An Algorithm for Computing Design Parameters of IFIR Filters with Low Complexity

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Abstract
This paper describes an algorithm for computing design parameters of Interpolated FIR (IFIR) digital filters with a low complexity (small number of products per output sample). The interpolation function is performed by a sharpened cascaded Recursive-Running Sum (RRS) filter, where the sharpening technique is used to improve the RRS frequency domain characteristics. Given the desired filter specifications and a chosen sharpening polynomial, the proposed algorithm computes the maximum IFIR filter interpolation factor and the minimum number of stages of the RRS filter in such a way that the overall structure meets the desired specifications with the minimum complexity.

Keywords: Narrowband filtering, Digital filters, IFIR, Sharpening technique, RRS filter.

1 Introduction

There are many applications in digital signal processing systems where a digital filter with a narrowband and sharp transition band is required. The use of a Finite Impulse Response (FIR) digital filter in such applications exhibits a number of desirable properties such as guaranteed stability and linear-phase. However due to the large length required in practical applications, the FIR filters are difficult and sometimes impossible to be implemented using conventional structures.

The interpolated FIR (IFIR) digital filter proposed by Neuvo et al. (1984) is one of the most promising approaches, [Mitra et al., 1993], for the design of low complex narrowband FIR filters and a rather well-developed topic in the field of digital filters [Mehrnia and Willson, 2004]. The IFIR filter $H(z)$ is a cascade of two filters

$$H(z) = G(z^M)I(z)$$

where $G(z^M)$ is an expanded model filter, $I(z)$ is an interpolation filter and $M$ is the interpolation factor. The narrow band FIR prototype filter $H(z)$ is designed using lower length filters, $G(z)$ and $I(z)$.
A linear increase in the interpolation factor $M$ results in the exponential growth of the interpolation filter order, as well as in the decrease of the model filter order. A key design point is to choose the right interpolation value $M$ in order to meet the desired specifications with the minimum complexity.

In order to decrease the model filter order as much as possible a recursive-running sum (RRS) filter, [Mitra, 2005], can be used to perform the interpolation function [Jovanovic-Gordana and Diaz-Carmona, 2002]. The transfer function of an RRS filter with length $M$ is given by

$$H_{RRS}(z) = \left( \frac{1}{M} \sum_{k=0}^{M-1} z^{-k} \right)^L = \left( \frac{1 - z^{-M}}{M - z^{-1}} \right)^L$$

where $L$ is the number of stages. The RRS filter is a linear-phase low pass filter that requires no multipliers and two additions per output sample, but it has a high pass-band droop and a low stop-band attenuation. The filter sharpening technique, [Kaiser and Hamming, 1984; Hartnett and Boudreaux, 1995; Samadi, 2000], can be used to improve the frequency characteristic of the RRS filter.

In this paper we propose an algorithm to find the interpolation factor $M$ and the number of the stages $L$, for a given sharpening polynomial, so that the specifications are satisfied with as low as possible complexity. Next section describes in a briefly way the filter sharpening technique. The proposed algorithm is described in section three, followed with a design example.

2 Filter Sharpening

The sharpening technique, [Kaiser and Hamming, 1984; Hartnett and Boudreaux, 1995], attempts to improve the pass-band and/or stop-band of a linear-phase FIR filter by using multiple copies of the same filter. This technique is based on the use of a polynomial approximation which maps a transfer function $H_{RRS}(\omega)$, before sharpening, to a transfer function $H_{SpRRS}(\omega)$. The sharpening polynomial is obtained using the closed formula [Samadi, 2000]:

$$H_{SpRRS}(\omega) = \delta H_{RRS}(\omega) + \sum_{j=n+1}^{R} \left( b_{j,0} - \sigma b_{j,1} - \delta b_{j,2} \right) H_{RRS}^j(\omega),$$

where:

$$b_{j,0} = \sum_{i=n+1}^{j} (-1)^{i-j} \binom{j}{i} R^i, \quad b_{j,1} = \sum_{i=n+1}^{j} (-1)^{i-j} \binom{j}{i} \left( 1 - \frac{i}{R} \right),$$

$$b_{j,2} = \sum_{i=n+1}^{j} (-1)^{i-j} \binom{j}{i} \frac{i}{R}. \quad R = n + m + 1, \quad j = n + 1, \ldots, R.$$

The design parameters are defined as:

- Parameter $n$ is the tangency order to the line of slope $\delta$ at $[H_{RRS}(\omega), H_{SpRRS}(\omega)] = (0,0)$.
- Parameter $m$ is the tangency order to the line of slope $\sigma$ at $[H_{RRS}(\omega), H_{SpRRS}(\omega)] = (1,1)$.

In this way the polynomial coefficients are always integers, which can be implemented as Shift-and-Add multipliers, [Parhi, 1999]. For instance if both line slopes are set to zero and both tangency orders are one, then the sharpening
polynomial is $H_{SpRRS}(\omega)=3H_{RRS}^2(\omega)-2H_{RRS}^3(\omega)$, which graph, shown in Figure 1, denotes an RRS pass-band improvement. The corresponding structure for this example is shown in Figure 2. In general to avoid a fractional delay branch for any sharpening polynomial the factor $(M-1)\ell/2$ must be integer, which implies to avoid even interpolation factors for odd number of stages.

![Fig. 1. Sharpening polynomial graph for $\sigma=\delta=0$ and $n=m=1$.](image1)

![Fig. 2. Sharpening polynomial structure for $\sigma=\delta=0$ and $n=m=1$.](image2)

3 Proposed Algorithm

The algorithm is based on an iterative design of the model filter $G(z)$ using following specifications:

$$\omega_{Gp} = M\omega_p, \quad \omega_{Gz} = M\omega_z,$$

$$\delta_{Gp} = \alpha\delta_p, \quad \delta_{Gz} = \delta_z,$$

$$\sigma = \delta = 0 \text{ and } n = m = 1.$$
where $\omega_{Gp}$ and $\omega_{Gs}$ are the normalized pass-band and stop-band frequencies, respectively, $\delta_{Gp}$ the maximum pass-band ripple and $\delta_{Gs}$ the minimum stop-band attenuation of the model filter $G(z)$, and $\omega_p$, $\omega_s$, $\delta_p$ and $\delta_s$ are the corresponding design specifications of the desired filter. The parameter $\alpha$ is a key factor in this design. The higher value of $\alpha$ the smaller model filter order, and vice versa. For a given value of $M$ the maximum value of $\alpha$ is determined by the sharpened RRS filter pass-band droop. Our experience says that it is useful to take the values of $\alpha$ in the range $0.5 \leq \alpha \leq 0.9$.

In Figure 3 is shown the flow diagram of the proposed algorithm. At first step, the maximum possible interpolation factor, $M = M_{max} = \pi / \omega_s$ and the minimum possible number of RRS stages $L = 1$, are selected. The choice of $M$ is a key factor to satisfy the pass-band specification, while the choice of $L$ is a key factor to satisfy the stop-band specification of the designed overall filter. Consequently, in the next steps the factor $M$ is increased until the pass-band specification is satisfied, interpolation factor searching loop, while the parameter $L$ is increased until the stop-band specification is reached, RRS stages searching loop. If as a result, an even value of $M$ and an odd value of $L$ are obtained from these searching loops, the factor $M$ is reduced by one to avoid a branch fractional delay in the sharpened RRS filter, [Webb and Munson, 1996]. In the following steps the algorithm looks for a possible further model filter order reduction by increasing the model pass-band ripple $\delta_{Gp}$, while keeping satisfied the given overall filter specification.

![Flow diagram of the proposed algorithm.](image)

The proposed algorithm complexity, number of operations performed, is strongly determined by the interpolation factor and RRS stages searching loops, this is the number of iterations required to meet the desired pass-band and stop-band specifications, respectively. The main iteration computational work load is the design of both the model filter, by
Parks McClellan algorithm, and the sharpened RRS filter, by the sharpening polynomial. Hence the total algorithm complexity depends on how fast the parameters are found for both pass-band and stop-band specifications. This speed is determined basically by two factors: 1. Desired filter specifications: in this sense the narrower pass-band the smaller number of iterations in the interpolation factor searching loop, and the smaller stop-band attenuation the smaller number of iterations in the RRS stages searching loop. 2. Sharpening polynomial structure: due the sharpening polynomial modifies the RRS pass-band and stop-band frequency magnitude response. For instance the higher the sharpening polynomial improves the RRS pass-band the smaller number of iterations in the interpolation factor searching loop, and the higher the sharpening polynomial improves the RRS stop-band the bigger number of iterations in the RRS stages searching loop.

Accordingly to the algorithm there may be next possible no convergence cases: 1. Any interpolation factor $M$ meets the pass-band specification. 2. The increment of $L$ in the RRS stages searching loop causes a not pass-band meeting for any $M$ in the interpolation factor searching loop. If the design is limited to narrowband filters the first case is completely avoided and the second case is less prone to occur. Both no convergence cases are detected with a fixed maximum number of iterations.

### 4 Design Example

The described algorithm was implemented in MATLAB and different filters have been designed in a PC PIV at 1.7 GHz. We show as design example the low pass linear-phase FIR filter, proposed in Saramaki et al. (1988), with next specifications: $\omega_p=0.05\pi$, $\omega_s=0.1\pi$, $\delta_p=0.01$, $\delta_s=0.001$. According to the proposal by Mehrnia and Willson, (2004) the optimal interpolation factor $M=4$, results in the total number of products per output sample of 25 and the total number of additions of 47.

The design parameters obtained using the proposed algorithm for the sharpening polynomial with $\sigma=\delta=0$, $m=n=1$ and $\alpha=0.6$ are: $L=2$, $M=3$ with a number of floating point operations of 112885030 FLOPS and a computation time of 3.4 seconds. The model filter order is $N_G=35$. The design parameters for the sharpening polynomial with $\sigma=\delta=0$, $m=2$, $n=1$ and $\alpha=0.6$ are: $L=3$, $M=5$, a number of floating point operations of 106420351 FLOPS and a computation time of 4.0 seconds. In this case we have $N_G=21$. In Figures 4 and 5 are shown the frequency magnitude responses for both cases.

As a matter of filter complexity comparison Table 1 shows the total number of products per output sample $NPS_T$, the total number of additions per output sample $NAS_T$, and the total delay units $NDel_T$ required for the proposed algorithm, and for algorithms proposed in [Saramaki et al.,1988], [Webb and Munson, 1996] and [Gustafsson et al., 2001]. Note that the smallest total number of products is obtained with the proposed algorithm using the sharpening polynomial with the parameters $m=n=1$. The method in [Saramaki et al., 1988] with 3 stages exhibits lower complexity than the proposed method with the sharpened polynomial $m=n=1$. However, the method [Saramaki et al., 1988] does not propose how to choose the design parameters.

<table>
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<tr>
<th>Reference</th>
<th>Method</th>
<th>$NPS_T$</th>
<th>$NAS_T$</th>
<th>$NDel_T$</th>
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<tr>
<td>[Saramaki et al., 1988]</td>
<td>1 stage IFIR</td>
<td>18</td>
<td>34</td>
<td>119</td>
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<tr>
<td></td>
<td>3 stages IFIR</td>
<td>15</td>
<td>24</td>
<td>127</td>
</tr>
<tr>
<td>[Gustafsson et al., 2001]</td>
<td>Two stages</td>
<td>20</td>
<td>36</td>
<td>90</td>
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<tr>
<td></td>
<td>Three stages</td>
<td>24</td>
<td>45</td>
<td>120</td>
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<tr>
<td>Proposed Algorithm</td>
<td>$m=n=1$</td>
<td>18</td>
<td>50</td>
<td>125</td>
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<td></td>
<td>$m=2$, $n=1$</td>
<td>11</td>
<td>55</td>
<td>198</td>
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</table>

Table 1. Filter complexity comparison between the proposed algorithm and other design methods.
Fig. 4. Design example magnitude response obtained with the proposed algorithm with sharpening polynomial parameters $\sigma=\delta=0$, $m=n=1$.

Fig. 5. Design example magnitude response obtained with the proposed algorithm with sharpening polynomial parameters $\sigma=\delta=0$, $m=2$, $n=1$. 
5 Conclusions

A direct algorithm to find the design parameters of an IFIR structure with a low complexity has been proposed, where the interpolation filter is a sharpened RRS filter. The proposal is based on an iterative design of the model filter $G(z)$ and the sharpened RRS filter until the filter desired specifications are met. As shown in the design example for a given sharpening polynomial, the proposed algorithm finds the maximum interpolator factor and the minimum number of RRS stages to meet desired specifications with small number of products per output sample. The proposed method is useful for narrowband FIR filter design.

References

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