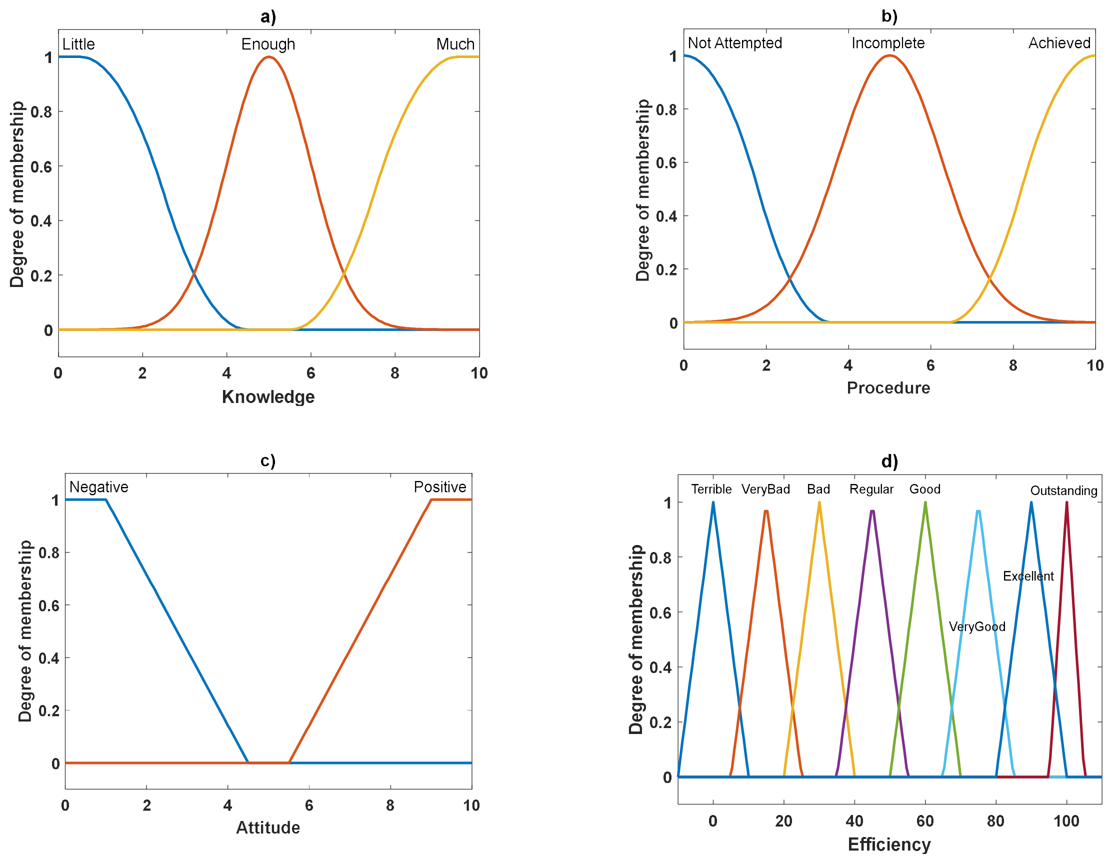

Fig. 3. Membership functions that characterize the fuzzy sets inherent to the fuzzy system's antecedents and consequent. (a) $\text{Little} = zmf(x, [0.5, 4.5])$, $\text{Enough} = gaussmf(x, [1,5])$ and $\text{Much} = smf(x, [5.5, 9.5])$ (Equations (A6), (A4) and (A5) respectively). (b) $\text{Not Attempted} = zmf(x, [0,3.6])$, $\text{Incomplete} = gaussmf(x, [1.274,5])$ and $\text{Achieved} = smf(x, [6.4,10])$ (Equations (A6), (A4) and (A5) respectively). (c) $\text{Negative} = tmf(x, [0,0,1,4.5])$ and $\text{Possitive} = tmf(x, [5,5.5,10,10])$ (Equation (A3)). (d) $\text{Terrible} = tmf(x, [-10,0,10])$, $\text{Very Bad} = tmf(x, [5,15,25])$, $\text{Bad} = tmf(x, [20,30,40])$, $\text{Regular} = tmf(x, [35,45,55])$, $\text{Good} = tmf(x, [50,60,70])$, $\text{Very Good} = tmf(x, [65,75,85])$, $\text{Excellent} = tmf(x, [80,90,100])$ and $\text{Outstanding} = tmf(x, [95,100,105])$ (Equation (A3))

The fuzzy inference system implements through Matlab's fuzzy logic toolbox (version 2016b). This aims to evaluate the efficiency given the input values x_1, x_2 and x_3 (see Appendix and Figure 4).

Building the fuzzy inference system relied on the Mamdani-Type method [34] (see Appendix). The minimum operator modeled the AND expression (fuzzy intersection in Equation (A9)), and correspondingly the OR expression by the maximum operator (fuzzy union in Equation (A9)).

In turn, the implication defined by the THEN expression modeled by the minimum operator and the aggregation by the maximum operator. The defuzzification step relied on the center of gravity procedure [35] defined in Equation (A14).

2.3 Alignment of Competencies Method

To line up the competencies with the objectives of the learning activities, an alignment matrix is initially constructed. This formalizes the mathematical relationship of the attributes of each competency with each one of the learning activities

Table 4. Relationship among variables knowledge, procedure and attitude and efficiency

IF							
	Knowledge is		**Procedure** is		**Attitude** is		**Efficiency** is
1	Little	AND	Not attempted	AND	Negative	THEN	Terrible
2	Little	AND	Incomplete	AND	Negative	THEN	Bad
3	Little	AND	Achieved	AND	Negative	THEN	Good
4	Enough	AND	Not attempted	AND	Negative	THEN	Terrible
5	Enough	AND	Incomplete	AND	Negative	THEN	Regular
6	Enough	AND	Achieved	AND	Negative	THEN	Very Good
7	Much	AND	Not attempted	AND	Negative	THEN	Terrible
8	Much	AND	Incomplete	AND	Negative	THEN	Good
9	Much	AND	Achieved	AND	Negative	THEN	Excellent
10	Little	AND	Not attempted	AND	Positive	THEN	Terrible
11	Little	AND	Incomplete	AND	Positive	THEN	Regular
12	Little	AND	Achieved	AND	Positive	THEN	Very Good
13	Enough	AND	Not attempted	AND	Positive	THEN	Very Bad
14	Enough	AND	Incomplete	AND	Positive	THEN	Good
15	Enough	AND	Achieved	AND	Positive	THEN	Excellent
16	Much	AND	Not attempted	AND	Positive	THEN	Bad
17	Much	AND	Incomplete	AND	Positive	THEN	Very Good
18	Much	AND	Achieved	AND	Positive	THEN	Outstanding

included in each unit of the study program. Relationship is made by indicating which attributes will be integrated in the evaluation of each learning activity, this relationship synthetically represents the alignment process.

From left to right, the attributes of the competencies are placed in the first column and from the second column the corresponding learning activities are placed in each unit. When an attribute is involved in a learning activity, the cell corresponding to the intersection of the attribute row with the column of the learning activity is marked, and this is done with the rest of the activities.

Once the alignment matrix is completed, it is easy to know which and how many attributes and competencies are integrated in a learning activity. This allows to calculate the value of the weight of each learning activity and each unit included in the study program. The grade of development of the competencies acquired by the student or by a group will be evaluated using named weight values. The grade is the percentage value of the development of the acquired competencies.

A study program can be structured by a number n of units and each one of them by another number m of learning activities (Figure 5). That is, let $\mathrm{U_i}$, for $i = 1 \dots n$, be the ith unit and let $\mathrm{H_{ij}}$, for $j = 1 \dots m$, be the jth learning activity of the unit $\mathrm{U_i}$.

To illustrate how the alignment method works, we will consider as an example a study program containing three units; unit $\mathrm{U_1}$ involving learning activities $\mathrm{H_{1j}}$, for $j = 1,2,3$, unit $\mathrm{U_2}$ with learning activities $\mathrm{H_{2j}}$, for $j = 1,2$, and unit $\mathrm{U_3}$ that integrates learning activities $\mathrm{H_{3j}}$, for $j = 1,2,3$. Similarly, the $h-th$ competence denotes by $\mathrm{C_h}$, for $h = 1,2,\ldots.,11$ (Table 1) and the $k-th$ attributes of a competence $\mathrm{C_h}$ represents through $\mathrm{C_{hk}}$, that is, the index k designates the kth attribute of the competence $\mathrm{C_h}$ (Table 5), we create the alignment matrix and make the alignment indicating with the value of 1 that attribute $\mathrm{C_{hk}}$ is integrated in the learning activity $\mathrm{H_{ij}}$, (Table 5).

So, let $V^{(\mathrm{H_{ij}})}$ be the number of competence attributes integrated in the learning activity $\mathrm{H_{ij}}$, which can be expressed as:

$$V^{(\mathrm{H_{ij}})} = \sum_{h=1}^{11}\sum_{k=1}^{p} \mathrm{C_{hk}^{(H_{ij})}}, \quad (1)$$

where the symbol $\mathrm{C_{hk}^{(H_{ij})}}$ denotes the attribute k of the competence $\mathrm{C_h}$ integrated in the learning activity $\mathrm{H_{ij}}$ (Table 5) and p represent the total number of attributes of the competence $\mathrm{C_h}$. Likewise, $V^{(\mathrm{U_i})}$ represent the total number of attributes integrated to the unit $\mathrm{U_i}$, which is obtained from the expression:

$$V^{(\mathrm{U_i})} = \sum_{j=1}^{m} V^{(\mathrm{H_{ij}})}, \quad (2)$$

and the total sum of attributes V integrated in a study program will be calculated by:

$$V = \sum_{i=1}^{n} V^{(\mathrm{U_i})}, \quad (3)$$

Considering the information presented in Table 5, we take Equations (1) and (2) to obtain the total number of competence attributes integrated in $\mathrm{H_{ij}}$ and $\mathrm{U_i}$ respectively (Table 6).

To finish the description of the alignment method, we calculate the values of the weights, v_{ij} and u_i, corresponding to the alignment matrix. The weight value v_{ij} is obtained through:

$$v_{ij} = \frac{100}{\sum_{j=1}^{m} V^{(\mathrm{H_{ij}})}} \times V^{(\mathrm{H_{ij}})}, \quad (4)$$

And the weight value u_i is obtained through:

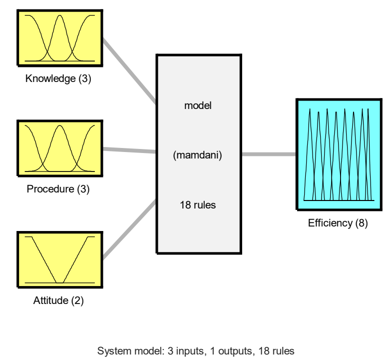

Fig. 4. Fuzzy inference system composing three antecedents: Knowledge, Procedure and Attitude, and a consequent, Efficiency, which are characterized by membership functions parameterized as describing in Figure 3. The 18 fuzzy rules integrated into conceived fuzzy inference system appear in Table 4

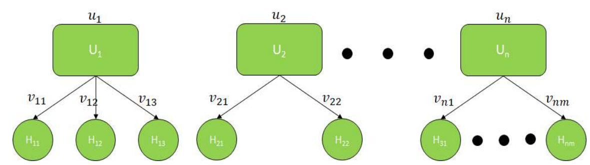

Fig. 5. Units ($\mathrm{U_i}$ for $i = 1,\ldots,n$) and learning activity ($\mathrm{H_{ij}}$ for $i = 1,\ldots,n$ and $j = 1,\ldots,m$) that comprise a study program. v_{ij} is the weight value corresponding to the jth learning activity of ith unit and u_i is the weight values corresponding to the unit ith

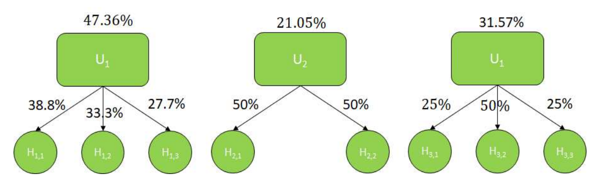

Fig. 6. Weight values obtained through the Equations (4) and (5) and corresponding to the alignment matrix presented in Table 5

$$u_i = \frac{100}{\sum_{i=1}^{n}\sum_{j=1}^{m} v_{ij}} \times V^{(\mathrm{U_i})}, \quad (5)$$

For the addressed example, we calculate the weight values, v_{ij} and u_{ij}, that correspond to the alignment matrix (Table 5) through the Equations (4) and (5) and considering the total number of competence attributes integrated in the learning activities $\mathrm{H_{ij}}$ and in the unit $\mathrm{U_i}$ from Table 6 (Figure 6).

Table 5. Alignment matrix: attribute $\mathrm{C_{hk}}$ with the learning activity $\mathrm{H_{ij}}$ included in a hypothetical study program taken as example

U₁				**U₂**			**U₃**			
C_{hk}	H_{11}	H_{12}	H_{13}	**C_{hk}**	H_{21}	H_{22}	**C_{hk}**	H_{31}	H_{32}	H_{33}
C_{31}	1	1		**C_{31}**		1	**C_{31}**	1		
C_{32}		1	1	**C_{32}**			**C_{32}**		1	1
C_{33}	1			**C_{33}**	1		**C_{33}**		1	
C_{51}	1		1	**C_{51}**			**C_{51}**	1	1	
C_{51}		1		**C_{52}**		1	**C_{52}**		1	
C_{53}			1	**C_{53}**			**C_{53}**			1
C_{81}		1	1	**C_{81}**			**C_{81}**		1	
C_{82}	1			**C_{82}**	1		**C_{82}**	1		1
C_{83}		1		**C_{83}**		1	**C_{83}**		1	
$C_{11\,1}$	1			**$C_{11\,1}$**	1					
$C_{11\,2}$	1	1		**$C_{11\,2}$**		1				
$C_{11\,3}$	1		1	**$C_{11\,3}$**	1					

Table 6. Total number of attributes integrated in $\mathrm{H_{ij}}$ and $\mathrm{U_i}$ corresponding to the information presented in Table 5

Unit	$\boldsymbol{V}^{(\mathbf{H_{i1}})}$	$\boldsymbol{V}^{(\mathbf{H_{i2}})}$	$\boldsymbol{V}^{(\mathbf{H_{i3}})}$	$\boldsymbol{V}^{(\mathbf{U_i})}$
1	7	6	5	**18**
2	4	4	0	**8**
3	3	6	3	**12**

The alignment method consists of establishing an implicit relationship between the attribute $\mathrm{C_{hk}}$ to be evaluated and the learning activity $\mathrm{H_{ij}}$ where it will be evaluated.

The teacher could select the most appropriate technique (Table 2) to evaluate the learning activity and could establish the criteria to collect the evidence (knowledge, procedure and attitude) required in our fuzzy system (Figure 4) to evaluate the efficiency with which the student developed the activity. Our method requires that the teacher indicates which competence attributes (into the alignment matrix, e.g., Table 5) are considered in each learning activity of the study program, then the educational evaluation system calculates the weights, v_{ij} and u_i.

Keep in mind that in no way will it be possible to change the weights values, v_{ij} and u_i, manually.

These values only can change when the alignment modifies. Also, it must be considered that the same competence attribute could be evaluated in one or more learning activities of one or more units.

Therefore, the student's progress, or in other words, the grade of development of the competencies acquired by the student is determined in a percentage way considering the tree forming by the resulting weight values (e.g., Figure 6) of the alignment matrix (e.g., Table 6).

2.4 Grade of Development of Acquired Competencies

The evaluation of the efficiency E_{ij} bases on the quantitative and synthetic conception of the components explaining the performance displayed by the student in the courses of a learning activity $\mathrm{H_{ij}}$.

Then, through the resulting weight values (e.g., Figure 6) of the alignment matrix (e.g., Table 6) and the efficiency E_{ij}, the educational evaluation system determines the grade of development of acquired competencies over a period within the study program. This is as follows: The grade of development of the competencies acquired by a student at the end of the learning activity $\mathrm{H_{ij}}$ is denoted by $\mathrm{D}_s^{(\mathrm{H_{ij}})}$ and calculated through:

$$\mathrm{D}_s^{(\mathrm{H_{ij}})} = E_{ij} \times v_{ij}, \tag{6}$$

where s denotes the sth student, the efficiency value, E_{ij}, is provided by the fuzzy system and v_{ij} is the weight value corresponding to the learning activity $\mathrm{H_{ij}}$. Likewise, $\mathrm{D}_s^{(\mathrm{U_i})}$ denotes the grade of development of the competencies acquired by the sth student while completing unit $\mathrm{U_i}$ and obtained by:

$$\mathrm{D}_s^{(\mathrm{U_i})} = \sum_{j=1}^{m} E_{ij} \times v_{ij} \times u_i, \tag{7}$$

where u_i is the weight value corresponding to the unit $\mathrm{U_i}$. In addition, the grade of development of the competencies acquired by the sth student while completing the study program, D_s, is obtained by:

$$\mathrm{D}_s = \sum_{i=1}^{n} \mathrm{D}_s^{(\mathrm{U_i})}. \tag{8}$$

Taking Equations (6-8) as a reference, it is possible to calculate a series of values that represent statistical parameters of the educational system that are relevant at obtaining information about students or groups.

For example, the number N^+ of students who acquired their competencies at the end of the study program with a grade of development D_s greater than P or less than P, and correspondingly the number N^- of those achieving a developmental grade smaller that P, can be calculated by:

$$\mathrm{N}^+ = \sum_{s=1}^{n_s} 1 \quad \forall\, \mathrm{D}_s > \mathrm{P} \text{ or } \mathrm{N}^- = \sum_{s=1}^{n_s} 1 \quad \forall\, \mathrm{D}_s < \mathrm{P}, \tag{9}$$

where P is a perceptual value and n_s is the number of students to be evaluated.

But if what we want is to calculate the percentage value associating to $\mathrm{N}_{\%}^+$ of N^+, or that one $\mathrm{N}_{\%}^-$ corresponding to N^-, then we can use the following expressions:

$$\mathrm{N}_{\%}^+ = \frac{\mathrm{N}^+}{n_s} \times 100 \text{ or } \mathrm{N}_{\%}^- = \frac{\mathrm{N}^-}{n_s} \times 100. \tag{10}$$

In addition, we can obtain the average percentage of the grade of development of the competencies acquired by a group is obtained through:

$$\mathrm{D} = \frac{\sum_{s=1}^{n_s} \mathbf{D}_s}{\boldsymbol{n_s}}. \tag{11}$$

Even more, it is possible to modify the Equation (9) if what we want is calculating the number $\mathrm{N_H^+}$ of students who acquired their competencies at the end of a learning activity $\mathrm{H_{ij}}$ with a grade of development greater than P, namely:

$$\mathrm{N_H^+} = \sum_{s=1}^{n_s} 1 \quad \forall\, \mathrm{D}_s^{(\mathrm{H_{ij}})} > \mathrm{P}. \tag{12}$$

Similarly, if we want is calculating the number $\mathrm{N_H^-}$ of students who acquired their competencies at the end of a learning activity $\mathrm{H_{ij}}$ with a grade of development less than P, then:

$$\mathrm{N_H^-} = \sum_{s=1}^{n_s} 1 \quad \forall\, \mathrm{D}_s^{(\mathrm{H_{ij}})} < \mathrm{P}. \tag{13}$$

Respectively. Likewise, if what we want is calculating the number $\mathrm{N_U^+}$ of students who

Table 7. Alignment matrix: attributes integrated within the learning activities that comprise the study program corresponding to the simulated data study case

	U_1				**U_2**			**U_3**		
C_{hk}	H_{11}	H_{12}	H_{13}	C_{hk}	H_{21}	H_{22}	C_{hk}	H_{31}	H_{32}	H_{33}
C_{11}		1		**C_{22}**	1		**C_{14}**	1		1
C_{16}	1			**C_{33}**		1	**C_{32}**	1		
C_{44}		1		**C_{61}**	1		**C_{53}**			1
C_{55}		1		**C_{73}**		1	**C_{95}**		1	
C_{93}	1		1	**C_{82}**		1	**$C_{10\ 3}$**	1		
$C_{11\ 1}$	1			**C_{91}**		1	**$C_{11\ 3}$**			1

Table 8. Total number of attributes integrated in the learning activities and units included in the alignment matrix presented in Table 7

Unit	$V^{(H_{i1})}$	$V^{(H_{i2})}$	$V^{(H_{i3})}$	V^{U_i}
1	3	3	1	**7**
2	2	4		**6**
3	3	1	3	**7**

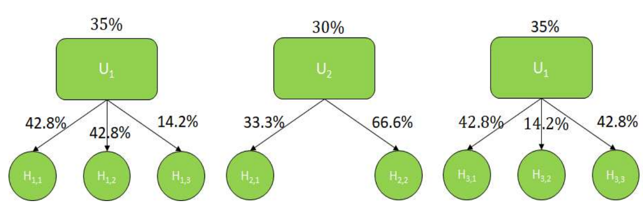

Fig. 7. Weights v_i and u_{ij} obtained through Equations (4) and (5) associating to the alignment matrix presented in Table 7

acquired their competencies at the end of the unit U_i with a grade of development greater than P, then:

$$N_U^{+} = \sum_{s=1}^{n_s} 1 \ \forall \ D_s^{(U_i)} > P, \quad (14)$$

or less than P:

$$N_U^{-} = \sum_{s=1}^{n_s} 1 \ \forall \ D_s^{(U_i)} < P, \quad (15)$$

Table 9. Simulated values $(n = 50)$ for x_1, x_2 and x_3 for each one of the $\mathrm{H_{ij}}$ learning activities corresponding to the unit $\mathrm{U_1}$

$\mathbf{U_1}$																	
H_{11} (1st to 25th)			**H_{12} (1st to 25th)**			**H_{13} (1st to 25th)**			**H_{11} (25th to 50th)**			**H_{12} (25th to 50th)**			**H_{13} (25th to 50th)**		
x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
9.49	3.45	5.92	2.51	9.99	8.01	4.29	8.43	8.38	9.68	2.54	6.92	2.60	9.62	8.36	3.67	6.55	8.03
9.72	3.21	7.70	2.82	9.56	9.44	3.62	7.87	7.75	9.70	3.01	6.89	2.84	9.25	9.70	4.13	7.98	8.30
9.61	2.63	6.42	2.53	9.60	9.41	4.01	7.35	8.42	9.90	2.59	7.76	2.92	8.72	9.06	4.14	7.48	8.01
9.46	2.83	6.97	2.29	8.60	9.07	4.22	8.12	8.43	9.02	2.87	7.47	2.88	7.78	8.42	3.89	7.52	9.46
9.83	2.53	7.68	2.56	7.90	9.93	4.97	6.17	9.86	9.35	2.45	6.57	2.99	9.55	9.75	4.96	8.37	8.13
9.67	2.71	7.12	2.30	8.44	9.29	3.73	6.54	9.31	9.44	2.36	6.05	2.53	9.78	9.93	3.97	6.33	9.82
9.86	2.88	7.83	2.99	9.16	9.63	4.58	7.37	9.68	9.59	2.66	7.99	2.71	8.93	9.72	4.05	7.97	9.72
9.81	2.65	6.72	2.79	8.80	8.58	3.99	7.92	9.87	9.63	2.50	7.37	2.88	8.33	10.0	4.35	7.43	8.97
9.33	2.92	7.28	2.55	9.80	9.58	4.08	7.58	8.31	9.52	2.30	6.78	2.67	8.36	9.00	3.86	7.76	8.25
9.56	2.79	7.28	2.75	9.91	9.33	4.09	7.01	8.03	9.30	2.83	6.98	2.56	9.54	10.8	4.00	7.37	8.39
9.02	2.27	6.46	2.72	8.91	9.25	4.94	6.73	9.52	9.81	2.47	7.60	2.66	8.50	9.93	4.85	6.80	8.24
9.26	3.15	6.97	2.78	9.50	9.76	3.98	8.05	9.09	9.98	2.67	7.29	2.75	10.0	9.37	3.66	6.90	8.23
9.49	2.52	7.54	2.43	8.77	9.23	3.62	8.90	8.69	9.44	2.88	7.43	2.54	9.17	10.0	4.30	8.09	10.0
9.92	3.01	6.70	2.42	9.37	9.70	3.55	6.39	8.53	9.75	2.67	6.53	2.60	9.68	10.0	3.79	6.29	8.61
9.02	2.83	6.34	2.46	8.79	9.91	3.64	6.67	8.28	10.0	2.63	7.89	2.64	10.0	8.99	4.15	7.23	8.84
9.83	2.87	7.31	2.59	8.80	9.67	4.09	7.28	8.65	9.70	2.89	7.05	2.75	8.58	10.0	3.74	6.90	9.03
9.51	3.16	7.43	2.63	7.80	8.63	3.60	8.29	9.15	9.67	2.74	7.77	2.82	8.89	9.69	4.03	7.57	8.60
9.75	2.67	7.43	2.60	9.58	9.94	4.27	6.42	9.44	9.46	2.44	7.79	2.42	8.43	9.21	3.83	6.61	8.72
9.33	2.61	7.52	2.61	9.07	9.70	3.76	7.46	8.95	9.97	2.39	7.71	2.20	9.54	10.0	4.09	7.32	7.66
9.07	2.40	7.86	2.91	8.25	9.51	3.61	8.58	7.74	9.34	3.36	6.54	2.84	9.54	10.0	4.47	6.25	9.96
9.09	2.36	6.76	2.75	7.77	9.57	4.42	6.39	7.98	9.72	2.82	6.65	3.36	7.97	10.0	3.59	6.84	9.62
9.62	2.86	7.61	2.50	7.90	9.69	4.38	7.87	9.26	9.22	2.74	7.65	3.09	9.32	9.44	3.87	6.48	8.35
9.30	2.33	7.06	2.36	8.43	9.29	4.20	7.61	8.63	9.15	2.59	7.91	2.71	8.56	10.0	4.03	7.58	9.17
9.15	2.37	6.86	2.66	8.28	8.85	4.62	7.33	8.91	9.25	2.50	7.44	2.57	8.27	10.0	4.48	8.11	9.10
9.95	2.80	6.72	2.76	8.20	9.70	4.10	7.14	9.48	10.0	3.00	7.35	2.48	8.61	9.26	4.45	7.12	10.0

3 Results

3.1 Study Case Based on Simulated Data

To illustrate the performance of offered evaluation scheme, we first consider a try based on a simulated data set. We conceive a hypothetical group of 50 students, assumed to have enrolled in a study program structured in three units: unit $\mathrm{U_1}$ involving learning activities $\mathrm{H_{1j}}$ for $j = 1,2,3$, unit $\mathrm{U_2}$ composing learning activities $\mathrm{H_{2j}}$ for $j = 1,2$ and $\mathrm{U_3}$ with $\mathrm{H_{3j}}$ for $j = 1,2,3$ learning activities.

Following the procedure in Section 2.3, we create the associating alignment matrix, but first,

Table 10. Simulated values $(n = 50)$ for x_1, x_2 and x_3 for each one of the $\mathrm{H_{ij}}$ learning activities corresponding to the unit $\mathrm{U_2}$

$\mathrm{U_2}$											
$\mathrm{H_{21}}$ (1st to 25th)			$\mathrm{H_{22}}$ (1st to 25th)			$\mathrm{H_{21}}$ (25th to 50th)			$\mathrm{H_{22}}$ (25th to 50th)		
x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
2.68	4.57	9.68	5.53	7.72	3.23	3.24	4.78	8.33	4.95	8.20	2.88
2.64	5.33	9.60	5.18	7.41	2.75	2.94	5.16	9.50	4.55	8.03	2.77
2.68	4.34	9.87	4.68	7.59	2.50	2.88	5.06	10.0	5.47	8.46	3.20
2.28	4.63	9.48	4.92	8.49	2.57	2.61	4.31	10.0	5.04	8.04	2.87
2.78	4.77	9.20	4.25	8.08	2.84	3.26	5.22	10.0	5.50	7.29	3.17
2.66	5.64	9.74	6.02	7.29	2.49	2.96	5.60	8.98	5.44	8.12	2.49
3.06	4.52	9.17	4.91	7.66	3.09	2.48	4.77	10.0	5.40	8.87	3.03
2.42	4.62	8.99	5.12	8.36	2.62	2.36	6.07	11.78	5.55	8.63	3.03
2.78	4.36	9.18	5.73	7.88	2.57	2.42	5.08	10.0	4.65	8.28	2.69
2.60	4.89	9.49	4.31	8.60	2.81	2.86	5.22	10.0	5.52	8.54	3.26
2.65	4.78	9.65	5.01	7.05	2.28	2.37	5.45	10.0	4.97	8.35	2.86
3.03	4.95	8.68	4.84	8.34	3.36	2.55	5.24	8.65	5.35	7.94	2.34
2.77	4.42	9.97	4.74	8.57	2.84	2.38	5.21	10.0	4.86	7.43	2.35
2.67	4.91	9.41	4.70	8.13	3.04	2.79	5.78	9.80	5.24	7.68	2.70
3.04	5.70	8.76	5.90	8.74	2.64	2.46	4.64	10.0	5.27	9.85	2.70
2.64	5.95	9.59	5.77	8.35	2.71	2.66	3.86	9.47	5.27	7.02	2.90
2.70	5.19	8.20	4.99	8.95	3.29	2.52	4.60	10.0	4.86	9.15	2.87
2.82	4.47	9.44	5.25	7.42	2.87	3.58	4.74	8.37	5.17	8.02	2.90
2.82	5.36	9.93	4.78	7.84	2.54	3.11	5.76	10.0	4.59	8.81	2.34
2.83	5.00	8.78	4.18	8.56	3.24	2.98	5.00	8.83	4.66	9.86	3.20
2.65	4.99	8.89	4.90	8.41	2.17	2.95	5.53	9.95	4.82	8.33	2.61
2.95	5.08	9.96	4.82	8.65	2.89	2.32	4.83	10.0	5.46	8.34	2.89
3.12	5.52	9.50	4.87	7.68	2.72	2.93	4.69	9.09	4.66	8.03	2.64
2.94	4.27	8.61	4.19	8.48	3.14	2.84	5.38	8.41	5.35	9.18	2.75
2.74	6.62	9.96	5.70	8.72	2.50	2.87	4.88	10.0	4.82	9.03	2.72

we achieve this task randomly, on its course, indicating the $\mathrm{C_{hk}}$ attributes integrated within the learning activity $\mathrm{H_{ij}}$ (Table 7). As we mentioned, the teacher ought to achieve the alignment systematically, thereby only in extreme cases could they only align the competencies at random. However, the analysis of random alignment requires case attains relevance because it stands for the worst expected scenario for an inconsistent evaluation. In this sense, we can explore the suitability of the proposed method even in extreme cases of subjective alignment.

According with the alignment matrix presented in Table 7, we obtained the total number of attributes $\mathrm{C_{hk}}$ integrated in $\mathrm{H_{ij}}$ and U_i through Equations (1) and (2), respectively (Table 8).

Table 11. Simulated values $(n = 50)$ for x_1, x_2 and x_3 for each one of the $\mathrm{H_{ij}}$ learning activities corresponding to the unit $\mathrm{U_3}$.

U₃																	
H₃₁ (1st to 25th)			**H₃₂ (1st to 25th)**			**H₃₃ (1st to 25th)**			**H₃₁ (25th to 50th)**			**H₃₂ (25th to 50th)**			**H₃₃ (25th to 50th)**		
x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
2.50	3.99	6.81	6.74	5.92	9.95	9.15	7.05	5.43	2.85	3.74	6.62	6.40	6.72	9.78	8.90	7.99	4.98
2.58	4.12	5.50	7.42	5.92	8.80	9.86	7.78	4.67	2.84	3.72	6.91	6.39	6.01	9.17	8.51	6.49	5.69
2.60	4.54	6.28	6.49	5.30	9.16	9.15	6.58	5.17	2.93	4.64	6.53	7.90	6.53	9.96	9.90	7.68	5.35
2.86	3.68	5.09	7.20	5.95	9.37	8.47	7.55	5.39	2.76	4.85	5.18	7.37	5.90	9.93	8.71	7.39	4.87
2.55	3.75	5.93	7.81	6.01	9.94	9.36	7.59	4.40	2.30	4.05	5.86	7.90	6.21	10.0	8.89	7.43	5.37
2.82	3.66	6.22	7.63	5.53	8.74	9.96	6.70	4.73	2.92	3.62	5.75	7.25	5.64	8.92	9.31	6.19	5.29
2.66	4.51	5.39	7.24	5.72	8.97	9.45	6.94	5.96	2.77	3.67	5.15	6.25	5.57	10.0	9.87	7.64	4.90
2.34	4.48	5.88	7.18	5.45	8.70	8.85	6.75	4.33	2.56	4.25	6.20	6.32	5.09	9.22	9.62	7.51	5.80
2.37	4.52	5.58	7.83	5.24	9.70	9.73	7.57	4.29	2.39	4.41	6.18	6.90	5.51	8.92	8.59	6.32	5.10
2.48	3.81	6.56	6.73	6.28	9.58	9.90	7.13	4.56	2.86	3.87	6.66	6.65	5.86	10.0	8.75	7.41	4.21
2.72	4.26	5.65	6.44	6.24	9.18	8.23	6.65	4.32	2.83	4.50	5.65	7.51	6.11	9.10	9.27	7.32	4.58
2.70	4.32	6.00	6.78	6.05	8.73	8.59	8.00	4.30	2.65	4.51	6.25	7.25	5.47	9.57	9.46	6.65	5.21
2.46	4.54	6.04	7.37	6.48	8.77	9.87	6.42	4.84	2.50	3.78	5.74	6.89	5.81	9.76	9.48	7.47	4.65
2.64	4.31	5.72	7.48	5.19	9.54	8.13	7.10	5.78	2.93	3.75	6.14	8.70	6.98	10.0	9.10	6.70	4.49
2.75	3.89	5.58	6.52	6.13	9.25	8.97	7.28	5.64	2.79	3.82	6.13	6.33	6.53	10.0	9.53	7.68	4.35
2.70	4.17	5.57	7.38	6.31	8.07	9.49	6.05	5.17	2.91	4.19	6.14	7.27	5.97	8.10	9.91	7.83	4.75
2.33	4.09	6.18	7.53	6.16	9.05	8.76	7.14	5.38	2.29	4.17	6.18	7.26	6.09	9.29	7.95	8.15	5.24
2.68	4.13	6.83	7.23	5.61	8.90	9.11	7.61	5.32	2.57	3.87	6.80	6.88	6.15	10.0	9.67	8.32	5.68
2.93	3.65	5.50	7.38	5.99	9.64	9.51	7.28	5.43	2.37	4.90	5.94	7.65	6.80	9.87	8.37	7.96	4.88
2.66	4.25	5.45	7.88	6.52	9.41	9.11	7.67	5.73	2.43	4.16	6.22	6.93	6.36	10.0	8.59	6.70	4.21
2.68	4.85	6.56	6.33	5.31	9.86	9.53	7.89	4.92	2.61	4.81	6.05	7.41	6.66	9.20	9.73	8.31	4.41
2.45	4.62	5.83	5.85	6.38	9.58	8.14	6.68	5.29	2.73	4.85	5.44	8.55	5.68	10.0	9.34	9.40	5.44
2.70	3.87	5.68	7.39	6.69	8.50	9.14	7.14	4.79	3.00	4.92	6.48	6.13	5.93	10.0	9.10	8.56	5.39
2.34	3.98	6.35	6.28	6.53	9.35	9.27	7.69	4.38	2.63	5.01	6.11	9.07	5.40	9.97	9.52	7.01	4.81
2.64	4.15	5.92	6.99	6.02	9.38	9.00	6.53	4.74	3.32	3.22	6.64	8.20	6.35	9.98	10.0	6.10	4.04

In turn, weight values v_{ij} and u_i calculated through Equations (4) and (5), respectively (Figure 7). Also, we considered simulated values for x_1, x_2 and x_3 generated so that the highest concentrations of values distributed around a given mean (Tables 9, 10 and 11).

Figure 8 shows the distributions of the x_1, x_2 and x_3 values fitted to a non-normal distribution curve. In Figure 8 domed lines differentiate the interval where the distributions of the input values x_1, x_2 and x_3 concentrates for each learning activity $\mathrm{H_{ij}}$.

Shown concentration intervals help the teacher visualizing the possible effects of the combinations of x_1, x_2 and x_3 values over the performance of the students during the development of their activities.

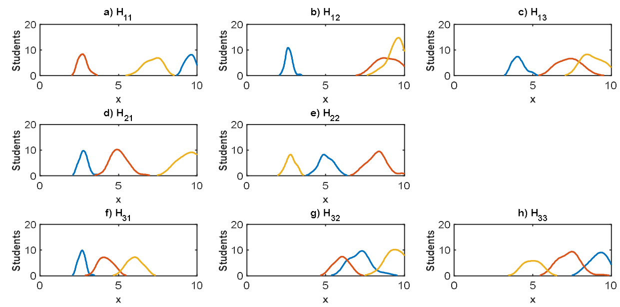

Fig. 8. Distribution of the x_1, x_2 and x_3 simulated inputs values. x_1 corresponds to $\text{knowledge} = \mathrm{X}_1$ variable (blue lines), x_2 corresponds to the $\text{procedure} = \mathrm{X}_2$ variable (red lines) and x_3 corresponds to the $\text{attitude} = \mathrm{X}_3$ variable (yellow lines)

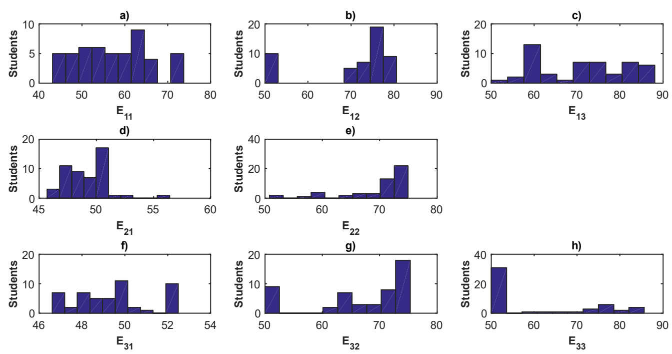

Fig. 9. Distribution of the E_{ij} evaluated by the fuzzy inference system (Figure 4) taking as an input the 50 simulated values presented in Tables 9, 10 and 11

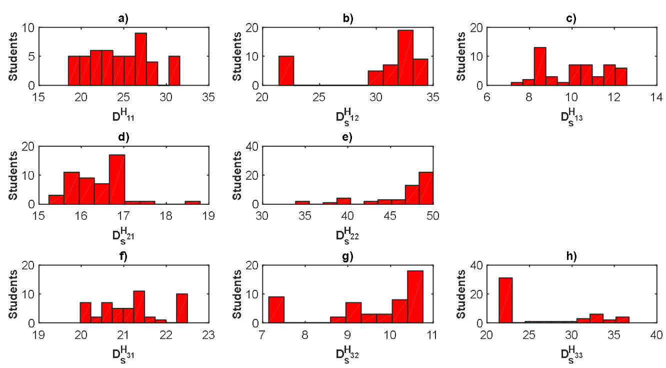

Fig. 10. Distribution of the $\mathrm{D}_s^{(\mathrm{H_{ij}})}$ grade of development of the competences acquired by the students in the learning activity $\mathrm{H_{ij}}$.

For example, we can observe in Figure 8b that in all cases knowledge rated insufficient, however, an attitude with very high values in x_2 associates to a completed procedure. Contrary to Figure 8b, it is possible to observe in Figure 8a a non-attempted procedure, despite high values in attitude and

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knowledge. In this case, the teacher can analyze and determine, if the technique and the teaching-learning strategy are adequate to evaluate the efficiency.

An efficiency value E_{ij} corresponding to each learning activity $\mathrm{H_{ij}}$ calculates from running our fuzzy inference system (Figure 4).

In performing such a task, input values are those as simulated for the hypothetical group of 50 students (Tables 9, 10 and 11). We show resulting E_{ij} values in Figure 9.

Figure 9 provides the teacher a clue on the performance of the students in each one of the learning activities $\mathrm{H_{ij}}$. For example, if most students obtained values of E_{ij} greater that 50% then this would mean that knowledge was at least sufficient, that the procedure was attempted in half of the associated learning activities associated, and that the attitude was more positive than negative.

But, in our hypothetical study case, there is a lot of variability in the distribution of the E_{ij} values presented in Figure 9. However, it is possible to identify the values x_1, x_2 and x_3 belonging to those students who had the lowest E_{ij} ratings, which allows to perform analytics and conceiving better teaching strategies aimed to improve learning.

Equation (6) allows calculation of $\mathrm{D}_s^{(\mathrm{H_{ij}})}$, interpreting as the grade of development of the competencies acquired by the sth student when performing the learning activity $\mathrm{H_{ij}}$, (Figure 10). Acquiring frequency distribution plots of resulting numbers permits visualizing the behavior of the grade of development of competencies.

Similarly, Equation (7) endures calculation of $\mathrm{D}_s^{(\mathrm{U_i})}$ standing for the grade of development of the competencies acquired by the qth student in unit $\mathrm{U_i}$, (Figure 11).

Additionally, D_s, identifying the grade of development of the competencies acquired by the sth student at the end of the study program, calculates by means of Equation (8) (Figure 12). Then, adding the results at the individual level, we obtain the grade of development of the competencies acquired by the student's group at the end of the study program.

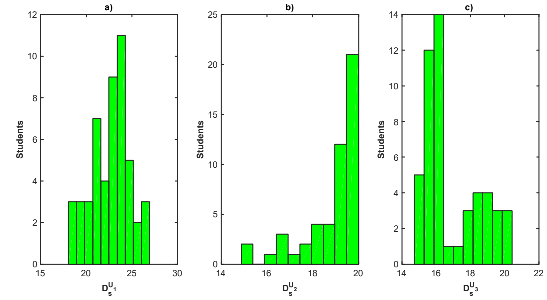

Fig. 11. Distribution of the $\mathrm{D}_s^{(\mathrm{U_i})}$ grade of development of the competences acquired by the students in unit $\mathrm{U_i}$

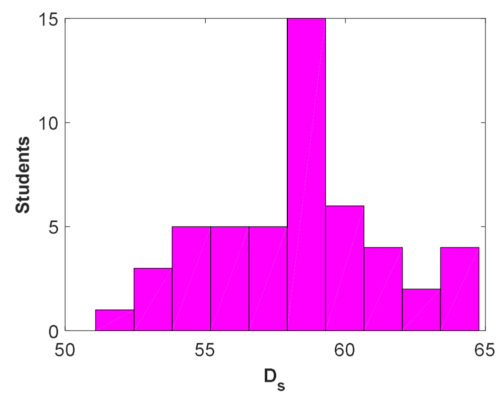

Fig. 12. Distribution of the $\mathrm{D_s}$ grade of development of the competences acquired by all students at the end of the study program

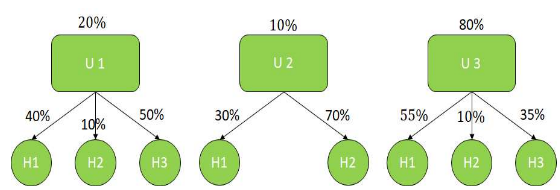

Fig. 13. Hypothetical weights v_i and u_{ij}

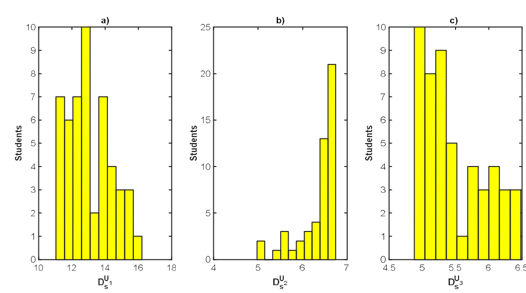

Fig. 14. Distribution of the $\mathrm{D}_s^{(\mathrm{U_i})}$ grade of development of the competences acquired by the students at the end to the unit U_i

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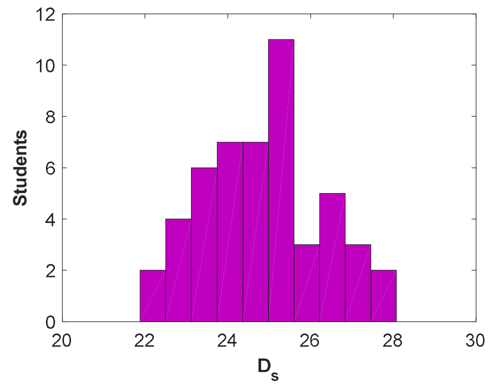

Table 12. Scores of students enrolling the subject “Operating Systems Management” presented as image in [12]

				Units	Unit 1									Unidad 2					
				Results of learning	RA 1				RA 2					RA 1			RA 2		
				Activities to evaluate	AE 1 (20%)				AE 1 (20%)					AE 1 (25%)			AE 1 (35%)		
				Indicators	1	2	3	4	1	2	3	4	5	1	2	3	1	2	3
%	1.1.1	1.2.1	2.1.1	2.2.1	30%	30%	35%	5%	20%	20%	20%	20%	20%	20%	20%	60%	20%	30%	50%
86.88	73.75	100	100	77.5	E	E	I	E	E	E	E	E	E	E	E	E	E	I	E
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
93.7	100	91	82	100	E	E	E	E	E	E	E	S	S	S	E	S	E	E	E
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
77.16	98.88	100	71.5	55.75	E	E	E	S	E	E	E	E	E	I	E	S	E	S	I
30.25	51.25	25	25	25	E	I	I	E	I	I	I	I	I	I	I	I	I	I	I
28.68	43.38	25	25	25	S	I	I	S	I	I	I	I	I	I	I	I	I	I	I
48.33	85.38	55	25	40	E	S	S	E	E	E	I	I	I	I	I	I	E	I	I
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
57.93	67	82	77.5	25	E	S	I	E	S	E	S	S	S	S	S	S	I	I	I
62.58	92.13	67	67	40	E	E	S	E	I	S	S	S	S	S	I	S	E	I	I
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
96.63	100	100	86.5	100	E	E	E	E	E	E	E	E	E	E	E	S	E	E	E
97.53	100	95.5	100	95.5	E	E	E	E	E	E	E	S	E	E	E	E	S	E	E
65.5	100	100	46	40	E	E	E	E	E	E	E	E	E	S	S	I	E	I	I
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
94.83	100	91	86.5	100	E	E	E	E	E	E	E	S	S	E	E	S	E	E	E
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
96.63	100	100	86.5	100	E	E	E	E	E	E	E	E	E	E	E	S	E	E	E
99.1	100	95.5	100	100	E	E	E	E	E	E	E	S	E	E	E	E	E	E	E
86.88	100	100	100	62.5	E	E	E	E	E	E	E	E	E	E	E	E	E	E	I
100	100	100	100	100	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E

Fig. 15. Distribution of the D_s grade of development of the competences acquired by all students at the end of the study program

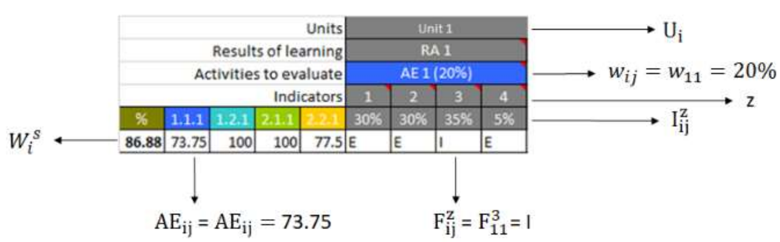

Fig. 16. Symbolic representation of the elements in Table 12

Figure 10b shows the distribution of the grade of development of competencies concentrated in an interval with values very close to the maximum weight value corresponding to the learning activity

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H_{12}. Only the grade of a few students places outside this range. Such a result implies that they all performed their activities with the best attitude but with little knowledge and with the procedure attempted most of the time (Figure 8b). All students completed the whole procedure due to the coincidence of a positive attitude that favored obstacle elimination, in this case the barely small knowledge, and which also promoted pursuing objectives.

Otherwise, in Figure 10h the distribution of the grade of development of competencies is concentrated in an interval with values above the mean of the maximum weight of corresponding to activity H_{33} (Figure 7), only the score of a few students is outside this range. This means that despite knowledge being predominantly sufficient, most of the students did not totally take on the procedure because the attitude was oscillating between the negative and the positive (Figure 8h).

There is a transition point between a negative and positive attitude, called a neutral attitude; it is not very common. Our fuzzy inference system characterizes the attitude with a small interval that defines the transition from negative to positive. The neutral attitude renders the subject's action ineffective, causing a loss of the notion of time or continuity from the left-off point.

The teacher can analyze the results of Figures 8-10 and being aware at detail what is happening with the elements that rate measure the performance of students while achieving the learning activities.

The teacher could identify problems in assessment techniques or at teaching-learning strategies with the advantage of modifying or changing them to achieve other objectives during the learning sequence. It is necessary to clarify that the results presented in Figure 10 and Figure 11 do not determine whether the students are competent or not. At the end of the study program, it is possible to assert know to what grade a student is competent or not competent (Figure 12).

In addition, it is possible to ascertain the competences that a student or a group developed through Table 7.

It is also possible to learn from Figure 12, that the maximum grade of development of the competences acquired by the students was less than 65%. Only 13 students who represent 26% of the total number in the group completed learning activities with a grade grater that 60% but less than 65%. A 75% of the total number of students developed the learning activities with an insufficient score, that is, with a grade of achievement lower than 60%. The conclusion is that the study program had many deficiencies in teaching-learning strategies around the development of learning activities and possibly in evaluation techniques. It would be necessary to review, what was what happened? This question can be only answered once the study program completes.

Moreover, it is recommended to answer this question after having completed each one of the learning activities. The average grade of development of the competences acquired by the group was 58.37% (Equation (9)), which is too low. Recall that this study case considers simulated input values, it is obvious for other different input values the average grade of development of the competencies acquired by the group will change. It could be higher or lower than that obtained in this test.

In addition, if the competencies are aligned with the learning activities in a different way to that presented in Table 7 then the results of alignment presented in Table 8 would attain other values, so the weights shown in Figure 7 would modify, this in turn will produce a different final result than presented in Figures 10, 11 and 12. In order to explain these effects, we consider different weights to those presented in Figure 7, which are hypothetically generated, that is, these weights are not associated with an alignment matrix but we assume that they should be (Figure 14).

Then, as described, we can obtain the grades of development of competencies. The task completes through Equation (6), Equation (7), and Equation (8) by using hypothetical weights (Figure 12) and simulated input data (Tables 9, 10, and 11). Then we proceed to compare them with those shown in Figure 11 and Figure 12 through their distribution in frequency graphs (see Figure 14 and Figure 15, respectively). This proves that whenever the weights v_{ij} and u_{ij} change the results also change. Furthermore, we obtained the average grade of development of the acquired group's competencies using Equation (11). We found a value of 24.88%, which turns out to be much lower than a calculated one of 58.34% obtained for the simulated data study case. Recall that the simulated data is generated randomly, thus precluding any significance or relationship either with the student's behavior or with the learning activities.

Equation (12) allowed calculation of the number $\mathrm{N_H^+}$ of students who acquired their competencies at the end of a learning activity $\mathrm{H_{13}}$ with a grade of development P greater than 60%, that is $\mathrm{P} > 60\%$ (30% of u_{13}).

If we take the D_s values corresponding to the study case based on simulated data, we obtain $\mathrm{N_H^+} = 40$, which typify 80% of n_s.

Likewise, if we aimed at calculating the number $\mathrm{N_H^+}$ of students who acquired their competencies at the end of a learning activity $\mathrm{H_{13}}$ with a grade of development P greater than 60%, that is, $\mathrm{P} > 60\%$ (8.52% of u_{13}), but taking D_s values corresponding to the hypothetical study case data, we would have obtained $\mathrm{N_H^+} = 38$, which represent 76% of n_s. In both the simulated data case and the hypothetical study cases, the $\mathrm{N_H^+}$ values are different.

Correspondingly, if the goal is obtaining the value of N_H^- with a grade of development P less than 60%, that is $\mathrm{P} < 60\%$, then the result will be the complement of the value of $\mathrm{N_H^+}$ obtained in both cases, but it is also possible to obtain N_H^- through Equation (13). This exercise can be illustrative whenever a comparison of the grade of development among learning activities $\mathrm{H_{ij}}$ of the same $\mathrm{U_i}$ requires. Also, if the aim is to make comparisons among the grades of development at the end of the units the task could be achieved through using D and Equations (14) and (15). In addition, to know the grade of development of the competencies acquired, it is mandatory knowing which competences the student developed. This is only possible through obtaining the alignment matrix.

3.2 Study Case Based on Real Data

In the addressed education evaluation system, a study program can compose several units (Unit), and each one can associate several learning outcomes (RA) associated with their corresponding learning activity (AE). However, it is possible that each learning activity (AE) be divided into one or more sub-activities called “indicators”. Adapting data from Romero-Escobar in [34], we can get a feel for an experimental study case.

The reference includes the results obtained in the study program corresponding to the subject "Operating Systems Management", which is pre-defined by the educational system in two units that comprise two learning outcomes (RA), and each one of them associating two activities to evaluate (AE) with 4 indicators each (Table 12). Percentage value of indicators is also pre-defined by the educational system.

In the Table 12, the scores of the 27 enrolled students identify each one by means of a letter. The letter 'E' stands for 'Competent', the letter 'S' signifies 'Sufficient' and the letter 'I' connotes 'Not Competent'. The official grading system does not consider other values for the score of the students. The meaning of 1.1.1 is: Unit 1, Result of learning 1 (RA 1) and Activity to evaluate 1 (AE 1), respectively. In addition, the report by [12], the evaluation of the activities (AE) was not carried out in such a way that the adopted techniques could be considered as a factor inducing evidence of the display of the indicators considered in the present study (knowledge, procedure, attitude).

In order to explain how the current education evaluation scheme works, we present in Figure 16 the symbolic representation of the elements of Table 12 describes as follows: the symbol $\mathrm{U_i}$, for $i = 1 \dots n$, stands for the ith unit, using $\mathrm{AE_{ij}}$, for $j = 1 \dots m$, we denote the jth activity of the unit $\mathrm{U_i}$ to evaluated, the symbol $\mathrm{F_{ij}^z}$, for $z = 1,2,3, \dots,$ will stands for the zth indicator of the jth activity of the unit $\mathrm{U_i}$ to be evaluated, w_{ij} standing for the weight of the activity $\mathrm{AE_{ij}}$, and finally, $\mathrm{I_{ij}^z}$ taken to symbolize the weight corresponds to $\mathrm{F_{ij}^z}$. W_i^s is the percentage value corresponding to $\mathrm{U_i}$

Using the listed notation convention, we go ahead and furnish the mathematical expressions that the education evaluation system takes into account at obtaining the results of Table 12. Alongside this, we elucidate what’s wrong with this evaluation scheme. Such a protocol assigns each evaluation result $\mathrm{AE_{ij}}$ of an activity of the sth student as follows:

$$\mathrm{AE_{ij}} = \sum_{z=1}^{n_z} G, \qquad (16)$$

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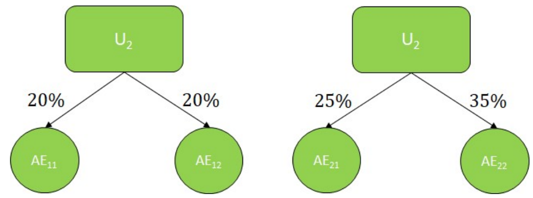

Fig. 17. Weight w_{ij} taken from Figure 16 corresponding to each activity $\mathrm{AE_{ij}}$

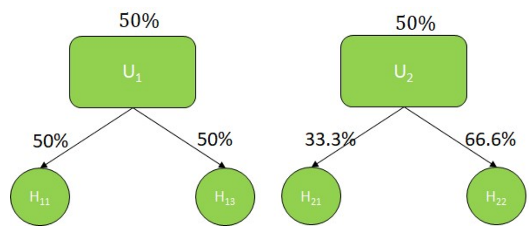

Fig. 18. Weight values, v_{ij} and u_i, obtained by Equations (4) and (5) corresponding to the alignment matrix presented in Table 13

Table 13. Alignment matrix: attributes as they integrate in the learning activities corresponding to the simulation study case

	U₁			**U₂**	
$\mathbf{C_{hk}}$	H11	H12	$\mathbf{C_{hk}}$	H21	H22
C11		1	**C22**	1	
C16	1		**C33**		1
C44		1	**C66**	1	
C55		1	**C73**		1
C93	1		**C82**		1
C11 1	1		**C91**		1

Table 14. Total number of attributes integrated in the leaning activities and units comprised in the alignment matrix presented in Table 13

Unidad	$V^{(\mathrm{H}_{i1})}$	$V^{(\mathrm{H}_{i2})}$	V^{U_i}
1	3	3	**6**
2	2	4	**6**

where $G = \begin{cases} \mathrm{I_{ij}^Z} & if \quad \mathrm{F_{ij}^Z} = \mathrm{E}, \\ 0.775 \times \mathrm{I_{ij}^Z} & if \quad \mathrm{F_{ij}^Z} = \mathrm{S}, \\ 0.250 \times \mathrm{I_{ij}^Z} & if \quad \mathrm{F_{ij}^Z} = \mathrm{I}, \end{cases}$

and where n_z is the total number of indicators in $\mathrm{AE_{ij}}$. In the Equation (16), we can be aware that the scheme of evaluation of the educational system considers only three qualitative values as rating, I, S, and E, the students who had a rating equal to 'I' will have only 25% of the value of $\mathrm{I_{ij}^Z}$.

Likewise, the students who had a rating equal to 'S' will credit for only 77.5% of the value of $\mathrm{I_{ij}^Z}$. It should be noted that the percentage represented by $\mathrm{I_{ij}^Z}$ determines by the teachers based on their own criteria and can be arbitrarily modified during the subject's program, if the teacher so wishes. In the other hand, If the teacher does not divide the activity to be evaluated (AE) into indicators then the official scheme evaluates the activity $\mathrm{AE_{ij}}$ as follows:

$$\mathrm{AE_{ij}} = \begin{cases} w_{ij} & if \quad \mathrm{F_{ij}} = \mathrm{E}, \\ 0.775 \times w_{ij} & if \quad \mathrm{F_{ij}} = \mathrm{S}, \\ 0.250 \times w_{ij} & if \quad \mathrm{F_{ij}} = \mathrm{I}, \end{cases} \quad (17)$$

where the $\mathrm{F_{ij}}$ value corresponding to rating to the activity $\mathrm{AE_{ij}}$. For the unit, W_i^s, the educational system evaluates the sth student as follows:

$$W_i^s = \left\{\sum_{j=1}^{m} \mathrm{AE_{ij}}\right\} \times w_{ij}. \quad (18)$$

The final evaluation, W^s, of the sth student is given by:

$$W^s = \sum_{i=1}^{n} W_i^s, \quad (19)$$

and the average rating of the group is obtained by:

$$W = \frac{\sum_{s=1}^{27} W^s}{27 \times 100}, \quad (20)$$

which is $W = 8.62$. Romero-Escobar [12] did not consider a formal relationship between competencies and learning activities to create the alignment matrix. Therefore, in contrast to the entries presented in Table 6, it is not possible to obtain the total number of attributes integrated into

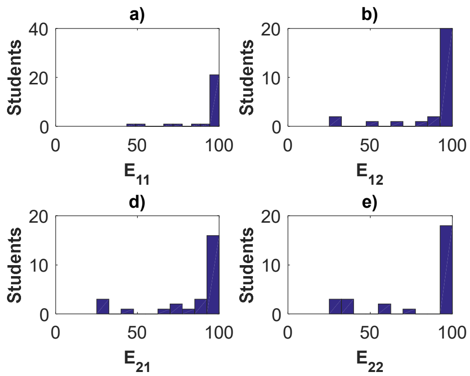

Fig. 19. Distribution of the scores of 27 students as presented in Table 12

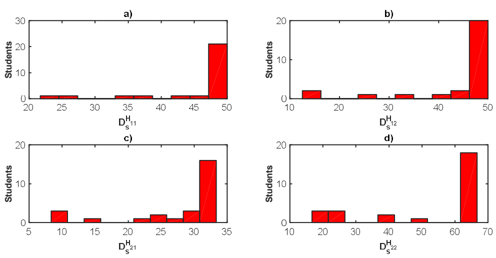

Fig. 20. Distribution of the grade of development of the competencies acquired by the sth student in the learning activity $\mathrm{H_{ij}}$

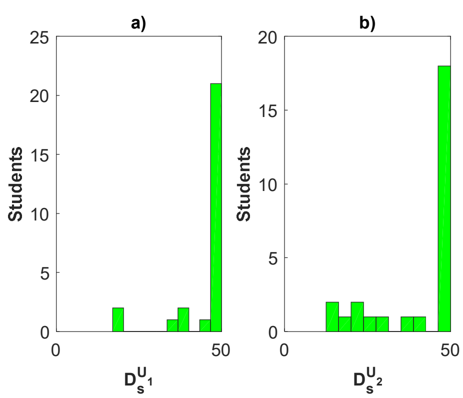

Fig. 21. Distribution of the $\mathrm{D}_s^{(\mathrm{U}_i)}$ grade of development of the competencies acquired by the students when ending all the learning activities of unit U_i

each activity to evaluate $\mathrm{AE_{ij}}$. Neither could we include the weights corresponding to each activity $\mathrm{AE_{ij}}$ to as shown in Figure 6. Besides, this restriction extends to the values of each unit $\mathrm{U_i}$. However, we can form a tree like diagram such as that appearing in Figure 6, provided we use as weights w_{ij} the ones taken from Figure 16.

The weights shown in Figure 17 are not the result of an alignment of competencies with the activities $\mathrm{AE_{ij}}$ to be evaluated. The official educational grading scheme relies on Equations (18) and (19) to obtain the rating after completing a unit and once the end the study program achieves (Figure 16), respectively. In other words, the rating of the unit W_i^s is the sum of ratings corresponding to each activity $\mathrm{AE_{ij}}$ and the W^s final rating is the sum of ratings corresponding to each unit W_i^s.

To acquire the comportments of the $\mathrm{AE_{ij}}$ values associating with the real data study case, we consider the E_{ij} efficiencies at which the students performed the learning activity $\mathrm{H_{ij}}$, as pertaining proxy values, that is, we take $E_{ij} = \mathrm{AE_{ij}}$. Recall that our method estimates E_{ij} values using a fuzzy inference system (Figure 4). Besides, since for the real data study case an alignment matrix is not available, we take the one corresponding to the simulated data study case as a possible surrogate (see Table 8). Given that, we only consider the first and second unit, both with the first and second corresponding activity $\mathrm{H_{ij}}$. Similarly, while analyzing the study case based on simulated data, the possibility of a random alignment was considered. It corresponds to the utmost subjective scenario compared to a consistent evaluation, so for this essay, taking as a base the random alignment associating with the case mentioned above renders convenient.

We calculate the total number of attributes integrated in the learning activity $\mathrm{H_{ij}}$ through Equation (1) and the total number of attributes integrated into U_i through Equation (2) (Table 14). And its weights v_{ij} and u_i obtained through Equations (4) and (5), respectively (Figure 18).

Let's show in Figure 19 the distribution of E_{ij} presented in a frequency graph. In panel (a) the distribution is concentrated in the highest value, 100%, and only three students presented evaluations lower than 100%. This means that the students performed the learning activity $\mathrm{H_{11}}$ skillfully and that the evaluation technique and the teaching-learning strategies were correct. In addition, more than 71% of the students developed their skills with a grade equal to 100% in the learning activities H_{12}, H_{21} and H_{22}. In general, evaluation techniques and teaching-learning strategies were the most appropriate. Values of E_{ij} less than 77% correspond only to 11% of the total number of students.

Considering the weight values presented in Figure 18, we calculated through Equation (6) the grade $\mathrm{D}_s^{(\mathrm{H_{ij}})}$ of development of the competencies acquired by the sth student, while achieving the learning activity $\mathrm{H_{ij}}$ (Figure 20).

In all the learning activities presented in Figure 20, a 71% of the total number of students developed their competencies with a grade equal to 100%. Furthermore, Figure 20 shows that an 18% of the total number of students developed the competencies with a grade lower than 100% but higher than 70%. A quite significant result for the educational system and for the curricular achievement of the teacher. If the evaluation techniques in [12] had considered the elements to evaluate presented in the simulated data study case, then, it could be said, that almost all the students acquired all the knowledge, that the procedures were achieved and that the attitude of the students was always positive throughout the study program. The results could always be biased towards the maximum grade of development using the qualifications of any of the groups or subjects presented in [12], due to the evaluation criteria that the educational system manages.

To calculate $\mathrm{D}_s^{(\mathrm{U}_i)}$ the grade of development of the competencies acquired by the sth student when ending all the learning activities of unit U_i, we use Equation (7) and the weight values presented in Figure 18 (see Figure 21).

In the same way, we obtain the $\mathrm{D_s}$ grade of development of competencies acquired by students throughout the study program (Figure 22) through the Equation (8).

Finally, we obtain the average percentage of the grade of development of the competencies acquired by the group through Equation (11), which is $\mathrm{D} = 8.67$. This value is scarcely differing to the one obtained by the educational evaluation system because the values of the weights u_i are in almost equal proportion (see Figure 17 and Figure 18).

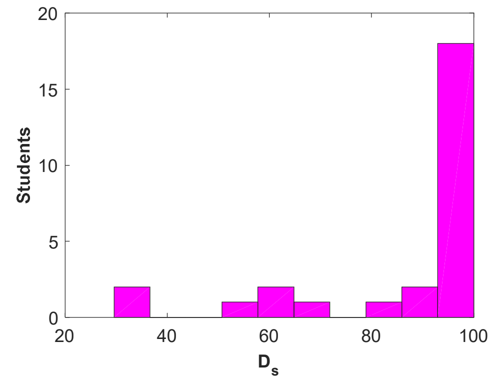

Fig. 22. Distribution of the $\mathrm{D_s}$ grade of development of the competencies acquired by the students throughout the study program

4 Discussion

The current scheme that the Mexican Middle Higher Educational System (MMHES) addresses at evaluating students overlooks the effort of the teacher and the students. This is mainly explaining because referred scheme standardizes the evaluation of $\mathrm{F_{ij}^z}$ (Figure 16) to a percentage value of either the weights $\mathrm{I_{ij}^z}$ (Equation (16)) or w_{ij} (Equation (17)).

We can interpret it being subjective that a rating with a value of 8 entailing a value very close to 10 could be associated with the letter 'E' that regularly links to an excellent performance. Moreover, emphasizing on subjectivity a rating value of 8 being close to 6 could be also associated with the letter 'S' linking to sufficient. By the same token, we can trust that a rating value of 7 could be either associated with the letter 'S' or the letter 'I' for incompetent. The point is that the ranges of rating values that can be associated with the letter 'E', 'S' or 'I' are not known by default. Therefore, there is no clear evidence that the ranges of scores associated with the letter 'E', 'S' and 'I' are not granted subjectively. As a results, we can expect that a student who gets a rating of 8 performed similarly to one achieving a rating of 10, if both are graded with the letter 'E', the one rating 8, would be too overvalued. In the worst case, in no way could it be known with the highest grade of certainty about the student's real progress in the development of their activities, whether they are based on the acquisition of competences or not. Furthermore, the overvaluation could be increased, for example: in the case of that a rating of 8 is associated to the letter $\mathrm{F}_{\mathrm{ij}}^{\mathrm{z}} = \mathrm{E}$ and its weight value $\mathrm{I}_{\mathrm{ij}}^{\mathrm{z}}$ (Figure 16) be the largest of all weights $\mathrm{I}_{\mathrm{ij}}^{\mathrm{z}}$ corresponding to $\mathrm{AE}_{\mathrm{ij}}$.

Our method conceives alignments as a formal mathematical procedure that establish a relationship between the attributes of the competences with the learning activities. In our approach the weights v_{ij} and u_{ij} assigned to each activity and unit are determined by Equations (4) and (5) one to one. Correspondingly, the attributes that ought to evaluate in each of the learning activities are known a priory (Table 5). Another feature of our method is that it allows a flexible slant for teachers as they are free to choose the most appropriate technique (Table 2) to evaluating each H_{ij} learning activity. In addition, presently offered evaluation scheme establishes the criteria to obtain evidence of student performance during the development of activities and achieves this task considering only three elements, knowledge, procedure and attitude.

Our fuzzy inference system also entails a synthetic evaluation of efficiency as it integrates the three elements in a single quantitative index value E_{ij}. Moreover, the knowledge that the student can handle or could display while achieving a given task can be evaluated both qualitatively and quantitatively, likewise, the procedures undertaken by the students can also be evaluated qualitatively and quantitatively. The evaluation of the attitude bases on the observation but it is valued by the effects that it may have on an individual [28], [32-33]. In summary our fuzzy system is built upon a set of rules so that it determines the relationship among the three essential elements: knowledge, procedure and attitude, thereby producing a result with less a subjective burden while assigning the rating of a student.

In the simulated data study case, efficiency under which students developed their competences evaluates by means of a fuzzy inference system. Through the conforming alignment matrix, that procedure identifies both the attributes that require an evaluation and their inclosing activities and units (Table 5). Then using Equations (6), (7), (8) and (11) it will be possible to determine to what grade the students acquired their competencies at the end of each learning activity, unit and study program, correspondingly. As we learnt from the real study case, the activities developed by students are evaluated by the teacher's own criteria, without clear evidence on how they achieved tasks. In addition, the evaluations can be overvalued or undervalued, this depends on the teacher and the educational evaluation system criterion. Certainly, it is not possible to know about the actual development of student learning.

Contributions by Romero-Escobar in [8] and Bedoya-Ruiz in [27] aim to evaluate the competencies acquired by students. But on spite of its relevance the approach by Romero-Escobar in [8] does not contemplate a formal mathematical method for the alignment of competencies, and neither involves a fuzzy system at sustaining the proposed method of evaluation. Bedoya-Ruiz [27] uses fuzzy logic to construct rubrics but does not develop a formal mathematical method to align competencies. Present approach attempts to fill these gaps. Blending the incipient approaches Romero-Escobar in [8] and Bedoya-Ruiz in [27] we integrate a Hybrid construct where the outstanding paradigm for management of vagueness provided by fuzzy logic integrates to a competencies-alignment matrix. The output composes a mathematically based tool for the evaluation of the grade of development of competencies by students enrolling at the MMHES.

According to obtained results, it turns out that present fuzzy system makes an acceptable interpretation if what it intends to be understood as an evaluation process for the establishment of competencies. Nevertheless, it is worth pointing out that the greatest difficulty that arises in the implementation of our method is the construction of the fuzzy system itself. If you want to change the rules to inference rules as to capture the effects induced by another type of attitude such as those presented in [28], special care must be taken in the choice of membership functions and their scale of values. They must be constructed through a fuzzy partition (see T_1 and T_2 in Appendix). Similarly, the assembly of the definition of the rule base to be used must consider all possible anticipating cases and must reflect the desired level of demand of the institution.

Our method allows to process the evidence collected in the different moments of assessment, regardless of the technique that teachers choose to evaluate the performance of the activities developed by the students. Our results show that teachers can monitor the development of students' competencies, know what their points of greatest development are, as well as, ascertaining which competencies they have difficulties acquiring.

5 Conclusion

This work presents a first version of a protocol aimed at evaluating the grade of development of the competencies acquired by students enrolling at the Mexican Higher Middle Education System. Entailed scheme contemplates the essential elements sustaining an objective evaluation. The associated alignment of competencies to study units and learning activities relies on a formal mathematical relationship. Offered construct establishes as a condition that the teacher typifies the alignment before starting a subject and precludes the possibility of subsequent later modifications.

Composing alignment method deters inconsistent practices by students or teachers that impair efficiency thereby, it attempts to amend deficiencies detected on this matter at the whole Mexican Middle Higher Educational System. In addition, the alignment method contemplates the estimation of the weight values, v_{ij} and u_{ij} (see Equation (4) and (5), respectively) that sustain evaluation depending on a numerical scale, while also precluding the possibility that they are modified by non-teaching personal.

Teaching practice focuses on transmitting “knowledge” to students through teaching and learning activities. The evaluation of competencies is a process of extracting suitable evidence. In accordance, the teacher's efforts should aim towards correctly identifying the contents to be considered. Relevant to his task becomes the organization and dosage in time, the conscientious selection of exercises, and the clarification of the correspondence that they keep with the selected content.

Besides, teachers must search for evaluation tools (Table 2) that offer greater reliability this thorough assigning numerical scores, which in our scheme typify as "evidence". Our fuzzy inference system integrates the evidence, which typify as elements supporting efficiency, namely knowledge, procedure and the attitude of the student.

The nature of attitudes allowed the design of fuzzy instruments for their evaluation, such as the attitudinal evaluation scale that implements through membership functions and fuzzy rules. The attitudinal assessment represents the availability of criteria agreed between experts in the area.

On this matter, we stood by requirements in [28]. In turn, we considered that knowledge concerns to facts, concepts and principles; that requires students to consider and "know how to say", for example: Is it? What is it? What is it like? What are its most significant characteristics? When did it happen? Where did it happen? How long did it last? Why did it happen or behave like this? How does it resemble? What differences does it have with? It can also be valuated qualitatively and quantitatively, which allowed the design of fuzzy evaluation tools.

In the same way, we can assess the procedure, for example: knowing how to add fractions, preparing a summary or essay, and so on. In general, fuzzy systems are a valuable tool because they model the experts' criteria through in linguistic variables and inference fuzzy rules.

Future work will attempt to develop a comprehensive evaluation system where fuzzy instruments' design is adapted and considered to appraise any attitudes pertaining to the educational process [28] and intended to assess acquisition of values.

Also, to address the design of fuzzy tools to assess other elements as presented in Figure 2 but including more details. And, in a more general way, to develop a system that addresses firsthand the Mexican Higher Middle Education System. On view of present results, we do not preclude the possibility that the foreseen comprehensive evaluation system could suit similar evaluation endeavors in educational systems elsewhere.

6 Appendix. Some Concepts of Fuzzy Set Theory

We present some concepts and definitions of fuzzy set theory that are relevant to the contents of the methods section, particularly on building the addressed fuzzy inference system that conforms to the approach in references [34] and [36-38].

6.1 Fuzzy Sets and Associating Memberships Functions

Let U be a universal set that contains all the possible elements of concern in each context or application. A fuzzy set A is defined through:

$$A = \{(x, \mu_A(x)) | x \in U\}, \quad \text{(A1}$$

where $(x, \mu_A(x))$ is an ordered pair composing an element x of U , and being $\mu_A(x)$ a membership function that assigns to x a numerical value in the interval $[0,1]$, namely:

$$\mu_A(x) = \begin{cases} 1 & if\ x \in A, \\ 0 & otherwise. \end{cases} \quad \text{(A2)}$$

An extensive collection of membership functions is available to choose from, but forms can conceive to suit specific requirements.

A triangular fuzzy number is a fuzzy set $A \subset R$ that characterizes by a membership function $\mu_A\colon \mathrm{R} \rightarrow [0,1]$ defined by:

$$\mu_A(x)\colon \begin{cases} 0 & if \quad x \le a \\ \dfrac{x-a}{m-a} & if \quad a \le x \le m \\ \dfrac{b-x}{b-m} & if \quad m \le x \le b \\ 0 & if \quad otherwise \end{cases} \quad \text{(A3)}$$

where a, b and m are real number such that a and b are the lower and upper limits of variation of x, that is, $a \le x \le b$, and m is the modal value. In the present settings, the triangular fuzzy number implements through the function $trimf(x, [a, m, b])$ of Matlab 2016b.

A Gaussian fuzzy number is a fuzzy set $A \subset R$ that identifies by a membership function $\mu_A\colon \mathrm{R} \rightarrow [0,1]$ defined as follows:

$$\mu_A(x)\colon e^{\frac{-(x-m)^2}{2\sigma^2}}, \quad \text{(A4)}$$

where m is the medium value and σ stands for the wideness of the bell, both m and σ are real numbers. Here a Gaussian fuzzy number achieves through the function $gaussmf(x, [\sigma, m])$ of Matlab 2016b.

An s-shaped fuzzy number is a fuzzy set $A \subset R$ that typifies by a membership function $\mu_A\colon \mathrm{R} \rightarrow [0,1]$, which define as follows:

$$\mu_A(x)\colon \begin{cases} 0 & if \quad x \le a, \\ 2\left[\dfrac{x-a}{b-a}\right]^2 & if \quad a \le x \le m, \\ 1 - 2\left[\dfrac{x-b}{b-a}\right]^2 & if \quad m < x < b, \\ 1 & if \quad x \ge b, \end{cases} \quad \text{(A5)}$$

where a and b stand one to one for the lower and upper limits of the range of x, and m picks out such that $a < m < b$ stands for the inflection point of $\mu_A(x)$. Usually, in practice a value $m = (a+b)/2$ chooses. The greater the distance $a - b$, the slower the growth of $\mu_A(x)$. In present work the s-shaped fuzzy number implements through the function $smf(x, [a, b])$ of Matlab 2016b.

A z-shaped fuzzy number is a fuzzy set $A \subset R$ associating with by a membership function $\mu_A\colon \mathrm{R} \rightarrow [0,1]$ defined as follows:

$$\mu_A(x)\colon \begin{cases} 1 & if \quad x \le a, \\ 1 - 2\left[\dfrac{x-a}{b-a}\right]^2 & if \quad a \le x \le \dfrac{a+b}{2}, \\ 2\left[\dfrac{x-b}{b-a}\right]^2 & if \quad \dfrac{a+b}{2} \le x \le b, \\ 0 & if \quad x \ge b, \end{cases} \quad \text{(A6)}$$

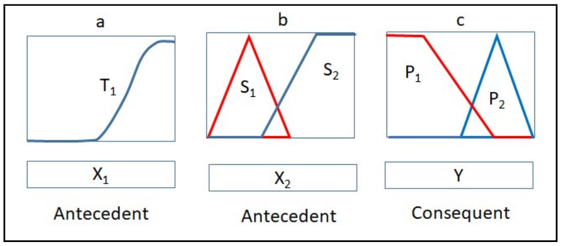

Fig. A1. Linguistic variables $\mathbf{X}_1, \mathbf{X}_2$ and $\mathbf{Y}$ characterized by fuzzy sets (a) $\mu_{\mathbf{T}_1}$ (b) $\{\mu_{\mathbf{S}_1}, \mu_{\mathbf{S}_2}\}$ and (c) $\{\mu_{\mathbf{P}_1}, \mu_{\mathbf{P}_2}\}$ respectively

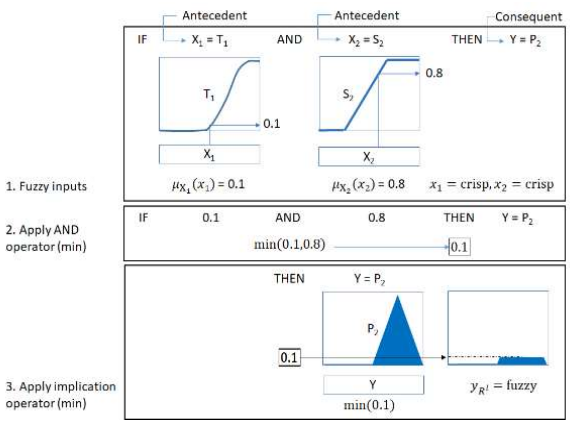

Fig. A2. Interpreting the if-then rule as a three-part process

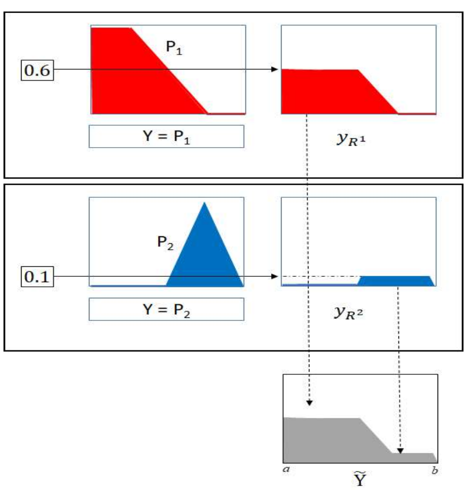

Fig. A3. The aggregation process illustrated for the case n_R=2

where a and b are real numbers typifying the lower and upper limits for x that is, $a < x < b$. The greater the distance $a - b$, the faster $\mu_A(x)$ grows. The z-shaped fuzzy number obtains here through the function $zmf(x, [a, b])$ in Matlab 2016b.

A trapezoidal fuzzy number is a fuzzy set $A \subset R$ that associates with a membership function $\mu_A\colon \mathrm{R} \to [0,1]$ defined as follows:

$$\mu_A(x)\colon \begin{cases} 0 & if \quad x \le a, \\ \dfrac{x-a}{b-a} & if \quad a \le x \le b, \\ 1 & if \quad b \le x \le c, \\ \dfrac{d-x}{d-c} & if \quad c \le x \le d, \\ 0 & if \quad d \le x, \end{cases} \tag{A7}$$

where a, b, c and d are real numbers defined such that $a < b < c < d$. The parameters a and b locate the “feet” of the trapezoid and the parameters d and c locate the associating “shoulders”. Trapezoidal fuzzy number computes here through the function $trapmf(x, [a, b, c, d])$ of Matlab 2016b.

6.2 Logical Operations

In fuzzy logic theory a proposition stating: if x is A then y is B, links to a characteristic function $\mu_{A \to B}(x, y)$ taking values in the interval [0.1]. Moreover, each if-then rule conceives as a fuzzy set with its characteristic function measuring the grade of truth of the implication relationship between x and y, formally:

$$\mu_{A \to B}(x, y) = \mu_A(x) \cdot \mu_B(y), \tag{A8}$$

For fuzzy sets A and B on a reference set U and characterizing by membership function $\mu_A(x)$ and $\mu_B(x)$, one to one, the standard set operations $A \cap B$ or $A \cup B$ define through:

$$\begin{aligned} \mu_{(A \cap B)}(x) &= \min[\mu_A(x), \mu_B(x)] \quad - \quad fuzzy\ intersecction, \\ \mu_{(A \cup B)}(x) &= \max[\mu_A(x), \mu_B(x)] \quad - \quad fuzzy\ union. \end{aligned} \tag{A9}$$

6.3 Fuzzy Rules

Fuzzy sets and operators are the subjects and verbs of a fuzzy inference system. The if-then rules compose conditional and consequent statements. Conditional ones define by means of a combination of one or more fuzzy input sets called antecedents. The consequent conceives as an associating output fuzzy set.

As an example, we can consider a fuzzy inference rule with a conditional statement that combines the conjunction of two antecedents, and with a consequent being a single fuzzy set, namely:

$$\text{If } \mathbf{X}_1 \text{ is } \mathbf{T}_1 \text{ and } \mathbf{X}_2 \text{ is } \mathbf{S}_2 \text{ then } \mathbf{Y} \text{ is } \mathbf{P}_2, \quad \text{(A10)}$$

where $\mathrm{X}_1, \mathrm{X}_2$ and $\mathbf{Y}$ are linguistic variables, $\mathrm{T}_1, \mathrm{S}_2$ and P_2 are linguistic terms defined by fuzzy sets $\mu_{\mathrm{T}_1}, \mu_{\mathrm{S}_2}$ and μ_{P_2} in the universe $\mathrm{X}_1, \mathrm{X}_2$ and Y respectively. Customarily, in the fuzzy inference systems terminology X_1 and X_2 refer as antecedents and Y as the consequent.

For instance, given that the antecedent X_2 and the consequent Y in Equation (A10) respectively describe by means of the linguistic terms S_2 and P_2, we could correspondingly assume that the linguistic variables X_2 and Y characterize one to one by means of pairs of linguistic terms (S_1 and S_2) and (P_1 and P_2), (see Figure A1).

6.4 Operation of the Mamdani Fuzzy System

A Mamdani fuzzy system conceives as an expert system with approximate reasoning that maps a vector of inputs to a single output (scalar).

In order to explain how such construct operates, we take the example represented by Equation (A10) and Figure (A1). The architecture of this system composes by fourth phases:

a. Phase of fuzzification: identified as the conversion of numeric input values into fuzzy sets. Upon input data the grade of belonging to each of the fuzzy sets $\mu_{\mathbf{T}_1}, \mu_{\mathbf{S}_2}$ and $\mu_{\mathbf{P}_2}$ determines by means of Equation (A3), Equation (A4), Equation (A5), Equation (A6), Equation (A7) or another) (see Figure A2 step 1).
b. Phase of inference: numeric values x_1 and x_2 are made correspond to the input variable X_1 and X_2 respectively (see Figure A2 step 1). The minimum (fuzzy intersection in Equation (A9)) is generally used to evaluate the "and" that connects the propositions associated to the antecedents (see Figure A2 step 2). If the fuzzy rule being evaluated is R^l, where $1 \leq l \leq n_R$, that is, n_R is the total number of rules in conceived fuzzy inference system, the grade of certainty or activation of the antecedents for the current values of the input variables is represented by y_{R^l} which for the case $n_R = 2$, defines as follows:

$$y_{R^l}(x_1, x_2) = \min\{\mu_{\mathbf{X_1}}(x_1), \mu_{\mathbf{X}_2}(x_2)\}, \quad \text{(A11)}$$

The execution of the rule y_{R^l} (Equation (A11)) is done by applying a fuzzy operator of implication (fuzzy union in Equation (A9)). Then, the output of each rule is the fuzzy set y_{R^l} resulting from the implication (see Figure A2 step 3).

c. Phase of aggregation: unification of the outputs of all rules. We take the membership function of all rule consequents previously clipped or scaled, and combine them into a single fuzzy set, $\widetilde{\mathrm{Y}} = \{(x, \mu_{\widetilde{\mathrm{Y}}}(x)) | x \in U\}$, this becomes (see Figure A3 for an illustration of the case $n_R = 2$)

$$\widetilde{\mathrm{Y}} = y_{R^1}(x_1, x_2) \oplus y_{R^2}(x_1, x_2) \oplus, \quad \text{(A12)}$$

where $\oplus$ means maximum operator (fuzzy union in Equation (A9)). Although, there are other operators available, the maximum is commonly used to perform this operation.

d. Phase of defuzzification: the input is the aggregate output fuzzy set $\widetilde{\mathrm{Y}}$ (Equation (A12) and the output is a single number y. We selected the centroid defuzzification method, which finds a point representing the center of gravity of the fuzzy set, $\widetilde{\mathrm{Y}}$, on the interval $[a, b]$ (see Figure 3A):

$$y = \frac{\sum_{i=a}^{b} x_i \, \mu_{\widetilde{\mathrm{Y}}}(x_i)}{\sum_{i=a}^{b} \mu_{\widetilde{\mathrm{Y}}}(x_i)}. \quad \text{(A13)}$$

The selection of the defuzzification method can play a decisive role in the synthesis of fuzzy models for many areas of application. It heavily relies on the expert's input.

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Corresponding author is Cecilia Leal-Ramírez.