A Systemic Rebuttal to the Criticism of Using the Eigenvector for Priority Assessment in the Analytic Hierarchy Process for Decision Making

Una Refutación Sistémica a la Crítica de Usar el Vector Propio para Calcular Prioridades en el Proceso Analítico Jerárquico para Toma de Decisiones

Claudio Garuti Anderlini[1], Valério Pamplona Salomon [2] and Isabel Spencer González[1]
Fulcrum Ingeniería[1]
Luis Thayer Ojeda 0180 Of.1004, Providencia, Santiago, Chile
São Paulo State University[2]
Av. Dr. A. P. Cunha 333, 12.516-410 Guaratinguetá, Brazil
claudiogaruti@fulcrum.cl, salomon@feg.unesp.br

Article received on March 06, 2008; accepted on August 12, 2008

Abstract
Arguments have been provided against the use of the eigenvector as the operator that derives priorities. A highlight of the arguments is that the eigenvector solution does not always respect the condition of ordinal preference (COP) based on the decision maker's judgments. While this condition may be reasonable when dealing with measurable concepts (such as distance or time) that lead to consistent matrices, it is questionable whether it is to be expected in all situations, particularly when the information provided by the decision maker is not fully consistent. The judgments that lead to inconsistency may also contain valuable information that must be considered in the priority assessment process as well. By the other hand, the analytic hierarchy process (AHP) use the eigenvector operator to derive the priorities that represent the cardinal decision maker preferences from a pairwise comparison matrix, which do not always respect the COP condition. The AHP and still deeper the ANP (the mathematical generalization of AHP) start from concepts of ordinal metric of dominance and system theory, which is well supported by graph theory and ordinal topology with the Cesaro sum as its fundamental pillar to build metric of dominance. These mathematic concepts has no relation with COP preservation moreover, this two way of thinking are in a course of collision since the second (COP) inhibit the first (Cesaro sum).
Systems theory claims that the whole is more than its standalone components, and that internal relationships provide additional information as well. Given that the pairwise comparison matrix is an interrelated system and not just a collection of standalone judgments, we plan to show that the eigenvector, because it is a systemic operator, is the most suitable to represent and capture the behavior of the whole system and its emerging properties.

Keywords: AHP/ANP, Eigenvector, Systems, Condition of Order Preservation, Ordinal Topology and Metric of Dominance.

Resumen
Se han entregado argumentos en la literatura contra el uso del vector propio para obtener prioridades. Uno de los principales argumentos dice que el vector propio no respeta la condición de ordinalidad de preferencia (COP) obtenida del decisor. Si bien, esta condición suena razonable cuando tratamos con conceptos clásicos de medida como distancia o tiempo, que conllevan intrínsicamente niveles de consistencia completa, es cuestionable que este comportamiento deba ser esperado en todo tipo de situaciones y variables, particularmente cuando la información entregada por el decisor no es completamente consistente. Los juicios que conllevan inconsistencia, normalmente contienen información valiosa, la que debe ser considerada en el proceso de evaluación. Por otro lado, el AHP usa el vector propio para derivar las prioridades cardinales que representan las preferencias del decisor a partir de una matriz de comparaciones a pares, la que no siempre respeta la condición COP. El AHP y con mayor fuerza aún el ANP, parten de los conceptos de métrica ordinal de dominancia y de la teoría de sistemas, las que son bien sustentadas por teoría de grafos y topología de ordinales, a través de la suma de Cesaro como su pilar fundamental para la construcción de esta métrica de dominancia. Estos conceptos matemáticos no guardan ninguna relación con la preservación de COP, mas aún, estas dos formas de pensamiento se hallan en curso de colisión, ya que la segunda (COP) coarta a la primera (suma de Cesaro).
Uno de los principales pilares de la teoría de sistemas corresponde al hecho indiscutible que el todo es más importante que la suma de sus partes aisladas, y que las relaciones internas del sistema, proveen información

adicional relevante. Dado que la matriz de comparaciones a pares es un sistema interrelacionado y no una colección de juicios sueltos, nosotros planteamos mostrar que el vector propio, como un operador eminentemente sistémico, es el más adecuado para capturar y representar el comportamiento del sistema como un todo, incluyendo sus propiedades emergentes.

Palabras Clave: AHP/ANP, vector propio, Sistemas, Condición de Preservación de Orden (COP), Topología Ordinal y Métricas de Dominancia.

1 Introduction

MCDM (Multiple-Criteria Decision-Making) is the study of the inclusion of conflicting criteria in decision-making, as defined by the International Society on MCDM (2006). It is a discipline that has produced a great number of theoretical and applied papers and books, since the 60's, according to Roy (2005). A recent survey by Salomon and Shimizu (2006) shows that AHP (Analytic Hierarchy Process) and MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique) are two of the MCDM methods most often applied to solve practical problems. The operator that derives priorities in AHP is the eigenvector, while in MACBETH it is LP (linear programming).

This article is mainly oriented to rebut the criticism over the eigenvector operator. This criticism, from Bana e Costa and Vansnick (2008), is based on the "condition of order preservation": for the objects A_1, A_2, A_3 and A_4 If A_1 dominates A_2 and A_3 dominates A_4, and the decision-maker's judgments indicate that the extent to which A_1 dominates A_2 is greater than the extent to which A_3 dominates A_4, then the vector of priorities w should be such that, not only $w_1 > w_2$ and $w_3 > w_4$ (preservation of order of preference) but also that $w_1 / w_2 > w_3 / w_4$ (preservation of order of intensity of preferences). This condition (constraint) is known as the Condition of Order Preservation (COP). This condition, that may sound logical on first thought, specially when dealing with criteria that posses fully consistent metric like time or distance, in a second thought when looking at the big picture with a systemic view, this criticism leads to using a way that is arbitrary to accommodate information in a rigid framework, (probably this arose because of, an intrinsic need of MACBETH, an LP based method, to obtain a non-empty set solution), this COP condition seems to be some kind of linear vision analysis instead of systemic one.

The paper is organized as follows: Section 2 contains a short explanation of the eigenvector operator and presents four arguments in favor of using the eigenvector operator to calculate the priority vector. In Section 3, there is some discussion of the negative criticisms against using the eigenvector. Despite the fact that this section takes the criticisms from Bana e Costa and Vansnick (2008) as a central issue, this criticism is not recent and it is based on the condition of order preservation, which is not directly related to the eigenvector use. Questions about the real need of this condition and what benefits and costs would be behind are raised, such as: Why is an order condition a must when dealing with decision making systems? Is it a requirement coming from the reality or from the methodology? What benefits does the decision maker receive if applied? What are the risks and costs behind enforcing an order condition?. We shall argue that imposing the COP condition on the original judgments of the decision maker is potentially dangerous because it may have negative impacts on the DM's comparison process that might be still worse when facing group decision process, and finally, in the form of assessment of the final priority vector. Section 4 develops arguments against the criticism of the use of the eigenvector from a systemic point of view, which includes analogies with real cases and physical applications as well as theoretical background from graph theory and the Cesaro summability condition. Also, a direct relation among the eigenvector and group decision making is established and the benefits obtained from using the eigenvector and multilinear composition as has traditionally been done in AHP/ANP are highlighted. In Section 5 are the concluding remarks in which we strongly recommend continuing to use the eigenvector to determine the priority vectors and multilinear composition for the synthesis process.

2 The use of the eigenvector within AHP

This section summarizes how and why the AHP uses the eigenvector. Four favorable reasons to use the eigenvector as the process for obtaining priorities are presented.

Let A be a matrix of pairwise comparisons among n objects A_i (they can be the criteria or the alternatives): a_{ij} is the comparison between A_i and A_j. As the AHP uses a ratio scale, if we let w_i be the priority of object A_i, then $a_{ij} = w_i / w_j$. To recover the vector w, Saaty (2005) introduces Equation 1, below, a "system of equations", since it is a system of n linear equations, of the form shown in Equation 2 for i = 1 to n.

$$A\,w = n\,w \tag{1}$$

$$a_{i1}\,w_1 + a_{i2}\,w_2 + a_{i3}\;\;w_3\ldots + a_{in}\;\;w_n = n\;\;w_i \tag{2}$$

The system of equations has an important implication: since w was obtained from A, w_i will not only depend on the judgments involving A_i, but also the judgments involving the other objects. For example, Equation 2 implies that a_{13} has an impact in w_3. This way, w can be considered as a systemic operator. This means that it does not depend on a single comparison, in particular. It depends on all the comparisons, simultaneously, and builds a metric of relations (based on a ratio scale), that represent the decision-maker's set of answers. To put it another way, with a system of linear equations, the process of obtaining w does not consider one comparison to be more or less important than any other.

Equation 1 will not hold if, for at least one comparison, $a_{ij} \neq w_i / w_j$. In this case, we have $A\,w < n\,w$. From the theory of Perron, $A\,w = n\,w$ becomes $A\,w = \lambda_{max}\,w$, where λ_{max} is the principal eigenvalue of A and w is a right eigenvector of A. The eigenvector is linked to the idea of dominance, or preference, among the objects and the eigenvalue is linked to the idea of consistency of the judgments provided by the DM.

The idea of dominance is the first justification presented by Saaty (2005) for why to use the eigenvector: dominance between the objects is obtained from the normalized sum of path intensities defined by the numerical judgments. The overall dominance of an object is the sum of the entries in its row. Fig. 1a shows a graphical representation for the matrix below:

$$\begin{bmatrix} 1 & 2 & a \\ 1/2 & 1 & 3 \\ 1/a & 1/3 & 1 \end{bmatrix}$$

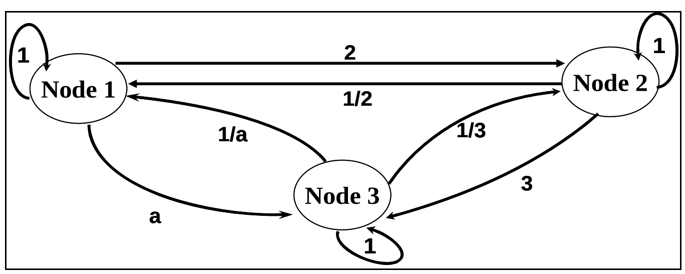


Fig. 1a. Graphical representation of pairwise comparisons

For the consistent case $a = 6$, $w = [0.6, 0.3, 0.1]^T$ and $\lambda_{max} = 3$. With $a \neq 6$, both w and λ_{max} will have small perturbations. The deviation between λ_{max} and n is an indicator of the inconsistency among the pairwise comparisons. Saaty (2005) claims a modicum of inconsistency, "is necessary to change our mind about old relations when we learn new things". Many examples, from business to sports, show that the real world allows inconsistency in the dominance of objects.

The inconsistency is related to the second justification for the use of the eigenvector: the need for the invariance of priorities. Using pairwise comparisons to determine w, means that $w_i = (a_{i1}\,w_1 + a_{i2}\,w_2 + a_{i3}\;\;w_3\ldots + a_{in}\;\;w_n) / n$. The non trivial solution of Equation 1, w, will be identical or proportional to the right eigenvector of A. As w can be normalized, the priorities are invariant only when the principal eigenvalue problem is solved.

The third justification for the use of the eigenvector is that it can be obtained with an LP optimization. Based on a field of mathematics known as *order topology*, and also using the Frobenius norm, Saaty (2005) has demonstrated that the eigenvector is obtained with the optimization of LP models shown in Equations 3 or 4.

$$\text{maximize } 1/n\ \Sigma_i\ \Sigma_j\ a_{ij}\ w_j/w_i \quad i,j=1,..,n \tag{3}$$

$$\text{maximize } 1/n\ \Sigma_j\ w_j \Sigma_i a_{ij} \quad i,j=1,..,n \ , \quad \text{subject to } \Sigma_j\ w_j = 1 \quad j=1,..,n \tag{4}$$

The last justification for the use of the eigenvector is that it is an operator with valuable and incontestable use in other practical areas, not only in decision-making. In fact, Garuti and Spencer (2007) comment on the parallelism between AHP/ANP theory and Fractal Geometry as systems. Also, an analogy of its use in decision making with the classical use of the eigenvector in the study of structural dynamics and vibrations in systems in Civil and Mechanical Engineering is presented.

3 On the negative criticism for the use of the eigenvector operator (presenting the problem)

The first negative criticism against the AHP was published in Omega, *The International Journal of Management Science* [Watson, 1982]. But the note from Belton and Gear (1983), published later in the same journal received greater notoriety. In fact, Prof. Belton became the president of the International Society on MCDM in the year 2000.

The criticism of Belton and Gear (1983) contains the fundamental aspects that Bana e Costa and Vansnick (2001 and 2008) also referred to: they are based on ideas about order preservation from classical measurement, which is not a requirement of the eigenvector operator, this is precisely one the best qualities of behavior of this system operator in order to perform a real system analysis.

As already presented in the introduction of this paper, the criticism from Bana e Costa and Vansnick (2008), is based on the "condition of order preservation": for the objects A_1, A_2, A_3 and A_4 If A_1 dominates A_2 and A_3 dominates A_4, and the decision-maker's judgments indicate that the extent to which A_1 dominates A_2 is greater than the extent to which A_3 dominates A_4, then the vector of priorities w should be such that, not only $w_1 > w_2$ and $w_3 > w_4$ (preservation of order of preference) but also that $w_1 / w_2 > w_3 / w_4$ (preservation of order of intensity of preferences). This condition (constraint) is known as the Condition of Order Preservation (COP). Some examples of the use of eigenvector violating the condition of order preservation are presented. The first example is from the following pairwise comparison matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 5 & 9 \\ 1/2 & 1 & 2 & 4 & 9 \\ 1/3 & 1/2 & 1 & 2 & 8 \\ 1/5 & 1/4 & 1/2 & 1 & 7 \\ 1/9 & 1/9 & 1/8 & 1/7 & 1 \end{bmatrix}$$

The normalized eigenvector obtained with the pairwise comparisons above is $w = [0.426, 0.281, 0.165, 0.101, 0.027]^T$. In particular, $w_1 / w_4 \approx 4.22 > w_4 / w_5 \approx 3.74$. But, as $a_{14} = 5 < a_{45} = 7$, there is a violation of the order of intensity of preference. Bana e Costa and Vansnick (2008) also criticize the rule for the *consistency index* significantly smaller than the threshold of .10, but according to the theory of the AHP the comparisons are satisfactory and do not need to be revised.

Surprisingly, if a_{45} were to be changed to 3, the order of intensity of preference will become preserved and the vector of priorities will have a small perturbation to $[0.432, 0.284, 0.170, 0.082, 0.031]^T$. The maximum difference between the elements from the eigenvectors will be less than 0.02, so that seems a small difference.

Before going into the details of the rebuttal, a basic question should be clarified: why is this condition of order preservation imposed? Or in other words, why do methods really need this condition to be preserved? For example in MACBETH, a method where priorities are derived using a linear programming (LP) approach. One of the key requirements when using LP is to ensure that the solution space is not empty, which may happen if inconsistency of order or inconsistency of intensity of order is allowed.

Because of this, it becomes necessary to preorder the alternatives. In this article, we will show that these constraints may produce an important cost to the decision making process when considering the pairwise comparison process as a system, particularly if group decision making is involved.

When we analyze the details of the criticism, several additional questions may also arise:

a.- Should a comparison provided by the DM be changed to satisfy a principle that is not necessary according to the AHP foundations?

b.- What is the reason for changing a_{45} and not some other judgment in the matrix? If a_{45} is modified, should any other judgment be reviewed to check the consistency of the DM?

c.- If the difference between the two vectors is small, "small in terms of closeness", (Garuti, 2007), and it does not change the results in practice, what is the need to make the change? What are the real costs for the DM to perform it?

To answer these and other questions raised when taking a systemic point of view, in the next section we will review the relationship between the eigenvector operator and the behavior of the system, using theory and examples that incorporate the main ideas for dealing with systems in a decision making process.

4 A Systemic Rebuttal to the Criticism of using the Eigenvector Operator for the Priority Vector Assessment

The so called COP condition, presented in the previous section is fully satisfied by using the principal eigenvector for situations where the pairwise comparison matrix is 100% consistent which of course includes all known physical measurements on ratio scales such as meters, pounds and so on. In this case the eigenvector of the matrix is obtained in the first step of the convergence process,

If the DM is not fully consistent (the normal situation), then the condition using the eigenvector may or may not be satisfied. But why should this condition be a must for the DM and hence for the system? Does it make sense to restrict the solution space through imposing this arbitrary condition in order to make the final solution vector totally compatible with the DM's initial answers? Why do the initial judgments provided by the DM have to be taken as conditions, or more specifically as constrains for the system?

4.1 Eigenvector as a systemic operator

As introduced in Section 3, the eigenvector is a systemic operator, obtained by raising the comparison matrix to powers until it converges to a limit. In this calculation, all of the individual initial comparisons provided by the DM are equally important regardless if provided at the beginning or at the end of the process of making the comparisons. The eigenvector engine treats all the initial comparisons in the matrix in the same way and simultaneously, building a metric of relations that results in a priority vector of relative absolute numbers. It represents the full set of connected judgments of the DM, without giving any one judgment more importance than any other. In this sense, the eigenvector does not include external boundary conditions. (External conditions should be handled through the modelling process).

Furthermore, from ordinal topology and the Cesaro summability property, the systemic approach of the eigenvector is analogous to path of dominance concept captured in Graph Theory. For a better understanding of the concepts related to the equilibrium stages and equilibrium conditions of a system that one wishes to solve by finding its eigenvector, the example already introduced in Fig.1a, is now reused in an extended form. In the graphical interpretation of the pairwise comparison matrix A, the length of the arc between nodes X and Y corresponds to the intensity of preference of X over Y, given by the DM. This matrix A^1 might be interpreted as the intensity matrix for paths of length 1 (one step from X to Y).

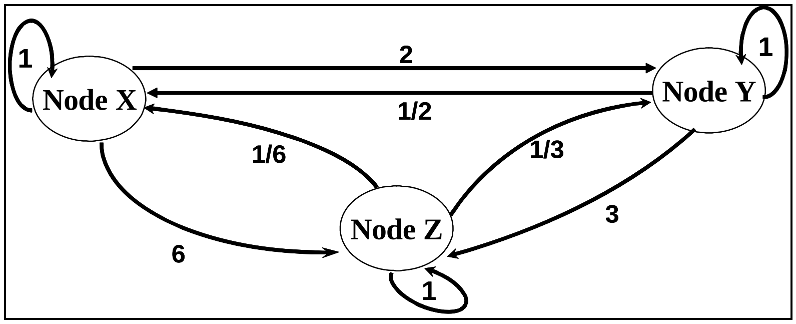


Fig. 1b. Graph representation of pairwise comparisons

If in Fig.1a, $a = 6$ (full consistency), then the intensity of preference from node *X* to node *Y* considering paths of length 2 between all the nodes is as follows:
path *X-X-X* : intensity = 1 * 1 = 1
path *X-Y-X* : intensity = 2 * (1/2) = 1
path *X-Z-X* : intensity = 6 * (1/6) = 1

Therefore the intensity of preference of node *X* to node *X* considering all paths of length 2 is: 1 + 1 + 1 = 3 $\rightarrow$ $a^2_{11} = 3$, and the average would be (3/3) = 1.

The intensity of preference of node *X* to node *Y*, considering all paths of length 2 is the sum of the intensities of all paths of length 2.
path *X-X-Y* : intensity = 1 * 2 = 2
path *X-Z-Y* : intensity = 6 * (1/3) = 2 $\rightarrow a^2_{12} = 2+2+2=6$
path *X-Y-Y* : intensity = 2 * 1 = 2

And taking the average we have (6/3) = 2 and we are back to the original judgment which we would expect for a consistent matrix.

Denoting $A^k = a^k_{ij}$ the matrix of the intensity of preferences with paths of length *k*: As *k* increases, paths of greater and greater length among the nodes are considered. The column *j* of A^k represents the global intensity of preference of k^{th} path in the k^{th} iteration (from node *j*). The column *j* of A^k represents the overall dominance of *j* over each of the other nodes after *k* iterations. The normalized column *j*, represents the relative importance of the rest of the nodes for all paths of length *k*, using as reference the node *j*. For the consistent case ($a = 6$), each normalized column of A^k corresponds to the preferences of A^1, normalized. ($A^1 = A^2 = \ldots = A^k$). As an example, Fig.2 presents the normalization values for the second iteration that correspond to the normalization values for the first iteration (with fully consistent judgments).

A^1			A^1, normalized		
1	2	6	1/(10/6)	6/(10/3)	6/10 = 0,6
1/2	1	3	1/2/(10/6)	1/(10/3)	3/10 = 0,3
1/6	1/3	1	1/6/(10/6)	1/3/(10/3)	1/10 = 0,1

A^2			A^2, normalized		
3	6	18	3/5 = 0,6	6/10 = 0,6	18/30 = 0,6
3/2	3	9	1,5/5 = 0,3	3/10 = 0,3	9/30 = 0,3
1/2	1	3	0,5/5 = 0,1	1/10 = 0,1	3/30 = 0,1

Fig. 2. The second iteration for the eigenvector computing

But, if A is not consistent (the normal situation), then the paths of length 1 will have different intensities since it will depend on the node used as reference. To compute the values in its equilibrium stage we use the Lim A^k, since for an inconsistent matrix, the sum of all the dominances along paths of length 1, 2, 3, and so on has a limit determined as a Cesaro sum. That limit is the principal eigenvector of the matrix of judgments. The total dominance $w(A_i)$, of alternative *i* over all other alternatives along paths of all lengths is given by the infinite series:

$$w(A_i) = \sum_{k=1}^{\infty} \frac{\sum_{j=1}^{n} a_{ij}^{(k)}}{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{(k)}}$$

whose sum is the Cesaro sum

$$\lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M} \frac{\sum_{j=1}^{n} a_{ij}^{(k)}}{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{(k)}}$$

Why? Note that the sums of different sets with k numbers in each, determines their ranks according to their total value. The average of each sum is obtained by dividing by k. The averages give the same ranks because they only differ by the same constant from the original sums. Often the sum of an infinite series of numbers is infinite, but if it is formed the average, that average as k tends to infinity may converge. In that case it converges to the same limit as that of the k^{th} term of the infinite sum. Thus taking the limit of the averages gives us a meaningful ranking of the objects. This is a profound observation proved by the Italian Mathematician Ernesto Cesaro. This notable result is the base of the metric of dominance, and is implemented in the AHP and deeply implemented in the ANP, the mathematical generalization of the AHP.

So, according to the Cesaro summability for dominance paths, the last iteration may replace or take into account all the previous ones (where each iteration k corresponds to all paths of length k that link node *X* with node *Y*). Hence, as k goes to infinity, A^k contains all possible paths that connect node *X* with node *Y*, considering all possible intermediate nodes. In this way, the eigenvector correctly computes the limiting dominance situation which represents the final equilibrium stage. Since: $\text{Lim}_{k\square\infty}\ [A]^k$ → principal eigenvector and this is equal to any column of matrix A normalized.

A corollary of Cesaro's summability is that when for the initial stage not all possible paths of all possible lengths are considered, (because some restriction or constraint was arbitrarily imposed – such as the COP), then the starting point of comparisons (the cell arbitrary selected as the first pairwise comparison in the matrix) may generate a difference in the results for the equilibrium stage. Applying Cesaro, and therefore using the eigenvector as the engine, the starting point of comparison becomes irrelevant, i.e. there is no point, no particular comparison that is more important than any other. And this is an important property for a process to have that is used to do system analysis in decision making. As an example, in MACBETH, which follow the COP condition, the set of comparisons that will be used to build the linear space solution complies with neither path dominance nor with Cesaro summability. This lack of being able to consider path dominance forces every comparison to strictly comply with all the others in terms of order, otherwise it is not possible to derive priorities from the space solution as defined. so the process forces the preorder step (called "precardinal information" in MACBETH), to establish a reference in further comparisons. Thus the comparison that is selected to be the initial comparison (that is, the starting point for this process) is crucial, as the following comparisons must then strictly adhere to the previous ones (in ordinal and intensity of order condition). In other words, depending on which comparison is arbitrarily selected to be first some comparisons might be more important than others, which is a clear bias. From a graphical point of view, given that not all possible paths or all possible lengths are being considered, there is less information being used than is available. Sometimes even during the comparisons in practical situations, the DM may decide that the initial ones were not accurate and back up to change them. The DM's willingness to do so will decline with this type of process, and the trend will be to simply adhere to what has been built. This anomalous situation will be strongly amplified when working with groups making decisions, because having once established an initial preordered list of elements (if possible), it becomes very difficult to change along the way, even if it becomes clear that it is needed. Participants are more likely to change the last specific comparison value which has been identified to cause trouble for the process, to simply adhere to what has been agreed upon so far. It becomes clear that this is not a systemic view kind of approach.

4.2 Physical interpretation of the eigenvector

It is interesting to notice that physically the eigenvector may be interpreted as being the equilibrium point of a system. Any system submitted to an external perturbation (no matter if energy or information), will oscillate around its equilibrium point when the perturbation is not large and in general will move into a new equilibrium point when the perturbation is large.

There are many examples in physics, biology, ecology (ecosystems), and engineering that illustrate this. Given the formal background of one of the authors, an example from the civil engineering field follows. It has analogies with decision making processes.

Consider a big building built with all sort of beams, columns and bars, constituting a real 3D complex grid that supports the weight over its base. If this building is subjected to a seismic event, i.e. a package of energy (a stimuli) is delivered in the base of the building, then the building has to respond in some way in order to dissipate that energy.

The way the building will express its response to the energy stimulus is by vibrating around its own equilibrium point, distributing the energy among its beams, columns and bars. Notice that this equilibrium point has been calculated in the last 30-35 years as the principal eigenvector of the building. In fact, the "*eigen*" name means "own" in German, and is a very appropriate name since the eigenvector represents the individual building's "own" form of response to the energy stimulus.

Buildings will vibrate (respond) in different ways when subjected to the same stimuli depending on their own characteristics: materials, geometry and foundations.

Also, resonance problems are solved through the eigenvector operator. The resonance corresponds to the "own" way of vibration of the object, building, bridge, etc... Fig.3 shows a graphic representation of the eigenvector as described above.

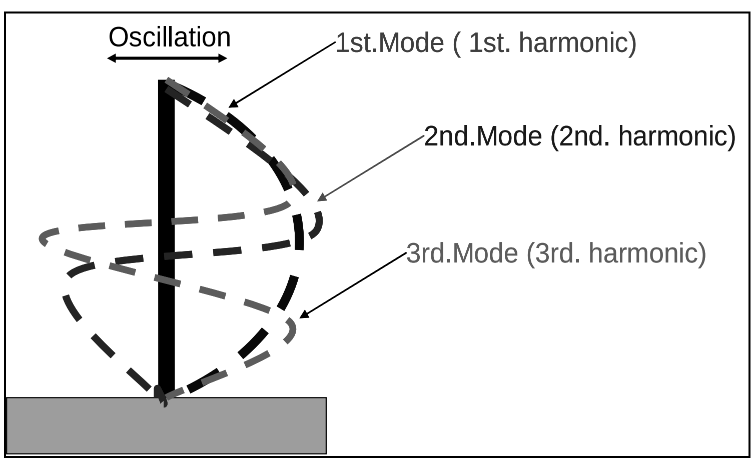


Fig. 3. Physical representation of the eigenvector for building oscillation

When the building (system) has a dominant eigenvector (main eigenvector), one of the oscillation modes that correspond to it will take the representation of the oscillation system and it will contains the greater part of the information to calculate the vibration (response) of the system. That oscillation mode becomes the vector that best represents the final equilibrium stage of the system as a whole.

The most interesting part of this analogy, extracted from physic reality, is that the response obtained through the eigenvector operator is the best approximation of the real distribution of stress along the elements of the building, as has been tested and proved with sensors measuring real stress over years.

Considering also that the seismic building response could be a very complex model, since the distribution of forces inside the building depends on many interrelated variables, it is outstanding that the eigenvector has the capability to capture all the interrelations within the building (the system). In a similar way, in a complex decision making problem, the eigenvector is able to capture the equilibrium point that represents the integrated responses of a DM to the stimuli (the set of pairwise comparisons). It is his/her "own" final vector of priorities that synthesizes all the interrelated preferences between the elements.

By the way, the use of eigenvector has been empirically tested in several numerical validation examples involving physical measurements, such as length, perimeters, areas, distances, brightness, etc., always returning correct results (Whitaker, 2007). Also the eigenvector operator has an additional advantage in that the result is a vector of absolute relative numbers, with full arithmetic capabilities, that allows the final numbers to be directly used in further calculations.

4.3 Why should the order of intensity of preferences be preserved?

Now it is possible to give a full answer to the questions raised at the beginning, including the benefits and costs, since preserving the order of intensity of preferences can be considered as an additional constraint, within the framework (system) of the DM's set of judgments. To add this constraint, a measure of closeness of two priority vectors will be introduced. This can be used to measure the real difference between two priority vectors and what this difference implies in terms of benefits and costs for the DM.

Garuti (2007) has introduced an index called the compatibility index (CI) for weighted environments, which is calculated as shown in Equation 5, where A and B are normalized priority vectors.

$$CI(A, B) = \Sigma_i (a_i + b_i)/2 \text{ x } \{Min(a_i, b_i) / Max(a_i, b_i)\} \qquad (5)$$

The idea of an index of compatibility between two vectors is a very important one, since it provides a definition for the concept of closeness and a way to measure it. In fact, Dr. Thomas Saaty has written several articles about this matter, summarized in his last book Group Decision Making: Drawing out and Reconciling Differences, (2008). (As a note, even Hilbert had defined his own Compatibility index as: Log {Max(a,b)/Min(a,b)} for non weighted space topology).

The suggested compatibility index CI may be interpreted graphically as the projection of one vector over the other. Notice that in weighted environments (where a_i and b_i represent weights), the compatibility index is well bounded and takes on values in the (0,1] range, with 0 being the situation of totally perpendicular vectors (total incompatibility) and 1 the situation of totally parallel vectors (total compatibility). If CI(A,B) takes on values equal to or greater than 90%, then for all practical purposes, vectors A and B are considered close enough to represent the same decision metric from the DM's perspective.

Let us see a numerical example: suppose that the DM in his/her own set of pairwise comparisons declares that *a* is preferable to *b*, *b* is preferable to *c*, and also that *c* is preferable to *d*; then he/she would like to see in the final priority vector that: $w(a)>w(b)>w(c)>w(d)$. (This is the condition of consistency about order of dominance).

Nevertheless, when the DM is invited to provide pairwise comparisons, the following matrix has intransitivities of order occurring, because the preference between *b* and *c* is not retained in some comparisons.

A =	*a*	*b*	*c*	*d*	
a	1	2	3	4	*a*>*b*, *a*>*c*, *a*>*d*
b	1/2	1	2/3	2	*b*<a, *b*<c, *b*>*d* (2/3: intransitivity of order)
c	1/3	3/2	1	4/3	*c*<*a*, *c*>*b*, *c*>*d* (3/2: reciprocal intransitivity)
d	1/4	1/2	3/4	1	*d*<*a, d*<*b, d*<*c*

Fig. 4a. Pairwise comparisons matrix A is not consistent

Note that the intransitivity of dominance between *c* and *b* (a_{23}) is produced **because of comparisons** in other possible paths that also relate *b* and *c*. For example, the path through element *a* indicated that *a*>*c* and also *a*>>*d*, meaning that in this specific path, *a* dominated *d* with greater intensity than *a* dominated *c*. However, in the direct comparison of elements *c* and *b*, *c* dominates *d*, which implies an order dominance inconsistency. Hence, the matrix A exhibits inconsistency of order, not following the COP condition.

If, on the other hand, the DM retains his/her initial order (initial comparison preferences) as a constraint for later pairwise comparisons, following the COP condition, then the matrix may look like:

A* =	*a*	*b*	*c*	*d*	
a	1	2	3	4	*a*>*b*, *a*>*c*, *a*>*d*
b	1/2	1	3/2	2	*b*<a, *b*>c, *b*>*d*
c	1/3	2/3	1	4/3	*c*<*a*, *c*<*b*, *c*>*d*
d	1/4	1/2	3/4	1	*d*<*a, d*<*b, d*<*c*

Fig. 4b. Pairwise comparisons matrix A* is now consistent

The eigenvector for A is w = $[0.480, 0.201, 0.199, 0.120]^T$ wit an inconsistence index of 0.03, and the eigenvector for A* is w* = $[0.480, 0.240, 0.160, 0.120]^T$ with an inconsistence index of 0.0 (full consistency).

The compatibility index for these vectors is CI(w, w*)=92.8% > 90%. So, w, w* the eigenvectors for the comparison matrices A and A*, can be considered compatible (topologically near) each other. This means that for all practical purposes the metric decisions of the DM for w and w* are the same in that they have the same rank order. That is, the decisions would be equivalent for w and w*, no matter that w present order intransitivity while w* does not. The last shows that, in the best case, the COP condition become irrelevant to compute the priority vector when using the eigenvector operator.

It is interesting to notice that with the suggested compatibility index, it is also possible to measure the closeness of two vectors associated with a rank reversal situation. For example, suppose that in vector w the positions 2 and 3 are reversed, then w = $[0.480, 0.199, 0.201, 0.120]^T$. For this new situation, the closeness measure is 92.6%, which is also sufficient to consider w and w* compatible vectors. Therefore, it may be possible that even in a rank reversal order situation (this is in a situation of totally COP violation), the properties of the whole in its final stage may overwhelm its components (individual comparisons) and this is indeed often typical of system behaviour.

Thus, in the kind of decision where the objective is to choose one alternative, if the two vectors represent two different DMs, and the first ranked alternatives are different, but the index shows the vectors to be compatible (the alternatives are close but reversed), then for practical matters it should be interpreted and meaning that, both solutions are similar and that the DMs should not make a final decision yet (if facing a non distributive type of problem). Additional steps such as performing sensitivity analysis or adding information by including additional criteria, should be taken to properly resolve the situation. It is not true that this situation will be well resolved by establishing conditions of order preservation (COP) on the DM answers.

Of course, if the first two positions in w are the ones to be shifted (0.480 and 0.201), the compatibility between the two vectors is lost and the result represents two entirely different priority vectors (that is, two different decisions or equilibrium points for the DM).

As shown in the examples above, <u>it is totally possible to build a cardinal decision metric compatible with the DM's judgments as a whole</u>, in spite of not satisfying the order of intensity of preferences among all the individual pairwise comparisons provided by the DM during the process, leaving the COP condition not only unnecessary but often dangerous for a well simulation of the system behaviour from a systemic point of view.

This example reinforces the idea that global properties of systems are stronger than the conditions or properties of isolated elements that are then compounded. In Systems Theory, this concept is related to "the emerging properties of the system" and it helps to support the idea that a systemic operator needs to be able to accommodate emerging properties. Any rule or constraint that does not allow the system to flow naturally, or behave integrally (without artificial constraints), threatens the ability of the results to really represent the system's behaviour and jeopardizes the whole system behaviour.

By the other hand, one might be tempted to replace vector w by w*, asking the DM to show a greater consistency on his/her judgments, or asking more and more questions, or using surveys and iterating until a 100% of consistency is achieved. But, as shown in the examples above, all this effort does not make any sense and could be very annoying and tiring for the DM. In fact, having a better consistency index, do not mean necessary better results (the index of consistency is a necessary condition not a sufficient one in AHP). The consistence index do not need to detect specific reversal situation or order preservation in the final priority vector, and this is because the consistency index assessment come from the greater eigenvalue of the matrix and like the eigenvector, the eigenvalue work as a systemic operator too, (this is: concerned about the matrix of comparison as whole a and not about any specific element or comparison). At the end, what really matters is to be sure that the final priority vector represents the DM's thinking and feelings without affecting his or her way of thinking and perceiving the properties of the system. As an observation, we have frequently noticed in practice that forcing the decision maker to improve their consistency can reduce the accuracy of the final priorities in representing the system. The authors have seen many students and practitioners become more worried about their consistency in providing judgments than to ensure their model is really capturing the DM's knowledge, feelings and thoughts (modelling the real system).

The criticism from Bana e Costa and Vansnick (2008) is that the eigenvector solution gives incorrect results because it does not satisfy the COP condition. It is interesting to point out that the compatibility index between the original AHP solution vector and the one suggested by Bana e Costa et.al., is within the acceptability ranges, which means that the modifications introduced were not important and produced no gain for the DM. But there is a very big cost involved, since a fundamental commandment in system analysis has been broken; that the whole is more important than its parts (even when the last are judgments).

Moreover, even if the changes produced quite different vectors in the sense that they are non compatible according to compatibility index calculations, how does one know which results are better? How can one be sure that the second set of results differ just because of the way that the DM is compelled to accommodate his/her early answers in the process to an arbitrary condition of ordinal preference, creating artificial dependencies on the system due to the starting point of comparisons?

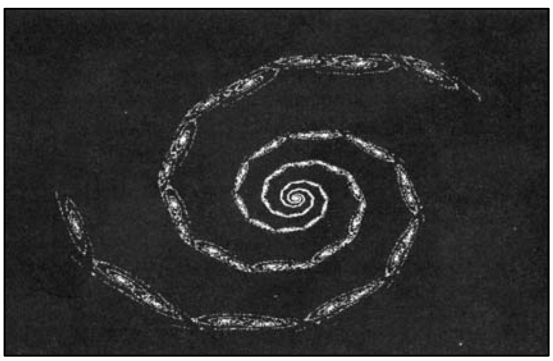
Finally, Bana e Costa reproduced the COP criticism using Saaty´s validation problem of ranking countries by their GNP, developed in 1972. But the eigenvector results were very similar to the 1972 GNP real values though the vector didn't follow their COP rule. It is interesting to note, that they give more relevance to preserving alignment to this arbitrary condition than to the fact that the process provided results close to reality, especially considering that the comparisons were provided by decision makers relying only on their common knowledge (no hard data were available when the exercise was carried out during a plane trip over the Atlantic Ocean).

Another way of answering the question raised in this section is to put it this way: "What is more important, to obey at any cost the initial inconsistencies in intensities of dominance, considering the elements more important than the whole, or focusing the efforts to best represent the DM's set of judgments as a whole, where the system represents more than its individual elements."

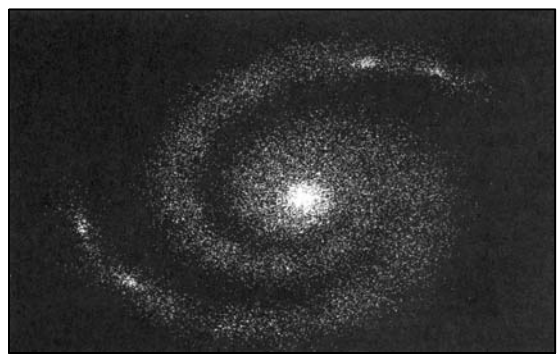
As mentioned before, the only way to ensure simultaneously the satisfaction of all the differences of dominance intensity in the eigenvector is to have a totally consistent matrix (IC=0.0). As soon as the DM is not fully consistent on his/her judgements, the priority eigenvector will reflect this (through the principal eigenvector result) and it may not fully adhere to this or any other constraint.

As practical experience shows, even in the case when the DM knows the final solution in advance, he/she may not necessary know in detail how each component of the solution behaves/relates to the rest. Somehow the scale of measuring must have been already built in his/her head (in gross terms), even though not all the information fits some predetermined format.

This might be clarified more by considering Charles Darwin's knowledge process: even if he did not know or at least did not include all the information of the evolution process (he didn't know the problem in all its components), he was able to build a theory that represented the evolution problem as a whole with its emerging properties included. So, the information of the system (as a whole) was richer and more valuable than the information of its compounding parts, even if some of the compounding parts were not available or did not perfectly fit in the big picture. In essence, what really matters when facing real life complex decisions is to comprehend and represent the whole problem, to be able to model it accurately, with no artificial constraints oriented more to the elements than the system as a whole. Of course, this approach begs for the use of a systemic operator. Nature is full of examples of systems that rely most on their ability to respond to internal/external stimuli as a whole, rather than their ability to preserve the characteristics of each individual component; in this sense, some level of internal "inconsistency", or randomness is always present. Moreover, there is actually a "need" for some inconsistency when modelling human and social systems to fit the new information that can be gathered in the decision process through for example, the different opinions of the participants. It is also interesting to note that even when modelling physical systems such as a galaxy through the use of fractal geometry, the introduction of some level of randomness improves the final representation of reality, as the following Fig.5 shows. At left, a galaxy picture built with a fractal function with no randomness, and at the right is the same fractal function with some randomness (inconsistency) included.



Fractal Galaxy without Randomness



Fractal Galaxy with some Randomness

Fig. 5. Inconsistency and Randomness Relation

In summary, considering that imposing additional constraints on any decision making problem may reduce the space solution (even to the extreme of no solution), the methodology for decision making has to strictly review which constraints are really intrinsic to the problem being analyzed so as not to jeopardize the solution.

4.4 The Eigenvector and Group Decision Making: Benefits and Risks

Even if this section is not directly related with validity of COP, it shows that the eigenvector is an operator much more appropriated to carry on a group decision making mathematically and logically. In fact, trying to apply COP condition to a group DM, may produce a rigid frame of work which at the time can produce important errors in the composite group priority vector.

From a group DM perspective, using an absolute scale for the judgments and the eigenvector to derive priorities provides several advantages, which include:

1.- The eigenvector is a system operator that seeks equilibrium stages, so it is not easy for any participant to anticipate final results and, therefore, try to manipulate them.

2.- Because it is oriented to system thinking, the eigenvector provides more flexibility in representing the full internal interactions between the individual components. This flexibility is very important when participants do not fully agree on some individual comparison, allowing them to arrive at a common grouped decision, accepted by all of them earlier and with less effort than other more rigid methods that impose arbitrary constraints on the process (like COP). Frequently, endless discussions over specific pairwise comparisons are avoided (participants may rank them in different orders) because from a system perspective, they have no significant impact on the final priority vector, due to the compatibility of the priority vectors.

3.- Since the eigenvector priority results belong to a relative absolute scale, in situations where each participant provides his or her individual priority vectors from the eigenvector, they may be combined through the use of the geometric mean to derive a compound solution. This is based on the fact the geometric mean is the only function that respects the conditions of symmetry, agreement, homogeneity, reciprocity and separability simultaneously (Saaty & Peniwati, 2008). The compatibility index can then be used to measure the deviation of any participant with respect to the group solution. When a significant individual deviation is detected, a debate can be held focused on the points that make the difference. This allows the group to understand if the individual has additional information to be shared and considered by the rest of the group, or is just generating "noise". This approach if used in a regular manner, provides a "fine tuning" option for the priorities calculation, and can produce an answer that really involves and represents the aims of the group.

In addition to these three benefits, the imposition of the COP (or other non intrinsic conditions to the process) in a group decision making session, implies at least the "getting stuck" risk. It is well known that ranking elements is not always an easy task, particularly when the list of element to be ordered is long or involves groups of people so, once a list is obtained (when possible), people will not be "receptive" to change their initial order even when knew is wrong, so this kind of situation may end up being a very rigid frame that participants will inclined to adhere to. (The getting stuck risk). So, to spend a lot of time "preordering" elements just because an arbitrary order condition has been imposed is not always a smart business, and it might be very expensive in terms of skill resources, painful for participants, and it may impact the quality of the final result. If appropriate, a more flexible approach of grouping elements in sets of higher, medium and lower importance may be used in AHP/ANP that provides the same general benefits of simplification and consistency (due to the improvement in the homogeneity degree of the sets), and with much less effort and time, while still allowing for flexibility within the groups.

5 Conclusions

In the Bana e Costa and Vansnick article criticizing the use of the eigenvector (2001 and again in 2008), they have represented the order of preference condition and the order of intensity of preference condition as fundamental conditions for every decision making process. But, as shown in this rebuttal article, these conditions are not aligned with Cesaro sum condition, graph theory and ordinal topology. This arbitrary order condition (COP) may produce a singularity over the system behaviour due to the effect of the primary comparisons over the latter, considering some path of dominance (the ones that keep COP), better or more relevant than others. Because of this situation, COP may produce significant costs to the decision making process, as well as unnecessary rigidness and actually divert the results away from being valid.

Since compatible (topologically near) decision metrics may perfectly support inconsistencies of order and also inconsistencies in intensity of order as well, it is clear that the introduction of this additional constraint of order preservation is mathematically not necessary, as they provide no relevant improvement over the eigenvector solution case. This always will happen for fully consistent situations (based on classic topology), when dealing with quantitative and measurable criteria.

On the other hand, when dealing with qualitative or intangible criteria, (the general case in decision making domain), the DM provides preferences, based on knowledge, perception, feelings and thoughts. In this case the problem relies on order topology, where dominance and intensity of dominance (paths of dominance) are more important than total consistency as there is no predefined metric to be used. We live in an imperfect metric world where inconsistency exists (of order and also of intensity of order), and it is important to understand that those inconsistencies may provide valuable information. Inconsistency must be handled within the system as it belongs to the system, it is not just noise to be deleted, as shown in the fractal galaxy example of section 4.3 (figure 5). We can't erase the randomness without loosing relevant information, one should not try to avoid/eliminated inconsistency by some arbitrary condition applied over the isolated components of the system like COP does. The inconsistency brings information inside (linked to the system behaviour).

We would like to reinforce the idea that the problem of representing the DM's preferences over a set of elements has to be seen and analyzed as a system, not as isolated comparisons that are hardly linked to initial conditions/comparisons, such as the constraints in a linear programming problem. There is no doubt that the initial conditions (the DM's initial judgments), are important to obtain a valid final system solution in the equilibrium stage, but these conditions should definitely not be treated as constraints nor imposed at any cost (as the COP implementation does). It is very important to understand that the DM is providing references, not precision or exactitude through the pairwise comparison judgments; he/she is not expressing the ultimate truth.

As it is well known, decision making problems are complex and require a global or systemic approach. The eigenvector is a systemic operator, mathematically well sounded and focused on system behaviour. Giving the same relevance to each component of the system (the pairwise comparisons) no dominance path is more relevant than other, no matter if one path obeys COP while other do not. Eigenvector operators allow some inconsistency (of order and also of intensity of order), within the judgements while still retaining the system's stability, being able to adequately represent the whole system's behaviour (the DM's set of judgments), allowing to its emerging properties to arise. Similar properties may be observed in natural evolution processes, as well as in modelling the physical world and social behaviour.

As the results of the eigenvector belong to a relative absolute scale, they may be used with any known complementary methods of optimization, such as linear programming, geographic information systems and in general any method that requires cardinal numbers of ratio scales. Also, from the perspective of group decision situations, the eigenvector and the multilinear composition strongly implemented in AHP/ANP provide interesting capabilities for controlling group discussions, consolidating judgments and optimizing the participation of skilled resources in the specific areas of their competency within the model, without imposing or needing any artificial condition of order preservation over the system.

The most important conclusion of this article is to recommend the continued use of the eigenvector along with multilinear composition as the main engine. Together they represent one of the best ways to capture all the richness in the system. Moreover, when the use of a standard metric is available, the results provided by the eigenvector are totally compatible with the ones perceived and measured from the real world. Along this line, is important to note that validation examples in AHP/ANP (and consequently the eigenvector operator) have been developed and registered in many applications over the years, complex and not complex problems, always getting results close (compatible) to the real problem solution, (Whitaker, 2007) no matter whether it follows or not the order preservation rule. This empirical verification also called "acid test", is a crucial issue for any theory that wants to survive, no matter if in the physical world (physical laws) or in the psychological or decision making world. The following phrase of L, Leshand and H, Margeneu extracted from their book "Einstein's Space and Van Gogh's Sky", 1982, clearly describes this concept: *"Scientific truth, that is to say the validity of an accepted theory, depends on two important kinds of factors: the guiding principles…and what we have called the process of empirical verification…these two factors are crucial in the establishment of any theory relating to any kind of knowledge"*.

Systems in nature have their own kind of order, and they may well accept a certain randomness or inconsistency; this apparent "lack of order" may actually be an intrinsic part of the order that provides the necessary degrees of freedom so the system can evolve. Because the eigenvector has a behaviour that allows these degrees of freedom, it will continue to be a favourite operator for determining priorities in decision making. Just as the whole is more relevant than its components, the priority vector (obtained in the final stage of dominance in the system), is more relevant than its components (that appear in the initial stage of dominance in the system) and this is a core concept of the eigenvector operator.

Given the above, care has to be taken not to apply to the system any external condition that may inhibit the emerging properties to arise, no matter how logical the condition may appear to be in a first sight. Imposing such an external condition may jeopardize the system's behaviour altering the final solution by forcing the decision maker to answer in a certain way, that contradicts the original intention producing a bias. This may happen, for instance, when the priority vector is built using a sequential process that forces the decision maker to preserve the consistency of later judgments over the prior ones, getting farther and farther away from the decision makers understanding element by element.

In synthesis, we have two clear different situations when dealing with COP:

a) when we are dealing with quantitative criteria that have a recognize and accepted metric scale (classic topology), and that the scale metric matches the DM feelings.
b) when we are dealing with qualitative criteria or quantitative criteria that have not a recognize and accepted metric scale

On the first case, to apply COP condition seems to be OK, because we are working in a zero inconsistency space. But, in the second case, COP condition is against the maths that may rule these situations, like ordinal topology and path theory. COP is also breaking one of the pillars of system theory that is the condition of superiority of the whole over its components and restricting the emerging properties of the system to arise free.

It seems that COP condition is based more in a human expectation than mathematical foundation, it has no mathematic background to contrast, is just a simple condition that *"seems"* to be right at first glance (in terms of expectation), but that is totally wrong in the DM domain.

References

1. **Bana e Costa, C.A. and J.C. Vansnick.** "MACBETH – an interactive path towards the construction of cardinal functions". *International Transactions in Operational Research*, 1: 489-500 (1994).
2. **Bana e Costa, C.A. and J.C. Vansnick.** "A critical analysis of the eigenvalue method used to derive priorities in AHP". *European Journal of Operational Research*, 187: 1422–1428 (2008).
3. **Belton, V. and A.E. Gear.** "On a shortcoming of Saaty's method of analytic hierarchies". *Omega*, 11: 228-230 (1983).
4. **Cho, K.T. and T.L. Saaty.** "Deciding on How Best to Decide: comparative evaluation of seven well-known multicriteria decision methods". Pittsburgh: Katz Graduate School of Business (1998).
5. **Garuti, C.** "Measuring Compatibility (Closeness) In Weighted Environments: When Close Really Means Close?". *9th International Symposium on the Analytic Hierarchy Process*. Viña del Mar, Chile, 2007.
6. **Garuti, C. and Spencer I..** "Parallels between the analytic hierarchy and network processes (AHP/ANP) and fractal geometry". *Mathematical and Computer Modelling*, 46: 926-934 (2007).
7. **International Society on MCDM.** *Introduction*. http://project.hkkk.fi/MCDM/intro.html, last updated in September 27, 2006.
8. **Oliver, D.** *Fractal Vision, Put Fractals to Work for You*. Indiana, Sams Publishing, 1992.
9. **Roy, B.** "Paradigms and Challenges". In: *Multiple Criteria Decision Analysis: State of the Art Surveys*, edited by **J. Figueira, S. Greco and M. Ehrgott**, pp. 3-24, New York, Springer, 2005.
10. **Saaty, T.L.** "Measuring the Fuzziness of Sets". *Journal of Cybernetics*, 4: 53-61 (1974).
11. **Saaty, T.L.** *Decision Making With Dependence and Feedback, The Analytic Network Process.* Pittsburgh, RWS, 2001.
12. **Saaty, T.L.** ''The analytic hierarchy and analytic network processes for the measurement of intangible criteria and for decision-making''. In: *Multiple Criteria Decision Analysis: State of the Art Surveys*, edited by **J. Figueira, S. Greco and M. Ehrgott**, pp. 345–407, New York, Springer, 2005.
13. **Saaty, T.L. and K. Peniwati** "Group Decision Making: Drawing out and Reconciling Differences pp.385, Pittsburgh, USA. RWS Publication (2008).
14. **Salomon, V.A.P.** "An example on the unreliability of MACBETH applications". *4th International Conference on Production Research*. Sao Paulo, Brazil, June, 2008.
15. **Salomon, V.A.P. and T. Shimizu.** "Performance of three different methods of multiple criteria decision making applied to the supplier selection". *18th International Conference on MCDM*, Chania, Greece, June, 2006.

16. **Vargas, L.G.** "Comparison of three multicriteria decision making theories: the analytic hierarchy process, multiattribute theory and outranking methods". Pittsburgh: Katz Graduate School of Business (2006).
17. **Watson, S.R.** "Assessing attribute weights". Omega, 10: 582–583 (1982).
18. **Whitaker, R.** "Validation examples of the Analytic Hierarchy/Network Processes". *9th International Symposium on the Analytic Hierarchy Process*. Viña del Mar, Chile, 2007.

Claudio Garuti Anderlini *1986, Degree of Structural Civil Engineering, at the Physics and Mathematics Faculty, University of Chile. 1988, Post Degree in "Architecture and Decision Making Process", University of Rome, La Sapienza, Roma. 20 years applying multicriteria decision making methodologies in private and publics projects. Classes and Seminars in many different Universities in Chile and around the world. Professor part-time of the MBA at the Federico Santa Maria University. Organizer of the 2007 International Symposium of AHP (ISAHP2007). Facilitator and manager for different decision making process projects.*

Valério Antonio Pamplona Salomon *obtained his Ph.D. in Industrial Engineering from the University of Sao Paulo (USP), Brazil. He is currently a full-time University Professor at the Sao Paulo State University (UNESP) when he teaches Industrial Engineering for graduate and undergraduate courses at the Engineering School of Guaratinguetá, Brazil. He also acts as ad-hoc consultant for the Foundation for the UNESP Development (FUNDUNESP), the Brazilian Association of Industrial Engineering (ABEPRO), and the Brazilian Operative Research Society (SOBRAPO). His areas of interest include the Analytic Hierarchy Process for Decision Making and the Fuzzy Sets.*

Isabel Spencer Gonzalez *1986, Degree of Mathematical Civil Engineering, at the Physics and Mathematics Faculty, University of Chile. 1988, Post Degree in "Architecture and Decision Making Process", University of Rome, La Sapienza, Roma. 20 years applying multicriteria decision making methodologies in private and publics projects. Classes and Seminars in many different Universities in Chile and around the world. Facilitator for different decision making process projects, mainly linked with resources allocation, human resources evaluation, environmental politics and army strategy decisions.*