Real-time Discrete Nonlinear Identification via Recurrent High Order Neural Networks

Identificación No Lineal en Tiempo Real usando Redes Neuronales Recurrentes de Alto Orden

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Abstract. This paper deals with the discrete-time nonlinear system identification via Recurrent High Order Neural Networks, trained with an extended Kalman filter (EKF) based algorithm. The paper also includes the respective stability analysis on the basis of the Lyapunov approach for the whole scheme. Applicability of the scheme is illustrated via real-time implementation for a three phase induction motor.

Keywords: Neural identification, Extended Kalman filtering learning, Discrete-time nonlinear systems, Three phase induction motor.

Resumen. Este artículo trata el problema de identificación de sistemas no lineales discretos usando redes neuronales recurrentes de alto orden entrenadas con un algoritmo basado en el filtro de Kalman extendido (EKF). El artículo también incluye el análisis de estabilidad para el sistema completo, en las bases de la técnica de Lyapunov. La aplicabilidad del esquema se ilustra a través de la implementación en tiempo real para un motor de inducción trifásico.

Palabras clave: Identificación neuronal, Aprendizaje usando filtro de Kalman Extendido, Sistemas no lineales discretos, Motor de inducción trifásico.

1 Introduction

Since the seminal paper [Narendra and Parthasarathy, 1990], Neural networks (NN) have become a well-established methodology as exemplified by their applications to identification and control of general nonlinear and complex systems. In particular, the use of recurrent high order neural networks (RHONN) has increased recently [Sanchez and Ricalde, 2003] due to their excellent approximation capabilities, requiring less units, compared to the first order ones; they are also more flexible and robust when faced with new and/or noisy data patterns [Ghosh and Shin, 1992]. Furthermore, several authors have demonstrated the feasibility of using these architectures in applications such as system identification and control [Ge, et al., 2004; Haykin, 1999; Kim and Lewis, 1998; Narendra and Parthasarathy, 1990; Rovithakis and Christodolou, 2000; Sanchez, et al., 2004; and references therein]. There are recent results which illustrate that the NN technique is highly effective in the identification of a broad category of complex discrete-time nonlinear systems without requiring complete model information [Yu and Li, 2003; Yu and Li, 2004].

The best well-known training approach for recurrent neural networks (RNN) is the back propagation through time learning [Singhal and Wu, 1989]. However, it is a first order gradient descent method and hence its learning speed could be very slow [Singhal and Wu, 1989]. Recently the Extended Kalman Filter (EKF) based algorithms has been introduced to train neural networks, in order to improve the learning convergence [Singhal and Wu, 1989]. The EKF training of neural networks, both feedforward and recurrent ones, has proven to be reliable and practical for many applications over the past ten years [Singhal and Wu, 1989].

In [Rovithakis and Christodolou, 2000], adaptive identification and control by means of on-line learning is analyzed; the stability of the closed loop system is established based on the Lyapunov function method. Lyapunov approach can be used directly to obtain robust training algorithms for continuous-time recurrent neural networks [Sanchez and Ricalde, 2003; Rovithakis and Christodolou, 2000]. For discrete-time systems, the problem is more complex due to the couplings among subsystems, inputs and outputs. Few results have been published in comparison with those for continuous-time domain [Yu and Li, 2003; Yu and Li, 2004]. By other hand discrete-time neural networks are more convenient for real-time applications.

For many nonlinear systems, it is often difficult to obtain their accurate and faithful mathematical models, regarding their physically complex structures and hidden parameters as discussed in [Chui and Chen, 1998]. Therefore, system identification becomes important and even necessary before control systems can be considered not only for understanding and predicting the behavior of the whole system, but also for obtaining an effective control law.

The identification problem consists of choosing an appropriate identification model and adjusting its parameters according to some adaptive law, such that the response of the model to an input signal (or class of input signals), approximates the response of the real system to the same input [Rovithakis and Christodolou, 2000]. A challenger problem for nonlinear systems identification is to select a suitable structure for the identifier, capable of approximating the unknown nonlinear dynamics. In this consideration, it is notable that recurrent neural networks offer the advantage of well approximating a nonlinear system to an arbitrarily accurate level [Cotter, 1990].

In this paper, a recurrent high order neural network (RHONN) is used to identify the plant model, under the assumption of all the state is available for measurement. The online learning algorithm for the RHONN is implemented using an Extended Kalman Filter (EKF). The respective stability analysis, on the basis of the Lyapunov approach, is included for the proposed scheme. The applicability of this scheme is illustrated by real-time implementation for an electric three phase induction motor.

2 Mathematical preliminaries

Through this paper we use k as the step sampling, $k \in 0 \cup Z^+$, $|\bullet|$ for the absolute value, $\|\bullet\|$ for the Euclidian norm for vectors and for any adequate norm for matrices. For more details related to this section see [Ge, et al., 2004]. Consider a MIMO nonlinear system:

$$\chi(k+1) = F(\chi(k), u(k)) \qquad \textbf{(1)}$$

where $\chi \in \Re^n$, $u \in \Re^m$ and $F \in \Re^n \times \Re^m \to \Re^n$ is nonlinear function.

Definition 1. The solution of (1) is semiglobally uniformly ultimately bounded (SGUUB), if for any Ω, a compact subset of $\Re^n$ and all $\chi(k_0) \in \Omega$, there exists an $\varepsilon > 0$ and a number $N(\in, \chi(k_0))$ such that $\|\chi(k)\| < \varepsilon$ for all $k > k_0 + N$.

In other words, the solution of (1) is said to be SGUUB if, for any apriori given (arbitrarily large) bounded set Ω and any apriori given (arbitrarily small) set Ω_0, which contains (0,0) as an interior point, there exists a control u, such that every trajectory of the closed loop system starting from Ω enters the set $\Omega_0 = \{\chi(k) \| \|\chi(k)\| < \varepsilon\}$ in a finite time and remains in it thereafter [Ge, et al., 2004].

Theorem 1 [Ge, et al., 2004]. Let $V(\chi(k))$ be a Lyapunov function for the discrete-time system (1), which satisfies the following properties:

$$\gamma_1(\|\chi(k)\|) \le V(\chi(k)) \le \gamma_2(\|\chi(k)\|)$$
$$V(\chi(k+1)) - V(\chi(k)) = \Delta V(\chi(k)) \le -\gamma_3(\|\chi(k)\|) + \gamma_3(\varsigma)$$

where ς is a positive constant, $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$ are strictly increasing functions, and $\gamma_3(\cdot)$ is a continuous, nondecreasing function. Thus if

$$\Delta V(\chi) < 0 \quad \text{for} \quad \|\chi(k)\| > \varsigma$$

then $\chi(k)$ is uniformly ultimately bounded, i.e. there is a time instant k_T, such that $\|\chi(k)\| < \varsigma \; \forall k < k_T$

3 Discrete-time Recurrent Neural Networks

Let consider the following discrete-time recurrent high order neural network (RHONN), depicted in Fig.1 which is described as:

$$x_i(k+1) = w^T z_i(x(k), u(k)), \quad i = 1, \cdots, n \qquad \textbf{(2)}$$

where x_i is the state of the i-th neuron, L_i is the respective number of higher-order connections,

$\{I_1, I_2, \cdots, I_{L_i}\}$ is a collection of non-ordered subsets of $\{1, 2, \cdots, n\}$, n is the state dimension, w_i is the respective on-line adapted weight vector, and $z_i(x(k), u(k))$ is given by

$$z_i(x(k), u(k)) = \begin{bmatrix} z_{i_1} \\ z_{i_2} \\ \vdots \\ z_{i_{L_i}} \end{bmatrix} = \begin{bmatrix} \prod_{j \in I_1} y_{i_j}^{d_j(1)} \\ \prod_{j \in I_2} y_{i_j}^{d_j(2)} \\ \vdots \\ \prod_{j \in I_{L_i}} y_{i_j}^{d_j(L_i)} \end{bmatrix} \quad \textbf{(3)}$$

with $d_{j_i}(k)$ being a nonnegative integers, and y_i is defined as follows:

$$y_i = \begin{bmatrix} y_{i_1} \\ \vdots \\ y_{i_n} \\ y_{i_{n+1}} \\ \vdots \\ y_{i_{n+m}} \end{bmatrix} = \begin{bmatrix} S(x_1) \\ \vdots \\ S(x_n) \\ u_1 \\ \vdots \\ u_m \end{bmatrix} \quad \textbf{(4)}$$

In (4), $u = [u_1, u_2, \cdots, u_m]^T$ is the input vector to the neural network, and $S(\bullet)$ is defined by

$$S(x) = \frac{1}{1 + \exp(-\beta x)} \quad \textbf{(5)}$$

Consider the problem to approximate the general discrete-time nonlinear system (1), by the following discrete-time RHONN series-parallel representation [Rovithakis and Christodolou, 2000]:

$$\chi_i(k+1) = w_i^{*T} z_i(x(k), u(k)) + \varepsilon_{z_i} \quad \textbf{(6)}$$

where χ_i is the *i*-th plant state, ε_{z_i} is a bounded approximation error, which can be reduced by increasing the number of the adjustable weights [Rovithakis and Christodolou, 2000]. Assume that there exists ideal weights vector w_i^* such that $\|\varepsilon_{z_i}\|$ can be minimized on a compact set $\Omega_{z_i} \subset \Re^{L_i}$. The ideal weight vector w_i^* is an artificial quantity required for analytical purpose [Rovithakis and Christodolou, 2000]. In general, it is assumed that this vector exists and is constant but unknown. Let us define its estimate as w_i and the estimation error as

$$\tilde{w}_i(k) = w_i^* - w_i(k) \quad \textbf{(7)}$$

The estimate w_i is used for stability analysis which will be discussed later.

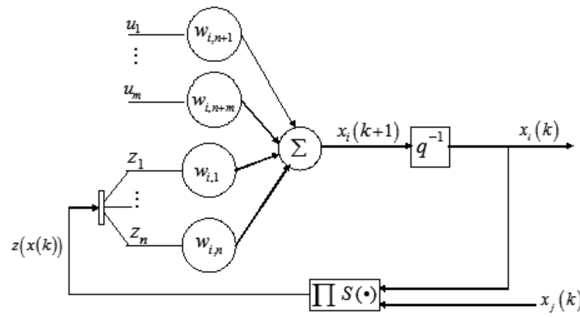

Fig. 1. Schematic representation for a discrete-time RHONN

4 The EKF Training Algorithm

Kalman filtering (KF) estimates the state of a linear system with additive state and output white noises [Chui and Chen, 1998; Grover and Hwang, 1992]. For KF-based neural network training, the network weights become the states to be estimated, with the error between the neural network output and the desired output; this error is considered as additive white noise. For identification, the desired output is information generated by the plant; in this paper, the respective state. Due to the fact that the neural network mapping is nonlinear, an extended Kalman Filtering (EKF)-type is required.

The training goal is to find the optimal weight values that minimize the prediction errors (the differences between the desired outputs and the neural network outputs). The EKF-based NN training algorithm is described by

$$\begin{aligned} K_i(k) &= P_i(k) H_i(k) M_i(k) \\ w_i(k+1) &= w_i(k) + \eta_i K_i(k) e_i(k) \\ P_i(k+1) &= P_i(k) - K_i(k) H_i^T(k) P_i(k) + Q_i(k) \\ i &= 1, \cdots, n \end{aligned} \quad \textbf{(8)}$$

with

$$M_i(k) = \left[R_i(k) + H_i^T(k) P_i(k) H_i(k)\right]^{-1} \quad \textbf{(9)}$$

$$e_i(k) = \chi_i(k) - x_i(k) \quad \textbf{(10)}$$

where $e_i(k)$ is the respective identification error, $P_i(k) \in \Re^{L_i \times L_i}$ is the prediction error covariance matrix at step k, $w_i \in \Re^{L_i}$ is the weight (state) vector, L_i is the respective number of neural network weights, χ_i is the *i*-th plant state, x_i is the *i*-th neural network state, n is the number of states, $K_i \in \Re^{L_i}$ is the Kalman gain vector, $Q_i \in \Re^{L_i \times L_i}$ is the NN weight estimation noise covariance matrix, $R_i \in \Re$ is the measurement noise covariance; $H_i \in \Re^{L_i}$ is a vector, in which each entry (H_{ij}) is the derivative of one of the neural network state, (x_i), with respect to one neural network weight, (w_{ij}), as follows

$$H_{ij}(k) = \left[\frac{\partial x_i(k)}{\partial w_{i_j}(k)}\right]^T_{w_i(k) = w_i(k+1)} \quad \textbf{(11)}$$

where $i = 1, 2, \cdots, n$ and $j = 1, 2, \cdots, L_i$. Usually P_i and Q_i are initialized as diagonal matrices, with entries $P_i(0)$ and $Q_i(0)$, respectively. It is important to remark that $H_i(k)$, $K_i(k)$ and $P_i(k)$ for the EKF are bounded; for a detailed explanation of this fact see [Song and Grizzle, 1995].

Then the dynamics of the identification error (10) can be expressed as

$$e_i(k+1) = \tilde{w}_i^T(k) z_i(x(k), u(k)) + \varepsilon_{z_i} \quad \textbf{(12)}$$

By the other hand the dynamics of (7) is

$$\tilde{w}_i(k+1) = w_i(k) - \eta_i K_i(k) e(k) \quad \textbf{(13)}$$

Now, we establish the main result of this paper in the following theorem.

Theorem 2: The RHONN (2) trained with the EKF-based algorithm (8) to identify the nonlinear plant (1), ensures that the identification error (10) is semiglobally uniformly ultimately bounded (SGUUB); moreover, the RHONN weights remain bounded.

Proof: Consider the Lyapunov function candidate

$$\begin{aligned} V_i(k) &= \tilde{w}_i^T(k) P_i(k) \tilde{w}_i(k) + e_i^2(k) \\ \Delta V_i(k) &= V_i(k+1) - V_i(k) \\ &= \tilde{w}_i^T(k+1) P_i(k+1) \tilde{w}_i(k+1) \\ &\quad - \tilde{w}_i^T(k) P_i(k) \tilde{w}_i(k) \\ &\quad + e_i^2(k+1) - e_i^2(k) \end{aligned} \quad \textbf{(14)}$$

Using (12) and (13) in (14)

$$\begin{aligned} \Delta V_i(k) &= \left[\tilde{w}_i(k) - \eta_i K_i(k) e_i(k)\right]^T \\ &\quad \times \left[P_i(k) - A_i(k)\right] \\ &\quad \times \left[\tilde{w}_i(k) - \eta_i K_i(k) e_i(k)\right] \\ &\quad + \left[\tilde{w}^T(k) z_i(x(k), u(k))\right]^2 \\ &\quad - \tilde{w}_i^T(k) P_i(k) \tilde{w}_i(k) - e_i^2(k) \end{aligned} \quad \textbf{(15)}$$

with $A_i(k) = K_i(k) H_i^T(k) P_i(k) + Q_i(k)$; then, (15) can be expressed as

$$\begin{aligned} \Delta V_i(k) &= \tilde{w}_i^T(k) P_i(k) \tilde{w}_i(k) \\ &\quad - \eta_i e_i(k) K_i^T(k) P_i(k) \tilde{w}_i(k) \\ &\quad - \tilde{w}_i^T(k) A_i(k) \tilde{w}_i(k) \\ &\quad + \eta_i e_i(k) K_i^T(k) A_i(k) \tilde{w}_i(k) \\ &\quad - \eta_i e_i(k) \tilde{w}_i^T(k) P_i(k) K_i(k) \\ &\quad + \eta_i^2 e_i^2(k) K_i^T(k) P_i(k) K_i(k) \\ &\quad + \eta_i e_i(k) \tilde{w}_i^T(k) A_i(k) K_i(k) \\ &\quad - \eta_i^2 e_i^2(k) K_i^T(k) A_i(k) K_i(k) \\ &\quad + \left(\tilde{w}_i^T(k) z_i(x(k), u(k))\right)^2 \\ &\quad + 2\tilde{w}_i^T(k) z_i(x(k), u(k)) \varepsilon_{z_i} + \varepsilon_{z_i}^2 \\ &\quad - \tilde{w}_i^T(k) P_i(k) \tilde{w}_i(k) - e_i^2(k) \end{aligned} \quad \textbf{(16)}$$

Using the inequalities

$$X^T X + Y^T Y \geq 2 X^T Y$$
$$X^T X + Y^T Y \geq -2 X^T Y$$
$$-\lambda_{\min}(P) X^2 \geq -X^T P X \geq -\lambda_{\max}(P) X^2$$

which are valid $\forall X, Y \in \Re^n$, $\forall P \in \Re^{n \times n}$, $P = P^T > 0$, then (16), can be rewritten as

$$
\begin{aligned}
\Delta V_i(k) \le & -\tilde{w}_i^T(k) A_i(k) \tilde{w}_i(k) \\
& -\eta_i^2 e_i^2(k) K_i^T(k) A_i(k) K_i(k) \\
& +\tilde{w}_i^T(k) \tilde{w}_i(k) + e_i^2(k) \\
& +\eta_i^2 e_i^2(k) K_i^T(k) P_i(k) P_i^T(k) K_i(k) \\
& +\eta_i^2 \tilde{w}_i^T A_i(k) K_i(k) K_i^T(k) A_i^T(k) \tilde{w}_i \\
& +\eta_i^2 e_i^2 K_i^T(k) P_i(k) K_i(k) \\
& +2\left(\tilde{w}_i^T z(x(k), u(k))\right)^2 + 2\varepsilon_{z_i}^2 - e_i^2(k)
\end{aligned} \quad \textbf{(17)}
$$

Then

$$
\begin{aligned}
\Delta V_i(k) \le & \left\|\tilde{w}_i(k)\right\|^2 - \left\|\tilde{w}_i(k)\right\|^2 \lambda_{\min}(A_i(k)) \\
& +\eta_i^2 \left|e_i(k)\right|^2 \left\|K_i(k)\right\|^2 \lambda_{\max}^2(P_i(k)) \\
& +\eta_i^2 \left\|\tilde{w}_i(k)\right\|^2 \lambda_{\max}^2(A_i(k)) \left\|K_i(k)\right\|^2 \\
& +\eta_i^2 \left|e_i(k)\right|^2 \left\|K_i(k)\right\|^2 \lambda_{\max}(P_i(k)) \\
& -\eta_i^2 \left|e_i(k)\right|^2 \left\|K_i(k)\right\|^2 \lambda_{\min}(A_i(k)) \\
& +2\left\|\tilde{w}_i(k)\right\|^2 \left\|z_i(x(k), u(k))\right\|^2 + 2\varepsilon_{z_i}^2
\end{aligned} \quad \textbf{(18)}
$$

Then, there exists η_i, Q_i and R_i such that $E_i > 0$ and $F_i > 0$, with

$$
\begin{aligned}
E_i(k) = & \lambda_{\min}(A_i(k)) - \eta_i^2 \lambda_{\max}^2(A_i(k)) \left\|K_i(k)\right\|^2 \\
& -2\left\|z_i(x(k), u(k))\right\|^2 - 1 \\
F_i(k) = & \eta_i^2 \left\|K_i(k)\right\|^2 \lambda_{\min}(A_i(k)) \\
& -\eta_i^2 \left\|K_i(k)\right\|^2 \lambda_{\max}^2(P_i(k)) \\
G_i(k) = & 2\varepsilon_{z_i}^2
\end{aligned}
$$

Therefore, (18) can be expressed as

$$
\Delta V_i(k) \le -\left\|\tilde{w}_i(k)\right\|^2 E_i(k) - \left|e_i(k)\right|^2 F_i(k) + G_i(k)
$$

Then $\Delta V_i(k) < 0$ when

$$
\left\|\tilde{w}_i(k)\right\| \ge \sqrt{\frac{G_i(k)}{E_i(k)}} \equiv \kappa_1 \quad \text{OR} \quad \left|e_i(k)\right| \ge \sqrt{\frac{G_i(k)}{F_i(k)}} \equiv \kappa_2
$$

Therefore, according to *Theorem 1*, the solution of (12) and (13) is stable, hence the identification error and the RHONN weights are SGUUB.

The neural identification is performed on-line, using a series-parallel configuration as illustrated in Fig. 2.

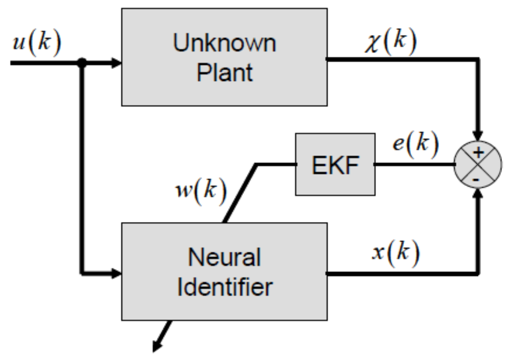

Fig. 2. Neural Identifier scheme

5 Application

In this section Real-Time results are presented for the for neural network identification scheme proposed above. The experiments are performed using a benchmark, which includes:

- *Computer Station*. A PC for supervision, with a DS1104 stand alone board for data acquisition and control, and the required software (Fig. 3).
- *Sensors*. One encoder, current sensors, and TTL to CMOS coupling (Fig. 4).
- *Electronic Power Station*. A three-phase driver, with the required IGBTs (Fig. 5).
- *Benchmark*. A three-phase squirrel cage induction motor (Fig. 4). It is important to remark that the induction motor parameters are unknown.

Fig. 3. View of the PC and the DS1104 board

Fig. 4. Encoder coupled with the induction motor

Fig. 5. PWM driver

5.1 Motor model

The six-order discrete-time induction motor model in the stator fixed reference frame (α,β), under the assumptions of equal mutual inductances and linear magnetic circuit, is given by [Loukianov, et al., 2002]

$$
\begin{aligned}
\omega(k+1) &= \omega(k)+\frac{\mu}{\alpha}(1-a)M\left(i^{\beta}(k)\psi^{\alpha}(k)\right.\\
&\quad \left.-i^{\alpha}(k)\psi^{\beta}(k)\right)-\left(\frac{T}{J}\right)T_L(k)\\
\psi^{\alpha}(k+1) &= \cos\left(n_p\theta(k+1)\right)\rho_1(k)\\
&\quad -\sin\left(n_p\theta(k+1)\right)\rho_2(k)\\
\psi^{\beta}(k+1) &= \sin\left(n_p\theta(k+1)\right)\rho_1(k)\\
&\quad +\cos\left(n_p\theta(k+1)\right)\rho_2(k)\\
i^{\alpha}(k+1) &= \varphi^{\alpha}(k)+\frac{T}{\sigma}u^{\alpha}(k)\\
i^{\beta}(k+1) &= \varphi^{\beta}(k)+\frac{T}{\sigma}u^{\beta}(k)\\
\theta(k+1) &= \theta(k)+\omega(k)T-\left(\frac{T_L(k)}{J}\right)T^2\\
&\quad +\frac{\mu}{\alpha}\left[T-\frac{(1-a)}{\alpha}\right]\\
&\quad \times M\left(i^{\beta}(k)\psi^{\alpha}(k)-i^{\alpha}(k)\psi^{\beta}(k)\right)
\end{aligned}
\tag{19}
$$

with

$$
\begin{aligned}
\rho_1(k) &= a\left(\cos(\Phi(k))\psi^{\alpha}(k)+\sin(\Phi(k))\psi^{\beta}(k)\right)\\
&\quad +b\left(\cos(\Phi(k))i^{\alpha}(k)+\sin(\Phi(k))i^{\beta}(k)\right)\\
\rho_2(k) &= a\left(\cos(\Phi(k))\psi^{\alpha}(k)-\sin(\Phi(k))\psi^{\beta}(k)\right)\\
&\quad +b\left(\cos(\Phi(k))i^{\alpha}(k)-\sin(\Phi(k))i^{\beta}(k)\right)\\
\varphi^{\alpha}(k) &= i^{\alpha}(k)+\alpha\beta T\psi^{\alpha}(k)+n_p\beta T\omega(k)\psi^{\alpha}(k)\\
&\quad -\gamma T i^{\alpha}(k)\\
\varphi^{\beta}(k) &= i^{\beta}(k)+\alpha\beta T\psi^{\beta}(k)+n_p\beta T\omega(k)\psi^{\beta}(k)\\
&\quad -\gamma T i^{\beta}(k)
\end{aligned}
\tag{20}
$$

with $b=(1-a)M$, $\alpha=\frac{R_r}{L_r}$, $\gamma=\frac{M^2R_r}{\sigma L_r^2}+\frac{R_s}{\sigma}$, $\Phi(k)=n_p\theta(k)$, $\sigma=L_s-\frac{M^2}{L_r}$, $\beta=\frac{M}{\sigma L_r}$, $a=e^{-\alpha T}$ and $\mu=\frac{Mn_p}{JL_r}$, besides L_s, L_r and M are the stator, rotor and mutual inductance respectively; R_s and R_r are the stator and rotor resistances respectively; n_p is the number of pole pairs; i^α and i^β represents the currents in the α and β phases, respectively; ψ^α and ψ^β represents the fluxes in the α and β phases, respectively and θ is the rotor angular displacement.

5.2 Neural network identification

The RHONN proposed for this application is as follows:

$$\begin{aligned} x_1(k+1) &= w_{11}(k)S(\omega(k)) \\ &\quad + w_{12}(k)S(\omega(k))S(\psi^\beta(k))i^\alpha(k) \\ &\quad + w_{13}(k)S(\omega(k))S(\psi^\alpha(k))i^\beta(k) \\ x_2(k+1) &= w_{21}(k)S(\omega(k))S(\psi^\beta(k)) + w_{22}i^\beta(k) \\ x_3(k+1) &= w_{31}(k)S(\omega(k))S(\psi^\alpha(k)) + w_{32}i^\alpha(k) \\ x_4(k+1) &= w_{41}(k)S(\psi^\alpha(k)) + w_{42}(k)S(\psi^\beta(k)) \\ &\quad + w_{43}(k)S(i^\alpha(k)) + w_{44}u^\alpha(k) \\ x_5(k+1) &= w_{51}(k)S(\psi^\alpha(k)) + w_{52}(k)S(\psi^\beta(k)) \\ &\quad + w_{53}(k)S(i^\beta(k)) + w_{54}u^\beta(k) \end{aligned} \qquad \textbf{(21)}$$

The training is performed on-line, using a series-parallel configuration as illustrated in Fig. 2. During the identification process the plant and the NN operates in open-loop. Both of them (plant and NN) have the same input vector $\left[u^\alpha, u^\beta\right]$; All the NN states are initialized in a random way as well as the weights vectors. It is important remark that the initial conditions of the plant are completely different from the initial conditions for the NN. The identification is performed using (8) with $i = 1, 2, \cdots, n$ with n the dimension of plant state $(n = 5)$.

5.3 Real-time results

In this subsection the neural network identification scheme proposed above for the discrete-time induction motor model is applied in real-time to the described benchmark. During the identification process the plant and the NN operates in open-loop. Both of them (plant and NN) have the same input vector $\left[u^\alpha, u^\beta\right]$; u^α and u^β are chirp functions with 200 volts of amplitude and incremental frequencies from 0 Hz to 150 Hz and 0 Hz to 200 Hz, respectively. The implementation is performed with a sampling time of 0.0005s. The results of the real-time implementation are presented as follows: Fig. 6 displays the identification performance for the speed rotor, plant signal is in solid line and neural signal is in dashed one, their overlap is due to the excellent performance of the neural identifier, the standard deviation for the identification error $\omega - x_1$ is $0.0896 rad/s$; Fig. 7 and Fig. 8 present the identification performance for the fluxes in phase α and β respectively, plant signal is in solid line and neural signal is in dashed one, their overlap is due to the excellent performance of the neural identifier, the standard deviation for flux identification errors $\psi^\alpha - x_2$ and $\psi^\beta - x_3$ are $0.0442 wb^2$ and $0.0263 wb^2$, respectively;. Fig. 9 and Fig. 10 portray the identification performance for currents in phase α and β respectively plant signal is in solid line and neural signal is in dashed one, their overlap is due to the excellent performance of the neural identifier, the standard deviation for current identification errors $i^\alpha - x_4$ and $i^\beta - x_5$ are $0.0840 A$ and $0.0995 A$ respectively. Finally the input signals are presented in Fig. 11.

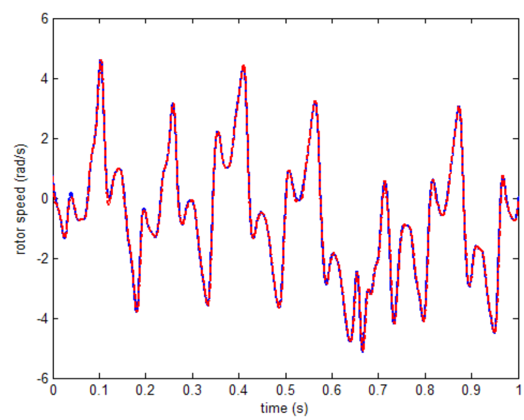

Fig. 6. Real time rotor speed identification (plant signal in solid line and neural signal in dashed line)

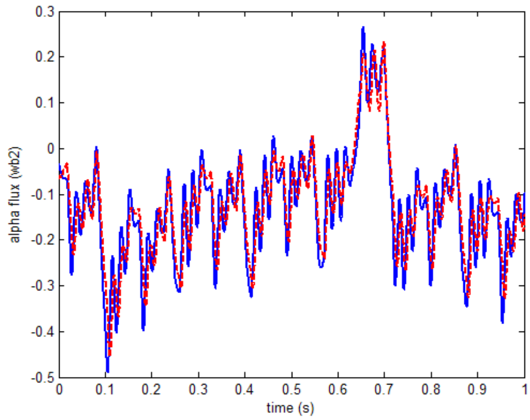

Fig. 7. Real time alpha flux identification (plant signal in solid line and neural signal in dashed line)

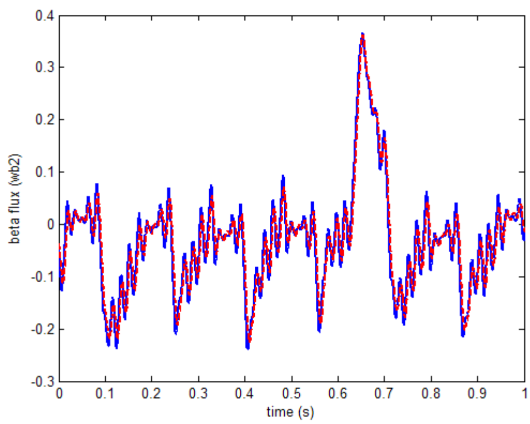

Fig. 8. Real time rotor beta flux identification (plant signal in solid line and neural signal in dashed line)

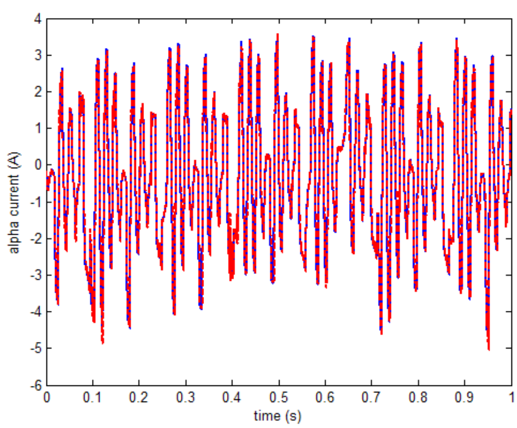

Fig. 9. Real time rotor alpha current identification (plant signal in solid line and neural signal in dashed line)

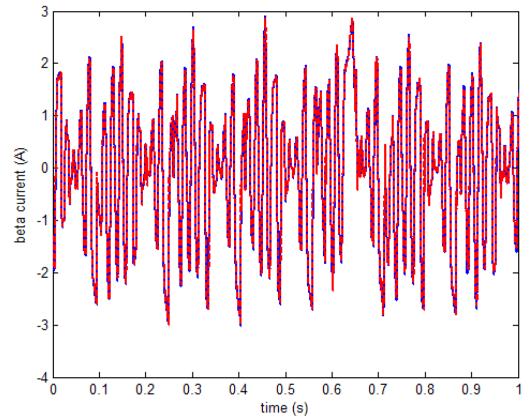

Fig. 10. Real time beta current speed identification (plant signal in solid line and neural signal in dashed line)

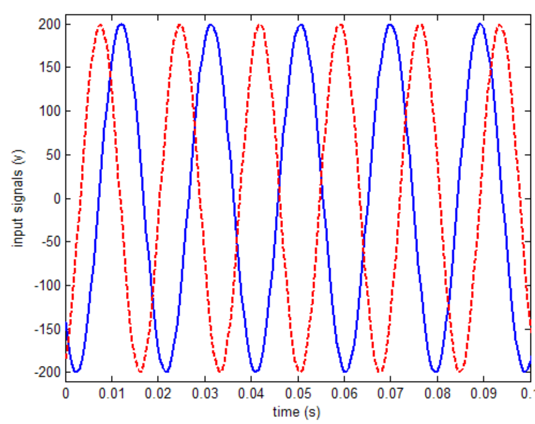

Fig. 11. Input signals applied during the identification process ($u^{\alpha}(k)$ in solid line and $u^{\beta}(k)$ in dashed line)

6 Conclusions

This paper has presented the application of recurrent high order neural networks to identification of discrete-time nonlinear systems. The training of the neural networks was performed on-line using an extended Kalman filter. The boundness of the identification error was established on the basis of the Lyapunov approach. The RHONN training with the EKF-based algorithm, presents good performance. Real-time results show the effectiveness of the proposed schemes, as applied to an electric three-phase squirrel cage induction motor. This paper deals only with on-line identification for a three phase induction motor, control synthesis and implementation based on the proposed approaches is considered as future work.

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