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Times of Execution of the Quantum NOT Gate Operating on One of Two Interacting Qubits

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Abstract. It is generally believed that entanglement speeds up Quantum Information Processing (QIP). However, we prove that for a system of two interacting qubits through a XXZ Hamiltonian which are maximally entangled it is not possible to execute a quantum NOT gate operating on one of these two qubits. The interaction between the two qubits means presence of noise in one of them. If the two interacting qubits are not entangled, the times of execution of the quantum NOT gate operating on one of the two qubits are not small enough. **Since the times of execution of the quantum NOT gate operating on one of the two interacting qubits is extremely large,** we conclude that the execution of the quantum NOT gate operating on one of two interacting qubits is not possible.

Keywords. XXZ Hamiltonian, qubits, quantum NOT gate.

1 Introduction

Entanglement and decoherence are susceptible of direct observations due to the fact that it is possible to create and manipulate single quantum states in the laboratory [5, 12, 10, 7, 9, 4, 11, 1]. Such topics play a fundamental role in quantum computation and QIP [6, 2, 3, 8]. A pure state $|\chi,\eta\rangle$ is said to be an entangled state if it cannot be decomposed as a product of individual states, that is,

$$|\chi,\eta\rangle \neq |\chi\rangle\otimes|\eta\rangle. \tag{1}$$

The *classical* NOT gate is such that

$$\begin{array}{ccc} |\chi\rangle|\eta\rangle & \rightarrow\ classical\ \text{NOT}\rightarrow & (\text{NOT}|\chi\rangle)\,|\eta\rangle, \\ |\chi\rangle & = & |\uparrow\rangle, |\downarrow\rangle \end{array} \tag{2}$$

without any entanglement between the individual states $|\chi\rangle$ and $|\eta\rangle$. In the above equation the NOT gate is such that NOT$|\uparrow\rangle = |\downarrow\rangle$ and NOT$|\downarrow\rangle = |\uparrow\rangle$.

Here we consider a couple of entangled subsystems S_1 and S_2 where the density matrix of S_2 is different from the projector, $\rho_2 \neq |\eta\rangle\langle\eta|$, furthermore it is different from its square, $\rho_2^2 \neq \rho_2$. Since Tr$\rho_2 = 1$ then Tr$\rho_2^2 \neq 1$.

On the other hand, we shall assume that the subsystems S_1 and S_2 are a two level system (qubits) generated by the states $\{|\uparrow\rangle, |\downarrow\rangle\}$. In the present work we calculate in an approximated way the execution times of the quantum NOT gate for a system of two interacting spins. In order to do the above we consider two spins, S_1 and S_2, in a constant magnetic field $\vec{B} = B_0\hat{z}$, interacting through the Hamiltonian

$$H = \epsilon_1 B_0 S_1^z + \epsilon_2 B_0 S_2^z + J S_1^x S_2^x, \tag{3}$$

which is similar to the XXZ Hamiltonian employed in statistical mechanics. According to the Heisenberg Uncertainty Principle, the uncertainty in the energy ΔE and the uncertainty in the time of measuring Δt are related according to [13]:

$$\Delta t \geq \frac{h}{\Delta E}. \tag{4}$$

In what follows we consider a system of units where $h = 1$ is employed. We study the following situations.

2 Case of two qubits: $s_1 = s_2 = 1/2$

First we study the situation where both qubits are maximally entangled, then we investigate the case where they are not entangled. The four maximally entangled Bell states are [8]

$$\begin{aligned} |\Phi^+\rangle &= \frac{1}{\sqrt{2}}\Big(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\Big), \\ |\Phi^-\rangle &= \frac{1}{\sqrt{2}}\Big(|0\rangle_1 \otimes |0\rangle_2 - |1\rangle_1 \otimes |1\rangle_2\Big), \\ |\Psi^+\rangle &= \frac{1}{\sqrt{2}}\Big(|0\rangle_1 \otimes |1\rangle_2 + |1\rangle_1 \otimes |0\rangle_2\Big), \\ |\Psi^-\rangle &= \frac{1}{\sqrt{2}}\Big(|0\rangle_1 \otimes |1\rangle_2 - |1\rangle_1 \otimes |0\rangle_2\Big). \end{aligned} \tag{5}$$

We assume that the quantum NOT operation acts exclusively in the spin s_1. Consequently from Eq. (5) we obtain that[1]

$$\begin{aligned} \text{NOT}_1|\Phi^+\rangle &= |\Psi^+\rangle, \\ \text{NOT}_1|\Phi^-\rangle &= -|\Psi^-\rangle, \\ \text{NOT}_1|\Psi^+\rangle &= |\Phi^+\rangle, \\ \text{NOT}_1|\Psi^-\rangle &= -|\Phi^-\rangle. \end{aligned} \tag{6}$$

The use of Eqs. (3), (5), and (6) yields (see Appendix for a detailed derivation)

$$\begin{aligned} \langle \text{NOT}_1\Phi^+|H|\Phi^+\rangle =&(\epsilon_1 B_0 + \epsilon_2 B_0)\langle\Psi^+|\Phi^-\rangle \\ &+ J\langle\Psi^+|\Phi^+\rangle = 0, \\ \langle \text{NOT}_1\Phi^-|H|\Phi^-\rangle =&-(\epsilon_1 B_0 + \epsilon_2 B_0)\langle\Psi^-|\Phi^+\rangle \\ &- J\langle\Psi^-|\Phi^-\rangle = 0, \\ \langle \text{NOT}_1\Psi^+|H|\Psi^+\rangle =&(\epsilon_1 B_0 - \epsilon_2 B_0)\langle\Phi^+|\Psi^+\rangle \\ &- J\langle\Phi^+|\Psi^+\rangle = 0, \\ \langle \text{NOT}_1\Psi^-|H|\Psi^-\rangle =&-(\epsilon_1 B_0 - \epsilon_2 B_0)\langle\Phi^-|\Psi^-\rangle \\ &- J\langle\Phi^-|\Psi^-\rangle = 0. \end{aligned} \tag{7}$$

Now it is possible for us to prove the following theorem.

Theorem 1. *If the system of two spins s_1 and s_2 is described initially by any of the four Bell states, then with the Hamiltonian of Eq. (3) it is not possible to perform a quantum* NOT *gate acting on one of the two entangled qubits.*

Proof. By using Eqs. (4) and (7) we observe that if the two spins are initially described by any of the four Bell states, the approximated times of execution of the quantum NOT gate are

$$t \sim \frac{1}{\langle f|H|i\rangle} = \frac{1}{\langle \text{NOT}_1\psi|H|\psi\rangle} \sim \frac{1}{0} \to \infty \\ |\psi\rangle = |\Phi^+\rangle,\ |\Phi^-\rangle,\ |\Psi^+\rangle,\ |\Psi^-\rangle. \diamond \tag{8}$$

In the general case where the two spin system is initially described by the general state

$$|\psi\rangle = A|00\rangle + B|01\rangle + C|10\rangle + D|11\rangle, \tag{9}$$

where $|A|^2 + |B|^2 + |C|^2 + |D|^2 = 1$ one obtains that

$$\text{NOT}_1|\psi\rangle = C|00\rangle + D|01\rangle + A|10\rangle + B|11\rangle. \tag{10}$$

On the other hand, Eq. (3) yields

$$\begin{aligned} H|00\rangle &= (\epsilon_1 B_0 + \epsilon_2 B_0)|00\rangle + J|11\rangle, \\ H|01\rangle &= (\epsilon_1 B_0 - \epsilon_2 B_0)|01\rangle + J|10\rangle, \\ H|10\rangle &= (-\epsilon_1 B_0 + \epsilon_2 B_0)|10\rangle + J|01\rangle, \\ H|11\rangle &= -(\epsilon_1 B_0 + \epsilon_2 B_0)|11\rangle + J|00\rangle. \end{aligned} \tag{11}$$

Now we prove the following two theorems.

Theorem 2. *If the initial state is described by the general two-qubit state of Eq. (10) then the matrix elements of the Hamiltonian (3) between the initial state $|\psi\rangle$ and the final state* $\text{NOT}_1|\psi\rangle$ *are*

$$\begin{aligned} \langle \text{NOT}_1\psi|H|\psi\rangle =&2(AC - BD)\epsilon_2 B_0 \\ &+ 2(CD + AB)J. \end{aligned} \tag{12}$$

Proof. Through the use of Eqs. (9), (10), and (11) the result follows. $\diamond$

One can conclude from Theorem 2 that the execution time of the quantum NOT gate is very sensitive to the initial state. The above can be seen from the following theorem.

Theorem 3. *If the initial state is described by a W state where $A = B = C = D = 1/2$, then the*

[1] Observe that the results do not change if the quantum NOT operation acts on the spin s_2.

time of execution of the NOT *gate on one of the two qubits is*

$$T_{GHZ} \sim \frac{1}{J}. \tag{13}$$

Proof. In this case the use of Theorem 2 implies that the matrix elements of the Hamiltonian (3) between the initial state and the final state are

$$\langle \mathrm{NOT}_1 \psi | H | \psi \rangle \quad = \quad J. \tag{14}$$

Thus, with Eqs. (4) and (14) we obtain

$$T_{GHZ} \sim \frac{1}{\langle f|H|i\rangle} = \frac{1}{|\langle \mathrm{NOT}_1\psi|H|\psi\rangle|} = \frac{1}{J}. \diamond$$

The main prediction of Theorem 3 is that the stronger the interaction term in the Hamiltonian (3), the smaller the time of execution of the quantum NOT gate. Obviously, experimentally it is not an easy task to reach a very large value of J.

In what follows we assume that the coefficients A, B, C, D are real. With Eq. (12) there arise some interesting particular cases. In the present work we consider only two special cases which we think to be most relevant. They are the following:

3 Case $AC = BD \neq 0$

Theorem 4. *In the particular case where $C = BD/A$, the times of execution of the quantum* NOT *are*

$$T_1 \sim \frac{1}{2\left(\frac{A}{B}\right)}\left[1+\left(\frac{A}{B}\right)^2\right]\cdot\frac{1}{J}. \tag{15}$$

Proof. In this case Eq. (12) takes the form

$$\langle \mathrm{NOT}_1\psi|H|\psi\rangle \quad = \quad 2\frac{A}{B}\left[1+\left(\frac{A}{B}\right)^2\right]^{-1} J, \tag{16}$$

where we used $C = BD/A$ and $A^2 + B^2 + C^2 + D^2 = 1$. With Eqs. (4) and (16) the result follows. $\diamond$

As a consequence of the above theorem, if $J > 0$ then necessarily $\frac{A}{B} > 0$. Figure 1 shows T_1 of Eq. (15) as a function of $x = A/B$ and J.

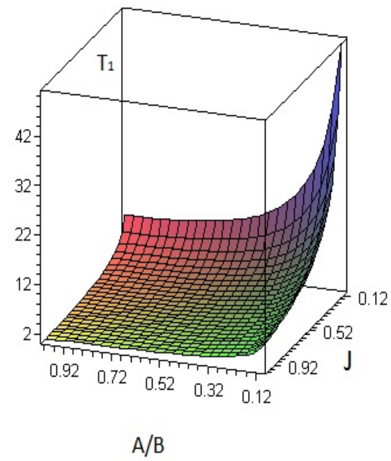

Fig. 1. Times of execution of the quantum NOT gate operating on one of two interacting qubits through the Hamiltonian (3) given by Eq. (15) (in units of seconds) as a function of A/B and J (in units of sec^{-1})

4 Case $CD = -AB \neq 0$

Theorem 5. *In the particular case where $C = -AB/D$, the times of execution of the quantum* NOT *are*

$$T_2 \sim -\frac{1}{2\left(\frac{B}{D}\right)}\left[1+\left(\frac{B}{D}\right)^2\right]\cdot\frac{1}{\epsilon_2 B_0}. \tag{17}$$

Proof. In this case Eq. (12) takes the form

$$\langle \mathrm{NOT}_1\psi|H|\psi\rangle = -2\frac{B}{D}\left[1+\left(\frac{B}{D}\right)^2\right]^{-1}\epsilon_2 B_0, \tag{18}$$

where we used $C = -AB/D$ and $A^2 + B^2 + C^2 + D^2 = 1$. With Eqs. (4) and (18) the result follows. $\diamond$

From the above theorem it can be concluded that if $\epsilon_2 B_0 > 0$ then $\frac{B}{D} < 0$. Figures 2-5 present T_2 of Eq. (17) as a function of two variables $y = B/D$ and $E_0 = \epsilon_2 B_0$ (in units of $1/(T \cdot sec)$ and the magnetic field B_0 (in units of T). i.e, times of execution of the quantum NOT gate operating on one of two interacting qubits through the Hamiltonian (3) given by Eq. (17) (in units of seconds) as a function of B/D .

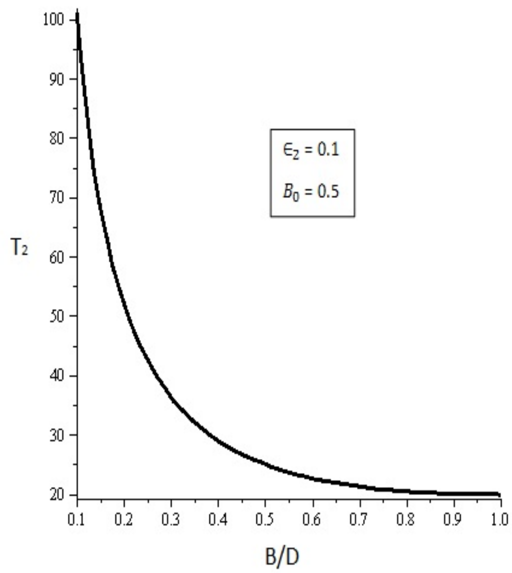

Fig. 2. Times of execution of the quantum NOT gate for $\epsilon = 0.1$ and $B_0 = 0.5$

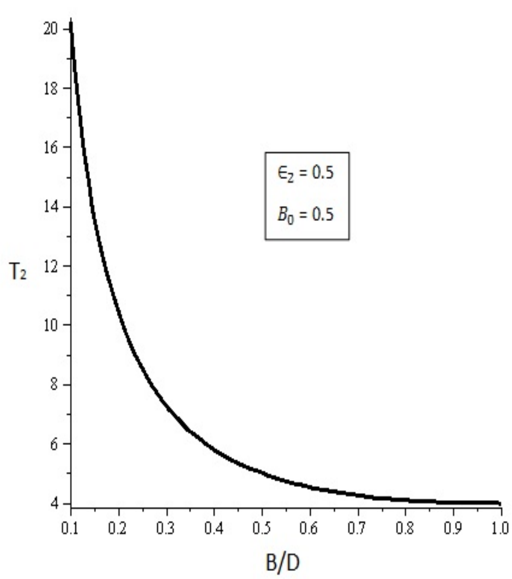

Fig. 3. Times of execution of the quantum NOT gate for $\epsilon = 0.5$ and $B_0 = 0.5$

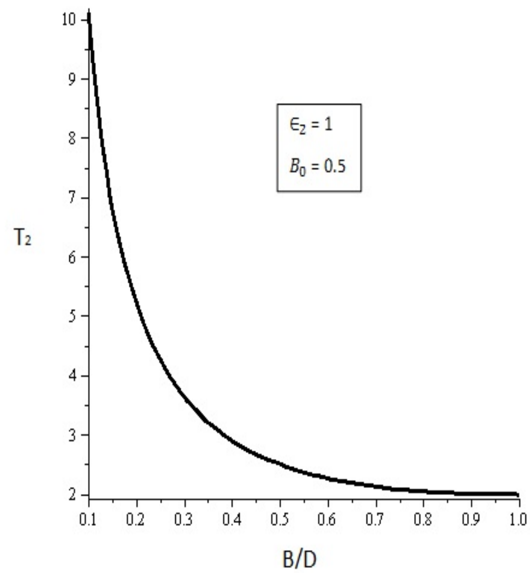

Fig. 4. Times of execution of the quantum NOT gate for $\epsilon = 1$ and $B_0 = 0.5$

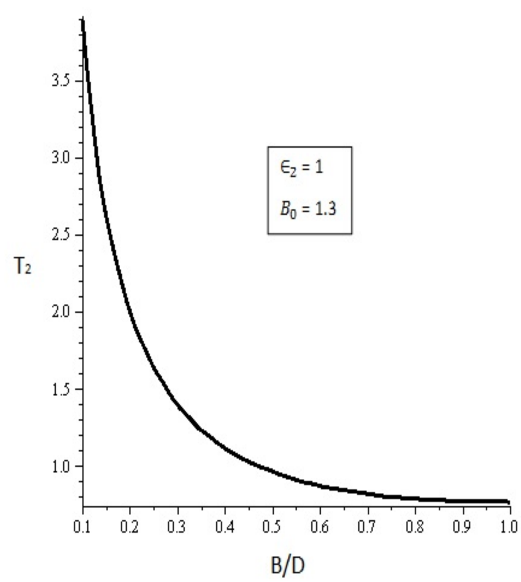

Fig. 5. Times of execution of the quantum NOT gate for $\epsilon = 1$ and $B_0 = 1.3$

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5 Discussion

The *classical* NOT gate operating on one of two spins is characterized by no interaction between the spins. On the other hand, a quantum NOT gate acting on one of the two qubits is differentiated from the classical NOT gate because in this case the two spins are interacting with each other through a given Hamiltonian such as the one of Eq. (3).

A very interesting situation occurs when the two interacting spins are entangled. It is commonly believed that entanglement is a necessary ingredient that speeds up QIP. However, Theorem 1 shows that if the two interacting spins are initially entangled, then it is not possible to execute a quantum NOT gate on one of them. Such a result is surprising but the nature of the interaction between the two spins and the general principle of Quantum Mechanics given by Eq. (4) confirm it. A very useful state for QIP is the so-called W state where the four different pure two-qubit states $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ of Eq. (9) are equally weighted.

What Theorem 3 says is that if initially the two-qubit state is a W state then the times of execution of the quantum NOT gate is inversely proportional to the interaction coupling constant J. In such a case the times of execution of the quantum NOT gate acting on one of the two interacting qubits can be made very small by making the value of J very large. However, experimentally it is not an easy task to make J large enough, so such times of execution are not necessarily small. As it can be appreciated from Figures 1 and 2 the times of execution of the quantum NOT gate acting on one of the two interacting qubits are not small. The latter shows that to execute a quantum NOT gate acting on one of two interacting spins is not so fast. One would expect that in a quantum system the execution of such a gate could be very fast. Here we have proved that this is not necessarily true.

In order that the time of executing of such a quantum gate becomes small enough, it is necessary that the value of the interaction coupling constant between the two spins J be very large. The above is not a trivial task from the experimental point of view. On the other hand, Theorem 2 predicts that if the external magnetic field B_0 is very large, the time of execution of the quantum NOT gate acting on one of the two interacting qubits would be very small. It is worth mentioning that the maximal values of experimental magnetic fields are of the order of 10 T, consequently the times of execution of the quantum NOT gate are not small. We conclude that due to the noise that induces one qubit on the other, the times of execution of the quantum NOT gate operating on one of two such qubits are not small. In other words, to execute the quantum NOT gate operating on one of two qubits interacting through the Hamiltonian (3) is not possible.

The pertinence of the present result consists in that it reveals that the unwelcome noise deteriorates the QIP. Noise is a central problem that currently attracts much effort to both understand and solve it. Consequently, the present work also helps the experimentalists to see possible routes of performing an efficient QIP in the presence of noise.

Appendix

In this section we derive Eqs. (7) and (11).

Derivation of Equation (7)

We shall derive only the first expression of Eq. (7); the remaining three expressions are derived in a similar way. In order to do the above, let us first note that the one-qubit states are orthonormal, that is,

$$\begin{aligned} {}_1\langle 0|0\rangle_1 &= {}_1\langle 1|1\rangle_1 = 1, \\ {}_1\langle 0|1\rangle_1 &= {}_1\langle 1|0\rangle_1 = {}_2\langle 0|1\rangle_2 = {}_2\langle 1|0\rangle_2 = 0. \end{aligned} \tag{19}$$

From the above equation it follows that

$$\begin{aligned} \langle \Phi^+|\Phi^-\rangle = &\frac{1}{\sqrt{2}}\Big({}_2\langle 0| \otimes_1 \langle 0| + {}_2\langle 1| \otimes_1 \langle 1|\Big)\cdot \\ &\frac{1}{\sqrt{2}}\Big(|0\rangle_1 \otimes |0\rangle_2 - |1\rangle_1 \otimes |1\rangle_2\Big) \\ =&\frac{1}{2}\Big({}_1\langle 0|0\rangle_{12}\langle 0|0\rangle_2 - {}_1\langle 0|1\rangle_{12}\langle 0|1\rangle_2 \\ &+ {}_1\langle 1|0\rangle_{12}\langle 1|0\rangle_2 \\ &- {}_1\langle 1|1\rangle_{12}\langle 1|1\rangle_2\Big) \\ =&1 - 1 = 0. \end{aligned} \tag{20}$$

In a similar way

$$\begin{aligned}\langle\Psi^+|\Phi^+\rangle =&\frac{1}{\sqrt{2}}\Big({}_2\langle 1|\otimes_1\langle 0| +_2\langle 0|\otimes_1\langle 1|\Big)\cdot\\ &\frac{1}{\sqrt{2}}\Big(|0\rangle_1\otimes|0\rangle_2+|1\rangle_1\otimes|1\rangle_2\Big)\\ =&\frac{1}{2}\Big({}_1\langle 0|0\rangle_{1\,2}\langle 0|1\rangle_2+{}_1\langle 0|1\rangle_{1\,2}\langle 0|0\rangle_2\\ &-{}_1\langle 0|1\rangle_{1\,2}\langle 1|1\rangle_2-{}_1\langle 1|1\rangle_{1\,2}\langle 0|1\rangle_2\Big)\\ =&0.\end{aligned} \tag{21}$$

We note that the Pauli operators satisfy $S^z|0\rangle = |0\rangle$, $S^z|1\rangle = -|1\rangle$, $S^x|0\rangle = |1\rangle$, and $S^x|1\rangle = |0\rangle$. Consequently, from Eq. (5) we obtain

$$\begin{aligned}S_1^z|\Phi^+\rangle &= \frac{1}{\sqrt{2}}\Big[\Big(S_1^z|0\rangle_1\Big)\otimes|0\rangle_2+\Big(S_1^z|1\rangle_1\Big)\otimes|1\rangle_2\Big]\\ &= \frac{1}{\sqrt{2}}\Big(|0\rangle_1\otimes|0\rangle_2-|1\rangle_1\otimes|1\rangle_2\Big)\\ &= |\Phi^-\rangle,\end{aligned} \tag{22}$$

$$\begin{aligned}S_2^z|\Phi^+\rangle &= \frac{1}{\sqrt{2}}\Big[|0\rangle_1\otimes\Big(S_2^z|0\rangle_2\Big)+|1\rangle_1\otimes\Big(S_2^z|1\rangle_2\Big)\Big]\\ &= \frac{1}{\sqrt{2}}\Big(|0\rangle_1\otimes|0\rangle_2-|1\rangle_1\otimes|1\rangle_2\Big)\\ &= |\Phi^-\rangle,\end{aligned} \tag{23}$$

$$\begin{aligned}S_1^xS_2^x|\Phi^+\rangle =&\frac{1}{\sqrt{2}}\Big[\Big(S_1^x|0\rangle_1\Big)\otimes\Big(S_2^x|0\rangle_2\Big)\\ &+\Big(S_1^x|1\rangle_1\Big)\otimes\Big(S_2^x|1\rangle_2\Big)\Big]\\ =&\frac{1}{\sqrt{2}}\Big(|1\rangle_1\otimes|1\rangle_2+|0\rangle_1\otimes|1\rangle_2\Big)\\ =&|\Phi^+\rangle.\end{aligned} \tag{24}$$

By using Eqs. (6), (20), (21), (22), (23), and (24) the first expression of Eq. (7) is derived

$$\begin{aligned}\langle \mathrm{NOT}_1\Phi^+|H|\Phi^+\rangle =&\langle\Psi^+|H|\Phi^+\rangle\\ =&\langle\Psi^+|\Big(\epsilon_1B_0S_1^z+\epsilon_2B_0S_2^z\\ &+JS_1^xS_2^x\Big)|\Phi^+\rangle\\ =&\epsilon_1B_0S_1^z\langle\Psi^+|S_1^z|\Phi^+\rangle\\ &+\epsilon_2B_0\langle\Psi^+|S_2^z|\Phi^+\rangle\\ &+J\langle\Psi^+|S_1^xS_2^x|\Phi^+\rangle\\ =&\epsilon_1B_0\langle\Psi^+|\Phi^-\rangle+\epsilon_2B_0\langle\Psi^+|\Phi^-\rangle\\ &+J\langle\Psi^+|\Phi^+\rangle\\ =&0.\end{aligned} \tag{25}$$

Through a similar procedure, the remaining three expressions of Eq. (7) are easily derived.

Derivation of Eq. (11)

We derive only the first expression of Eq. (11), the remaining three expressions are derived in a similar way. Using $S^x|0\rangle = |1\rangle$, $S^x|1\rangle = |0\rangle$, $S^z|0\rangle = |0\rangle$, and $S^x|1\rangle = -|1\rangle$, it follows that

$$\begin{aligned}H|00\rangle =&\Big(\epsilon_1B_0S_1^z+\epsilon_2S_2^z+JS_1^xS_2^x\Big)|00\rangle\\ =&\epsilon_1B_0\Big(S_1^z|0\rangle\Big)|0\rangle+\epsilon_2B_0|0\rangle\Big(S_2^z|0\rangle\Big)\\ &+J\Big(S_1^x|0\rangle\Big)\Big(S_2^x|0\rangle\Big)\\ =&\epsilon_1B_0|0\rangle|0\rangle+\epsilon_2B_0|0\rangle|0\rangle+J|1\rangle|1\rangle\\ =&\epsilon_1B_0|00\rangle+\epsilon_2B_0|00\rangle+J|11\rangle\\ =&\Big(\epsilon_1B_0+\epsilon_2B_0\Big)|00\rangle+J|11\rangle v.\end{aligned} \tag{26}$$

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