PID Control Law for Trajectory Tracking Error Using Time-Delay Adaptive Neural Networks for Chaos Synchronization

Joel Perez P., Jose P. Perez

Universidad Autonoma de Nuevo Leon (UANL), Facultad de Ciencias Fisico Matematicas, Monterrey, Mexico

joelperezp@yahoo.com, josepazp@gmail.com

Abstract. This paper presents an application of Time-Delay adaptive neural networks based on a dynamic neural network for trajectory tracking of unknown nonlinear plants. Our approach is based on two main methodologies: the first one employs Time-Delay neural networks and Lyapunov-Krasovskii functions and the second one is Proportional-Integral-Derivative (PID) control for nonlinear systems. The proposed controller structure is composed of a neural identifier and a control law defined by using the PID approach. The new control scheme is applied via simulations to Chaos Synchronization. Experimental results have shown the usefulness of the proposed approach for Chaos Production. To verify the analytical results, an example of a dynamical network is simulated and a theorem is proposed to ensure the tracking of the nonlinear system.

Keywords. Lyapunov-Krasovskii function stability, chaos synchronization, trajectory tracking, time-delay adaptive neural networks, PID control.

1 Introduction

In order to solve problems in the fields of optimization, pattern recognition, signal processing, and control systems, among others, recurrent neural networks have to be designed with the property that there is only one equilibrium point, and this point has to be globally asymptotically stable. Therefore analysis of this kind of stability has been intensively investigated (see [2] and references therein). In biological and artificial neural networks, time delays arise in the processing of information storage and transmission. It is known that these delays can create oscillatory or even unstable trajectories [1]. Hence, lately a lot of results regarding stability of delayed neural networks have been published.

In this paper we present the design of a control law to ensure tracking of a general nonlinear system by a delayed recurrent neural network. Following the ideas of [9], we propose a PID control law that considers the effect of time delay in the system.

The tracking error stability is analyzed by means of a Lyapunov-Krasovkii functional (see [3, 8, 7]). The proposed adaptive control scheme is composed of a recurrent neural identifier and a controller, see Fig. 1.

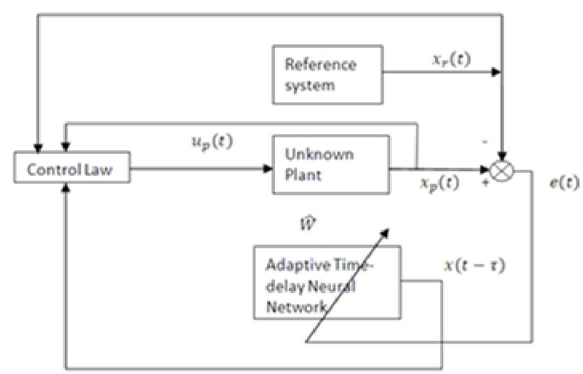

Fig. 1. Adaptive time-delay recurrent neural network scheme

We further improve the design by adapting it to systems with fewer inputs than states (see [11] and [10]) in which the control law is optimal with respect to a well-defined Lyapunov-Krasovskii function.

ISSN 1405-5546
doi: 10.13053/CyS-19-2-1908

The applicability of the approach is illustrated by one example of complex dynamical systems: Chaos Synchronization.

2 Plant Modeling

An unknown nonlinear plant is given by

$$\dot{x_p} = F_p(x_p, u) \triangleq f_p(x_p) + g_p(x_p)u, \tag{1}$$

where $x_p,\ f_p \in R^n,\ u \in R^m$, and $g_p \in R^{nxm}$, x_p is the plant, u is the control input, and both f_p and g_p are unknown. We propose to model (1) by the time-delay neural network state space representation $\dot{x} = Ax + W^*\Gamma_z[x(t-\tau)] + \Omega u$, plus one more term modeling error (as in [5]). A is a $n \times m$ matrix; without loss of generality we can assume that $A = -\lambda I,\ \lambda > 0$. x are the neural states, giving an approximation to the real plant by a neural network, W^* is the weights matrix, $\Gamma_z(x)$ is the hyperbolic tangent $Gamma_z(x) = (\tanh(x_1(t)), \ldots, \tanh(x_n(t)))^t$ and Ω is a $n \times m$ matrix that modifies the input u, and τ is the time delay.

We define the modeling error between the time-delay neural network and the plant by

$$w_{per} = x - x_p. \tag{2}$$

We assume the following.

Hypotheses 1. (Objective of Modeling): The modeling error is exponentially stable, that is,

$$\dot{w}_{per} = -kw_{per}. \tag{3}$$

In this work we consider $k = 1$. From (2) we have $\dot{w}_{per}$= $\dot{x} - \dot{x_p}$ and using the hypothesis we get $\dot{x_p}$= $\dot{x}$+ w_{per}.

The unknown plant can be modeled as

$$\dot{x_p} = \dot{x} + w_{per} = Ax + W^*\Gamma_z[x(t-\tau)] + w_{per} + \Omega u, \tag{4}$$

where W^* are the fixed but unknown weights from the neural network. They minimize the modeling error.

3 Trajectory Tracking

We proceed now to analyze the modeling error between the unknown plant modeled by (4) and the reference signal defined by

$$\dot{x_r} = f_r(x_r, u_r), \text{ with } u_r \text{ and } x_r \in R^n, \tag{5}$$

where x_r are the reference states, u_r is the input and f_r is a nonlinear function.

For this purpose, we define the control error between the plant and the reference signal by

$$e = x_p - x_r, \tag{6}$$

whose derivative with respect to time is

$$\begin{aligned} \dot{e} &= \dot{x_p} - \dot{x_r} = \\ &Ax + W^*\Gamma_z[x(t-\tau)] + w_{per} + \Omega u - f_r(x_r, u_r). \end{aligned} \tag{7}$$

Adding and subtracting to the right hand side of (7) the terms $\widehat{W}\Gamma_z[x_r(t-\tau)], \Omega\alpha_r(t, \widehat{W}), Ae,$ where $\widehat{W}$ is the estimate of W^*, α_r is defined below, and considering that $e = x_p - x_r$, we have

$$\begin{aligned} \dot{e} \ &= \ Ax + W^*\Gamma_z[x(t-\tau)] + x - x_p + \Omega u \\ &\quad -f_r(x_r, u_r) + \widehat{W}\Gamma_z[x_r(t-\tau)] \\ &\quad - \widehat{W}\Gamma_z[x_r(t-\tau)] + \Omega\alpha_r(t, \widehat{W}) \\ &\quad -\Omega\alpha_r(t, \widehat{W}) + Ae - Ae, \end{aligned}$$

$$\begin{aligned} \dot{e} =& Ae + W^*\Gamma_z[x(t-\tau)] + \Omega u - f_r(x_r, u_r) \\ &+ \widehat{W}\Gamma_z[x_r(t-\tau)] + \Omega\alpha_r(t, \widehat{W}) \\ &- \Omega\alpha_r(t, \widehat{W}) - e - x_r - Ae + x + Ax \\ &- \widehat{W}\Gamma_z[x_r(t-\tau)]. \end{aligned} \tag{8}$$

At this point, we consider the following supposition:

the time-delay neural network will follow the reference signal, even with the presence of disturbances, if

$$\begin{aligned} &Ax_r + \widehat{W}\Gamma_z[x_r(t-\tau)] + x_r - x_p + \Omega\alpha_r(t, \widehat{W}) \\ &= f_r(x_r, u_r). \end{aligned}$$

ISSN 1405-5546
doi: 10.13053/CyS-19-2-1908

Then

$$\Omega\alpha_r(t,\widehat{W}) = f_r(x_r,u_r) - Ax_r - \widehat{W}\Gamma_z[x_r(t-\tau)] - x_r + x_p \quad (9)$$

and we get

$$\dot{e} = Ae + W^*\Gamma_z[x(t-\tau)] - \widehat{W}\Gamma_z[x_r(t-\tau)] - Ae + (A+I)(x-x_r) + \Omega(u-\alpha_r(t,\widehat{W})). \quad (10)$$

Now, adding and subtracting in (10) the term $\widehat{W}\Gamma_z[x(t-\tau)]$ we have

$$\begin{aligned}\dot{e} =& Ae + (W^* - \widehat{W})\Gamma_z[x(t-\tau)] \\ &+ \widehat{W}(\Gamma_z[x(t-\tau)] - \Gamma_z[x_r(t-\tau)]) \\ &+ (A+I)(x-x_r) - Ae + \Omega(u-\alpha_r(t,\widehat{W}))\end{aligned} \quad (11)$$

We define

$$\widetilde{W} = W^* - \widehat{W} \text{ and } \widetilde{u} = u - \alpha_r(t,\widehat{W}) \quad (12)$$

and substituting (12) in (11), we obtain

$$\begin{aligned}\dot{e} =& Ae + \widetilde{W}\Gamma_z[x(t-\tau)] + \widehat{W}(\Gamma_z[x(t-\tau)] \\ &- \Gamma_z[x_r(t-\tau)]) \\ &+ (A+I)(x-x_r) - Ae + \Omega\widetilde{u},\end{aligned}$$

$$\begin{aligned}\dot{e} =& Ae + \widetilde{W}\Gamma_z[x(t-\tau)] \\ &+ \widehat{W}\left\{\begin{array}{c}\Gamma_z[x(t-\tau)] - \Gamma_z[x_p(t-\tau)] + \\ \Gamma_z[x_p(t-\tau)] - \Gamma_z[x_r(t-\tau)]\end{array}\right\} \\ &+ (A+I)(x - x_p + x_p - x_r) - Ae + \Omega\widetilde{u}.\end{aligned} \quad (13)$$

Now we can set

$$\widetilde{u} = u_1 + u_2 \quad (14)$$

and define

$$\begin{aligned}\Omega u_1 =& -\widehat{W}(\Gamma_z[x(t-\tau)] - \Gamma_z[x_p(t-\tau)]) \\ &- (A+I)(x-x_p),\end{aligned} \quad (15)$$

so (13) is reduced to

$$\begin{aligned}\dot{e} &= Ae + \widetilde{W}\Gamma_z[x(t-\tau)] \\ &\quad + \widehat{W}(\Gamma_z[x_p(t-\tau)] - \Gamma_z[x_r(t-\tau)]) \\ &\quad + (A+I)(x_p - x_r) - Ae + \Omega u_2.\end{aligned}$$

By (6), shortening notation a little by setting $\sigma = \Gamma_z$ and defining

$$\phi_\sigma(t-\tau) = \sigma(x_p(t-\tau)) - \sigma(x_r(t-\tau)),$$

we get

$$\dot{e} = (A+I)e + \widetilde{W}\sigma[x(t-\tau)] + \widehat{W}\phi_\sigma(t-\tau) + \Omega u_2. \quad (16)$$

Now, the problem is to find the control law Ωu_2 that stabilizes the system (16). We will obtain the control law by using the Lyapunov-Krasovskii methodology.

4 Stability of the Tracking Error

Once (16) is obtained, we consider its stabilization in feedforward. We note $(e,\widehat{W}) = 0$ is an asymptotically stable equilibrium point of the undisturbed autonomous system ($A = -\lambda I$ and $\lambda > 0$). For its stability, we propose the following PID control law:

$$\begin{aligned}\Omega u_2 =& K_p e + K_v\dot{e} + K_i\int_0^t e(\tau)d\tau \\ &- \Upsilon(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e.\end{aligned} \quad (17)$$

The parameters K_p, K_v and K_i will be determined later, L_σ^2 is the Lipschitz constant of ϕ [6], $\Upsilon > 0$. This control law (17) is similar to the control law in [5].

We will show that the feedback system is asymptotically stable. Substituting (17) in (16) and setting $a = (1-K_v)$, we get

$$\begin{aligned}\dot{e} =& \frac{1}{a}(A+I)e + \frac{1}{a}\widetilde{W}\sigma[x(t-\tau)] + \\ &\frac{1}{a}\widehat{W}\phi_\sigma(t-\tau) + \frac{1}{a}K_p e + \\ &\frac{1}{a}K_i\int_0^t e(\tau)d\tau - \frac{\Upsilon}{a}(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e\end{aligned} \quad (18)$$

and assuming $A = -\lambda I$

$$\begin{aligned}\dot{e} =& \frac{-1}{a}(\lambda - 1 - K_p)e + \frac{1}{a}\widetilde{W}\sigma[x(t-\tau)] \\ &+ \frac{1}{a}\widehat{W}\phi_\sigma(t-\tau) + \frac{1}{a}K_i\int_0^t e(\tau)d\tau \\ &- \frac{\Upsilon}{a}(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e.\end{aligned} \quad (19)$$

ISSN 1405-5546
doi: 10.13053/CyS-19-2-1908

If we set $w = \frac{1}{a}K_i\int_0^t e(\tau)d\tau$, then $\dot{w} = \frac{1}{a}K_i e(t)$, and (19) can be written as

$$\begin{aligned}\dot{e} =& \frac{-1}{a}(\lambda - 1 - K_p)e + \frac{1}{a}\widetilde{W}\sigma[x(t-\tau)] \\ &+ \frac{1}{a}\widehat{W}\phi_\sigma(t-\tau) \\ &+ w - \frac{\Upsilon}{a}(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e. \end{aligned} \quad (20)$$

We will show that the new state $(e^T, w^T)^T$ is asymptotically stable and that the equilibrium point is $(e^T, w^T)^T = (0,0)^T$, when $\widetilde{W}\sigma[x_r(t-\tau)] = 0$, which is taken as an external disturbance.

Let V be the candidate Lyapunov-Krasovskii function [3, 7] given by

$$\begin{aligned}V =& \frac{1}{2}(e^T, w^T)(e^T, w^T)^T + \frac{1}{2a}tr\left\{\tilde{W}^T\tilde{W}\right\} \\ &+ \frac{1}{a}\int_{t-\tau}^{t}[\phi_\sigma^T(s)\widehat{W}^T\widehat{W}\phi_\sigma(s)]ds. \end{aligned} \quad (21)$$

The time derivative of (21) along the trajectories of (20) is

$$\begin{aligned}\dot{V} =& e^T\dot{e} + w^T\dot{w} + \frac{1}{a}tr\left\{\dot{\tilde{W}}^T\tilde{W}\right\} \\ &+ \frac{1}{a}[\phi_\sigma^T(t)\widehat{W}^T\widehat{W}\phi_\sigma(t) \\ &- \phi_\sigma^T(t-\tau)\widehat{W}^T\widehat{W}\phi_\sigma(t-\tau)], \end{aligned} \quad (22)$$

$$\begin{aligned}\dot{V} =& e^T[\frac{-1}{a}(\lambda - 1 - K_p)e + \frac{1}{a}\widetilde{W}\sigma[x(t-\tau)] \\ &+ \frac{1}{a}\widehat{W}\phi_\sigma(t-\tau) + w \\ &- \frac{\Upsilon}{a}(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e] + \frac{1}{a}w^T K_i e \quad (23) \\ &+ \frac{1}{a}tr\left\{\dot{\tilde{W}}^T\tilde{W}\right\} + \frac{1}{a}[\phi_\sigma^T(t)\widehat{W}^T\widehat{W}\phi_\sigma(t) \\ &- \phi_\sigma^T(t-\tau)\widehat{W}^T\widehat{W}\phi_\sigma(t-\tau)]. \end{aligned}$$

At this point, we select the next learning law for the neural network weights as in [11] and [4]:

$$tr\left\{\dot{\tilde{W}}^T\tilde{W}\right\} = -e^T\widetilde{W}\sigma[x(t-\tau)] \quad (24)$$

Then (23) is reduced to

$$\begin{aligned}\dot{V} =& \frac{-1}{a}(\lambda - 1 - K_p)e^Te + \frac{e^T}{a}\widehat{W}\phi_\sigma(t-\tau) \\ &+ (1 + \frac{K_i}{a})e^Tw - \frac{\Upsilon}{a}(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e^Te \\ &+ \frac{1}{a}[\phi_\sigma^T(t)\widehat{W}^T\widehat{W}\phi_\sigma(t) - \phi_\sigma^T(t-\tau)\widehat{W}^T\widehat{W}\phi_\sigma(t-\tau)]. \end{aligned} \quad (25)$$

If we apply the inequality

$$x^Ty \leq \frac{1}{2}x^Tx + \frac{1}{2}y^Ty \quad (26)$$

to the second term in the right hand side of (25) then

$$\begin{aligned}\dot{V} \leq& \frac{-1}{a}(\lambda - 1 - K_p)e^Te \\ &+ \frac{1}{a}[\frac{e^Te}{2} + \frac{1}{2}\phi_\sigma^T(t-\tau)\widehat{W}^T\widehat{W}\phi_\sigma(t-\tau)] \\ &+ (1 + \frac{K_i}{a})e^Tw - \frac{\Upsilon}{a}(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e^Te \\ &+ \frac{1}{a}[\phi_\sigma^T(t)\widehat{W}^T\widehat{W}\phi_\sigma(t) - \phi_\sigma^T(t-\tau)\widehat{W}^T\widehat{W}\phi_\sigma(t-\tau)], \end{aligned}$$

$$\begin{aligned}\dot{V} \leq& \frac{-1}{a}(\lambda - 1 - K_p)e^Te + \frac{1}{a}(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e^Te \\ &+ (1 + \frac{K_i}{a})e^Tw - \frac{\Upsilon}{a}(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e^Te. \end{aligned} \quad (27)$$

Here, we select $(1 + \frac{K_i}{a}) = 0$, so $K_V = K_i + 1$, and $K_V \geq 0$ when $K_i \geq -1$. With this selection of parameters (27) is reduced to

$$\begin{aligned}\dot{V} \leq& \frac{-1}{a}(\lambda - 1 - K_p)e^Te \\ &- \frac{1}{a}(\Upsilon - 1)(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e^Te. \end{aligned} \quad (28)$$

Observe $\lambda - 1 - K_p > 0$, $a > 0$ and $\Upsilon - 1 > 0$, then $\dot{V} < 0$, $\forall\ e, w, \widehat{W} \neq 0$, the error tracking is

ISSN 1405-5546
doi: 10.13053/CyS-19-2-1908

asymptotically stable and it converges to zero for every $e \neq 0$, i.e., the plant will follow the reference asymptotically. Finally, the control law which affects the plant and the time-delay neural network is given by

$$
\begin{aligned}
u =&\Omega^{\dagger}[-\widehat{W}\Gamma(z[x(t-\tau)] - z[x_p(t-\tau)]) \\
&-(A+I)(x-x_p) + K_p e + K_v \dot{e} + K_i \int_0^t e(\tau)d\tau \\
&-\Upsilon(\frac{1}{2}+\frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e + f_r(x_r,u_r) - Ax_r \\
&-\widehat{W}\Gamma_z[x_r(t-\tau)] - x_r + x_p]
\end{aligned} \tag{29}
$$

where $\Omega^{\dagger}$ is the Penrose pseudo-inverse of Ω. This control law gives asymptotic stability of error dynamics and thus ensures the tracking of the reference signal. The results obtained can be summarized as follows.

Theorem 1 *For the unknown nonlinear system modeled by (4), the on-line learning law (24) and the control law (29) ensure the tracking of the nonlinear reference model (5).*

Remark 2 *From (28) we have*

$$
\begin{aligned}
\dot{V} \leq & \frac{-1}{a}(\lambda - 1 - K_p)e^T e \\
&-\frac{1}{a}(\Upsilon - 1)(\frac{1}{2}+\frac{1}{2}\left\|\widehat{W}\right\|^2 L_\sigma^2)e^T e < 0, \\
&\forall e \neq 0, \forall \widehat{W}.
\end{aligned}
$$

Therefore V is decreasing and bounded from below by $V(0)$, where $V = \frac{1}{2}(e^T, w^T)(e,w)^T + \frac{1}{2a}tr\left\{\widetilde{W}^T\tilde{W}\right\} + \int_{t-\tau}^{t}[\phi_\sigma^T(s)\widehat{W}^T\widehat{W}\phi_\sigma(s)]ds.$

We conclude that $e, \widetilde{W} \in L_1$; this means that the weights remain bounded.

5 Simulations

In order to demonstrate the applicability of the proposed adaptive control scheme, the following example is tested.

In this example, the unknown plant considered is a Lorenz´s chaotic attractor generated by

$$
\begin{aligned}
\dot{x}_{p1} =& 10x_{p2} - 10x_{p1}, \quad x_{p1}(0) = 10, \\
\dot{x}_{p2} =& -x_{p2} - x_{p1}x_{p3} + 28x_{p1}, \quad x_{p2}(0) = 0, \\
\dot{x}_{p3} =& x_{p1}x_{p2} - (\frac{8}{3})x_{p3}, \quad x_{p3}(0) = 10.
\end{aligned} \tag{30}
$$

The goal is to force the chaotic Lorenz´s attractor to track the reference, the Chen´s attractor generated by

$$
\begin{aligned}
\dot{x}_{r1} =& 35x_{r2} - 35x_{r1}, \quad x_{r1}(0) = -10, \\
\dot{x}_{r2} =& -7x_{r1} - x_{r1}x_{r3} + 28x_{r2}, \quad x_{r2}(0) = 0, \\
\dot{x}_{r3} =& x_{r1}x_{r2} - 3x_{r3}, \quad x_{r3}(0) = 37.
\end{aligned} \tag{31}
$$

In the simulations, the following time-delay neural network was used with $\tau = 5$ sec:

$$\dot{x} = Ax + W^*\Gamma_z(x) + \Omega u + w_{per} \tag{32}$$

and

$$
A = \begin{pmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{pmatrix},
\Gamma = \begin{pmatrix} 150 & 0 & 0 \\ 0 & 150 & 0 \\ 0 & 0 & 150 \end{pmatrix}.
$$

Here

$$\Gamma_z(x) = [\tanh(0.5x_1), \tanh(0.5x_2), \tanh(0.5x_3)]^T$$

with $\Omega = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

W^* is estimated using the learning law given in (24), and u is calculated using (29).

The results of the simulations are shown in Figures 2-6, where the time evolution of the states and phase portraits are presented.

We can see that the Time-Delay Adaptive Neural Controller ensures rapid convergence of the system outputs to the reference trajectory. In our approach, direct control is considered; the learning laws for the time-delay neural networks depend explicitly on the tracking error instead of the identification error. This approach results in a faster response of the system.

ISSN 1405-5546
doi: 10.13053/CyS-19-2-1908

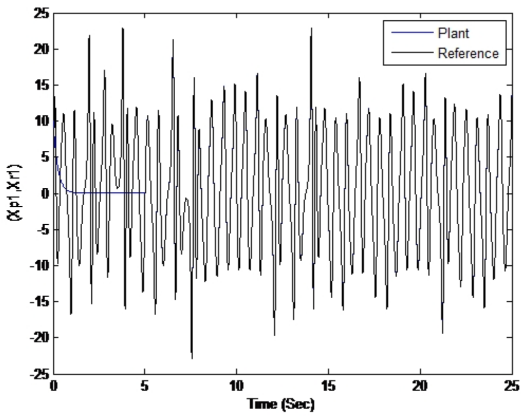

Fig. 2. Time evolution for state 1

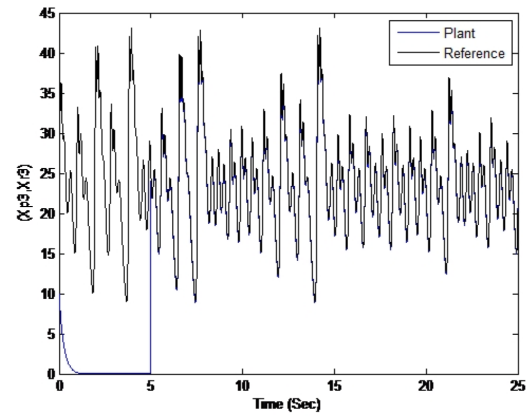

Fig. 4. Time evolution for state 3

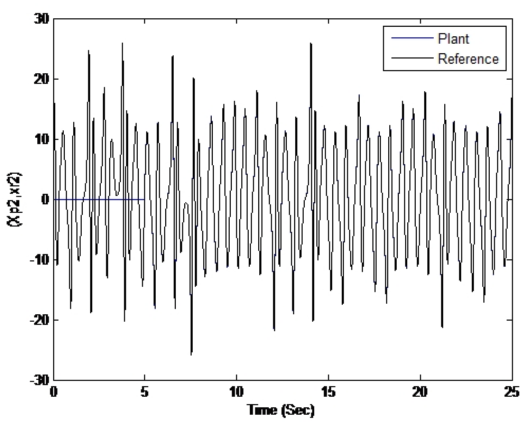

Fig. 3. Time evolution for state 2

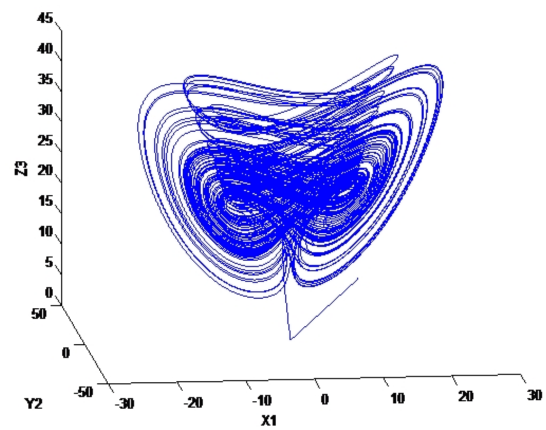

Fig. 5. Plant phase portrait

6 Conclusions

We have extended the Time-Delay Adaptive Neural Network Control previously developed in [12] to the trajectory tracking control problem in order to consider fewer inputs than states. Stability of the tracking error is analyzed via Lyapunov-Krasovskii control functions and the control law is obtained based on the PID approach. A new adaptive control structure based on a time-delay dynamic neural network for chaotic orbit tracking of unknown nonlinear systems has been developed. This structure is composed of a Time-Delay neural network identifier and a control law for orbit tracking. Stability of the tracking control system has also been established by means of the Lyapunov-Krasovskii function method. An open question that remains to be further investigated is the tolerance of a general system to the magnitude of the delay.

Acknowledgements

The authors appreciate the support from CONACYT and the Dynamical Systems Group of the Faculty of Physical and Mathematical Sciences of the Autonomous University of Nuevo Leon, Mexico.

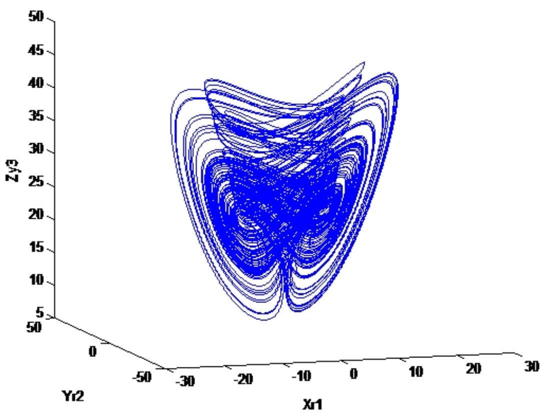

Fig. 6. Reference phase portrait

References

1. **Baldi, P. & Atiya, A. F.** (**1994**). How delays affect neural dynamics and learning. *IEEE Trans. on Neural Networks*, Vol. 5, pp. 612–621.

2. **Forti, M.** (**1994**). Necessary and suffiient conditions for absolute stability of neural networks. *IEEE Trans. on Circuits and Systems*, Vol. 41, pp. 491–494.

3. **Hale, J. K. & V.Lunel, S. M.** (**1991**). *Introduction to the Theory of Functional Differential Equations*. Springer Verlag, New York, USA.

4. **Ioannou, P. A. & Sun, J.** (**2003**). *Robust Adaptive Control*. Prentice Hall, Upper Saddle River, New Jersey, USA.

5. **Kelly, R., Guerra, R. H., & Reyes, F.** (**1999**). Lyapunov stable control of robot manipulators: a fuzzy self-tuning procedure. *Intelligent Automation and Soft Computing*, Vol. 5, pp. 313–326.

6. **Khalil, H.** (**1996**). *Nonlinear System Analysis*. Prentice Hall, Upper Saddle River, New Jersey, USA, 2 edition.

7. **Liao, X., Chen, G., & Sanchez, E. N.** (**2002**). Delay-dependent exponential stability analysis of delayed neural networks: an LMI approach. *Neural Networks*, Vol. 15, pp. 855–866.

8. **Liao, X., Chen, G., & Sanchez, E. N.** (**2002**). Lmi-based approach for asymptotically stability analysis of delayed neural networks. *IEEE Trans. on Circuits and Systems*, Vol. 49, pp. 1033–1039.

9. **Perez, J., Perez, J. P., Rdz, F., & Flores, A.** (**2013**). Trajectory tracking for the chaotic pendulum using pi control law. *Revista Mexicana de Fisica*, Vol. 59, pp. 471–477.

10. **Sanchez, E., Perez, J., Ricalde, L., & Chen, G.** (**2001**). Chaos production and synchronization via adaptive neural control. *Proceeding of IEEE Conference on Decision and Control*, Orlando, USA.

11. **Sanchez, E. N., Perez, J. P., Ricalde, L., & Chen, G.** (**2001**). Trajectory tracking via adaptive neural control. *Proceeding of IEEE Int. Symposium on Intelligent Control*, Mexico City, Mexico, pp. 286–289.

12. **Yazdizadeh, A. & Khorasani, K.** (**2002**). *Adaptive time delay neural network structures for nonlinear system identification*. Elsevier Science.

Joel Perez Padron obtained his B.Sc. degree in Mathematics in 1991, from the Faculty of Physical and Mathematical Sciences, a Master's degree in Electrical Engineering with specialization in Control in 2001, from the Faculty of Mechanical and Electrical Engineering, and a Ph.D. in Industrial Physics Engineering with specialization in Control in 2008, from the Faculty of Physical and Mathematical Sciences of Nuevo Leon Autonomous University.

Jose Paz Perez Padron obtained his B.Sc. degree in Mathematics in 1990, from the Faculty of Physical and Mathematical Sciences, a Master's degree in Electrical Engineering with specialization in Control in 1999, from the Faculty of Mechanical and Electrical Engineering, and a Ph.D. in Science in Electrical Engineering in 2004, from the Cinvestav-Gdl.

*Article received on 12/11/2013, accepted on 01/09/2014.
Corresponding author is Joel Perez P.*