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Dynamics of the mixing system of mexico: An application of phase diagrams

Dinámica del sistema afianzador de México: una aplicación de diagramas de fase

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Abstract

Since the application of Solvency II in Mexico, it has been implanted significant changes in regulations for insurers and sureties, which implied an increase in costs and expenses, particularly for the surety sector, since it represents a more significant regulatory burden in regard to the previous Law. This new regulation has a direct impact on insurance premium and its profits. In that sense, the aim of this paper is to analyze the dynamics between total insurance premiums and net income reported of the surety sector in Mexico; this allow to determine whether the collection of insurance premiums is enough to increase net incomes. Phase diagrams are used to recognize the temporal dynamics between variables and know their trajectories in relation to the Steady State. Based on the proposed methodology, it is noted that surety companies present globally unstable behaviors. Likewise, it is verified cases where increased risk aversion of the sureties (represented by the premiums) do not always reflect higher earnings.

JEL Code: C02, C39, C62, G22
Keywords: Solvency II; Mexican surety; Phase diagrams

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Resumen

A partir de la implementación de Solvencia II en México, se generó un cambio significativo en la normatividad para aseguradoras y afianzadoras, lo que implicó incremento en sus costos y gastos, en particular para el sector afianzador, ya que les representa una mayor carga regulatoria con respecto a su Ley anterior; teniendo un impacto directo en el costo de las primas y la generación de utilidades. El objetivo del presente trabajo es analizar la dinámica entre el total de primas emitidas por el sector afianzador y la utilidad bruta reportada por las mismas; lo anterior permite determinar si la captación de primas es suficiente para incrementar las utilidades reportadas por el sector. Se utilizan diagramas de fase con la finalidad de comprender la dinámica temporal entre las variables y conocer sus trayectorias en relación al Estado Estacionario. A partir de la metodología propuesta se señala que las afianzadoras mexicanas, a pesar de sus utilidades, presentan comportamientos globalmente inestables, asimismo, se verifican los casos donde una mayor aversión al riesgo de las afianzadoras (representadas por las primas emitidas) no siempre reflejan mayores utilidades.

Código JEL: C02, C39, C62, G22
Palabras clave: Solvencia II; Afianzadoras mexicanas; Diagramas de fase

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Introduction

The Solvency project began with the First Non-Life Directive (*Primera Directiva de No Vida*) (PDC, 1973), in which the requirements for the solvency margin are established, and in 1979 with the First Life Directive (*Primera Directiva de Vida*) (PDV, 1979). Both directives had to be modified on several occasions, until 1992 when the single regulation for the insurance market, called the Solvency Treaty, was created with the aim of establishing rules for insurance institutions to have an adequate solvency margin and to establish minimum capital requirements. Thus, at the time of the enactment of the Treaty on European Union[1], one of the most competitive insurance markets in the world already existed.

In the search to homologate the principles of Basel III with what is established in Solvency and after several years of work, on September 30th, 2015, the European Commission in charge of Solvency formulated a series of amendments, with which Solvency II was created (Commission, 2017)—this encompassed the draft revision of the capital adequacy regime for the European insurance industry. Solvency II is a series of standards that compile, among other things: the valuation of assets and liabilities, capital levels according to the risks assumed, risk management and governance, the assessment of equivalence between third country solvency regimes, and the internal model framework.

[1] Which is part of the treaties under which the European Union would subsequently be governed.

In the case of Mexico, in January 1935, the Insurance Contract Law comes into force (Cámara de Diputados d. H., 1935.) and in December 1950, came the Federal Law of Bail Institutions (Cámara de Diputados d. H., 1950), which for decades remained in force and, after a significant number of reforms, on April 4th, 2013, the Law of Insurance and Bonding Institutions (LISF for its acronym in Spanish) was published in the Official Gazette of the Federation (Cámara de Diputados d. H., 2013), based on Solvency II, which came into force in Mexico, gradually, from April 4th, 2015, for the entire insurance and bonding sector. This guideline establishes that the National Insurance and Bonding Commission (CNSF for its acronym in Spanish) is responsible for issuing the provisions referred to in the Law, before it enters into force, so on December 9th, 2014, the Single Circular of Insurance and Bonds (CUSF for its acronym in Spanish) is published in the Official Gazette of the Federation (Comisión Nacional de Seguros y Fianzas, 2014). This implementation in Mexico represents a significant change with regard to the aforementioned bonding companies, as this does not exist in Solvency II.

Since the new regulation for bonding companies is a problem, as it represents a greater regulatory burden, we propose an analysis from the perspective of optimal control for the bonding sector, more specifically, using phase diagrams. This tool has generally been used in chemical systems and robotics as shown in the work carried out by (Belmonte, Morales, Fernández-Caballero, & Somolinos, 2016), where they present the design of a multivariate, non-linear, and decentralized control for a multivariate, non-linear laboratory helicopter. (Zoujun, 2016) designs models for a web control system with an uncertain time lag. (Saini, Kumar, & Rajput, 2016) present a cascade control methodology for the non-linear exothermal chemical reactor. (Hongbin, Jian, & Yuelieng, 2016) apply an open-ended iterative learning control (ILC) algorithm to the non-linear discrete system of a mobile robot. (Chessab, 2016) suggests using a diffuse controller for the determination and control of kufasat equipped with three magnetic coils. While (Álvarez & Londoño, 2014) propose an angular speed control method to improve the action of a controller in a paper converting process. Concerning economic topics, they are widely used in rational expectation models, specifically in monetary issues where it is a matter of explaining the dynamics of prices based on monetary policy decisions. Such works can be found in (Muth, 1961), (Lucas & Sargent, 1981), (Sargent & Wallace, 1973), (Chow, 1983), as well as in the proposal of the extended and dynamic IS-LM model proposed by (Blanchard, 1981); similar to how (Laitner, 1990) uses phase diagrams to model tax changes. However, no related researches of solely the financial sector in particular have been presented.

Among the studies that deal with the analysis of the bonding sector based on the changes proposed in Solvency II, there is that of (Sirmans & Mccullough, 2017). In their study, they indicate the need to increase the information of users, who purchase services from

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insurance and bonding companies in order to improve the ratings or considerations that are taken into account for two issues: 1) to provide the insurance/guarantee service, and 2) to determine the amount or percentage that should be delivered from the information provided by customers. Similarly, (Eling & Pankoke, 2014) study the regulation of the minimum capital requirements of the insured sector in general in the European Union through the implementation of Solvency II; among its most noteworthy findings—implementing the Value at Risk (VaR) methodology—there are substantial deviations that end up causing reductions in the capital requirements for selected European companies.

In the case of (Zimmer, Grundl, Schade, & Glenzer, 2016), they focused on analyzing the reaction of a sample of 281 people who were insured to the payment of a contract that presented different levels of risk. Their conclusions highlight that individuals, rather than being risk averse, make their hiring decisions by making ambiguous decisions, which can be reflected in an increase in the probability of default and a change in the demand for insurance.

In another study, (Braun, Schmeiser, & Schreiber, 2017) evaluate the expected returns and diversification effects of a sample of different European companies based on the requirements of Solvency II. The results suggest that the measurement posed by the Solvency II standard may impact on the fulfilment of contractual obligations of insurers, causing their investments to be inclined towards risky assets in order to comply with the minimum capital reserve requested of them. However, in the case of Mexico, the studies of the bonding sector[2] are reduced to a handful of investigations that can be found, for the most part, in the bulletins and reports of the CNSF, as well as the analyses of the risk rating agencies. For example, the FitchRatings report prepared by (Martínez & Turrubiartes, 2017) indicates that, from the changes implemented in Solvency II, the technical utility of insurers and bonding companies has benefited. However, one of the causes is that the same shareholders have injected resources so that it can increase the capital of stock holders.

In that sense, the objective of the present work is to analyze the dynamics between the total of premiums issued by the bonding sector and the gross profit reported by the same, in order to determine if the capture of premiums is sufficient to increase the profits reported by the bonding sector in Mexico. Phase diagrams are used in order to understand the temporal dynamics between the variables, as well as to know their trajectories in relation to the stable or stationary state. The gross profit is considered since the aim is to analyze the economic impact that the increase of technical reserves will have in the bonding sector due to the requirements of the new regulation in the bonding sector and, according to the Single Circular of Insurance and Bonds (CUSF) (CNSF, 2018), in its annex 22.1.2 *"Accounting standards applicable to institutions, mutual societies, and holding companies"*, in Series III, *"Criteria relating to the consolidated basic financial*

[2] Unlike the insurance sector, where more research is contemplated, there is not an abundance of literature specializing in the subject.

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statements", section "a" Minimum Catalogue (C-1), where the concepts that are a part of the Profit and Loss Statement of Mutual Societies and Institutions are listed, in section 2 *"Results statement"*, it is stated that:

Technical Profit (Loss) = Premiums Issued - Premiums Relinquished - Retention Premiums - Net Increase in the Reserve for Current Risks and Bonds in Force - Accrued Retention Premiums - Acquisition Net Cost - Losses Net Cost, Claims, and Other Obligations Pending Performance - Claims Recovered from Reinsurance and Non-Proportional Reinsurance Reinforcement

Gross Profit (Loss) = Technical Profit (Loss) - Net Increase of Other Technical Reserves - Results of Analogous and Related Operations

Furthermore, the CUSF (CNSF, 2018), in its Title 21, Chapter 21.1, indicates that analogous and related operations belong only to insurance institutions. Thus, in the case of bonding companies:

Gross Profit (Loss) = Technical Profit (Loss) - Net Increase in Other Technical Reserves

The research is divided into the following sections: first, the characteristics of the bonding sector in Mexico, as well as the way Solvency II affects them, are described; the following section deals with the methodological aspects, and a dynamic system that captures the relation between the premiums issued and the gross profit of each bonding company is captured and finally reflected in the phase diagrams; the analysis of results is shown in the subsequent section; finally, the conclusions of the document are presented.

Integration of the Bonding System in Mexico

According to the LISF, a bonding institution is a corporation authorized to organize and operate as such, its object being the granting of bonds for valuable consideration[3]. A bond is a contract by which a bonding institution (guarantor) guarantees the fulfilment of obligations with economic content, contracted by a natural or legal person (client) before another private or public natural or legal person, in the event that the former does not comply.

According to figures published in the Analysis Bulletin of Insurance and Bonds as of December 31st, 2015 (Fianzas, 2015), the bonding sector in Mexico is made up of 15 companies; the branches in which they operate are shown in Table 1:

[3] According to the definition of the Insurance and Bonding Institutions Law, article 2, section XVII.

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Table 1
Institutions from the bonding sector, name and area

COMPANY	Fidelity	Judicial	Administrative	Credit
ACE Fianzas Monterrey, S.A.	Yes	Yes	Yes	Yes
Afianzadora Aserta, S.A. de C.V., Grupo Financiero Aserta	Yes	Yes	Yes	Yes
Afianzadora Fiducia, S.A. de C.V.	No	No	Yes	No
Afianzadora Insurgentes, S.A de C.V., Grupo Financiero Aserta	Yes	Yes	Yes	Yes
Afianzadora Sofimex S.A.	Yes	Yes	Yes	Yes
CESCE Fianzas México. S.A. de C.V.	No	Yes	Yes	No
Chubb de México, Compañía Afianzadora, S.A. de C.V.	Yes	Yes	Yes	Yes
Crédito Afianzador, S.A., Compañía Mexicana de Garantías	Yes	Yes	Yes	Yes
Fianzas Asecam, S.A.	Yes	Yes	Yes	Yes
Fianzas Atlas, S.A	Yes	Yes	Yes	Yes
Fianzas Dorama, S.A.	Yes	Yes	Yes	Yes
Fianzas Guardiana Inbursa, S.A., Grupo Financiero Inbursa	Yes	Yes	Yes	Yes
Mapfre Fianzas S.A.	Yes	Yes	Yes	Yes
Primero Fianzas, S.A. de C.V.	Yes	Yes	Yes	Yes
Zurich Fianzas México	No	Yes	Yes	Yes

Source: Own elaboration with data from the National Insurance and Bonding Commission (CNSF).

The objective of the new Law is to preserve the solvency and stability of the institutions, to promote an adequate market conduct considering the users, and to improve the calculations of valuation of the reserves and investments, for a greater dissemination of the respective information to the directors, the shareholders, the market, and the supervisory authorities. In other words, the new regulation implies the imposition of a relatively high cost associated with the requirements of corporate governance and its functioning (Gavira & Castillo, 2017). Under this scheme, small bonding companies, which are usually entirely national capital and do not belong to financial

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groups, are the ones with the greatest impact when operating in accordance with the new regulation; this in turn has a direct impact on consumers and, therefore, on the cost of premiums. Institutions with foreign matrices, on the other hand, have certain advantages when implementing the new regulation, with which they will be able to take greater advantage of small institutions.

At the end of 2015, of the 15 bonding institutions registered with the CNSF, 6 are affiliated institutions that do not belong to a financial group, 3 institutions have national capital and belong to a financial group, and 6 are national capital institutions that do not belong to a financial group. The 10 institutions that have national capital and the 6 that do not belong to financial groups are shown in Table 2:

Table 2

Institutions with a majority of national capital, Bonding Sector, at the end of 2015

COMPANY	2015
Afianzadora Aserta, S.A. de C.V., Grupo Financiero Aserta	CNGF
Afianzadora Punto Aserta, S.A.	CNGF
Afianzadora Fiducia, S.A. de C.V.	CN
Afianzadora Insurgentes, S.A de C.V., Grupo Financiero Aserta	CNGF
Afianzadora Sofimex, S.A.	CN
Crédito Afianzador, S.A., Compañía Mexicana de Garantías	CN
Fianzas Asecam, S.A.	CN
Fianzas Atlas, S.A	CN
Fianzas Dorama, S.A.	CN
Fianzas Guardiana Inbursa, S.A., Grupo Financiero Inbursa	CNGF
CN: Institutions with a majority of national capital.	
CNGF: Institutions with a majority of national capital and which are a part of the Financial Group.	

Source: Own elaboration with data from the National Insurance and Bonding Commission (CNSF).

Regarding the Mexican insurance sector, at the end of 2015, the total premiums issued was of 395,082.7 million pesos (98.3% direct insurance), direct premiums presented a real annual increase of 7.3% over the end of the previous year, above the GDP growth of 2.5%. The net profit of the insurance sector for the year was of 21,214.4 million pesos, 17.4% lower in real terms compared to the end of the previous year. At the end of 2015, total premiums issued by the sector amounted to 9,494.1 million pesos (96.2%).

The direct premiums of the sector had a real annual increase of 1.1% at the end of 2015, with respect to the same period for the previous year. This result is below the growth of national GDP, which was 2.5% in the same period. The net profit of the sector for the year was of 1,467.2 million pesos, which represents a decrease, in real terms, of 11.6% with respect to that reported at the end of December 2014. Considering the premiums issued in 2015, the bonding sector represents 2.4% of the insurance sector and that the legal, technical, systems, regulatory reports, constitution of reserves, capital, etc., represented by the new Mexican regulation is the same for both sectors. Hence, the implementation of this Law represents a greater economic impact for bonding companies.

Methodological Aspects

The methodological proposal of this work consists of using phase diagrams with the purpose of understanding the temporal dynamics between the total premiums issued by the bonding sector and the gross profit reported by them, likewise, the trajectories in relation to the stable or stationary state are exposed. Table 3 presents the Mexican bonding companies used in this investigation:

Table 3
Bonding companies considered for the study

Bonding Company	Short name
ACE Fianzas Monterrey, S.A.	Monterrey
Afianzadora Aserta, S.A. de C.V., Grupo Financiero Aserta	Aserta
Afianzadora Fiducia, S.A. de C.V.	Fiducia
Afianzadora Insurgentes, S.A. de C.V., Grupo Financiero Aserta	Insurgentes
Afianzadora Sofimex, S.A.	Sofimex
Chubb de México, Compañía Afianzadora, S.A. de C.V.	Chubb
Crédito Afianzador, S.A., Compañía Mexicana de Garantías	Mexicana
Fianzas Asecam, S.A.	Asecam
Fianzas Atlas, S.A.	Atlas
Fianzas Dorama, S.A.	Dorama
Fianzas Guardiana Inbursa, S.A., Grupo Financiero Inbursa	Inbursa
Mapfre Fianzas, S.A.	Mapfre
Primero Fianzas, S.A. de C.V.	Primero

Source: Own elaboration with data from the National Insurance and Bonding Commission (CNSF).

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The data used in the study was taken from the National Insurance and Bonding Commission (CNSF) from the second quarter of 2007 to the fourth quarter of 2015, with a total of 36 quarterly observations for each bonding company, resulting in a total of 468 available observations (32 units for 11 moments in time). The variables used are shown in Table 4:

Table 4

Variables proposed for the study

Variable	Description	Units
Premium	Premiums Issued	Thousands of pesos
Profit	Gross Profit	Thousands of pesos

Source: Own elaboration with data from the National Insurance and Bonding Commission (CNSF).

The variables expressed in Table 4 are deflated at prices of the last quarter recorded in 2015. Subsequently, the variables are transformed into fixed base indices, the base period also being the last quarter of the temporality of the study; the purpose of transforming monetary amounts into indices is to facilitate the comparison and handling of quantities.

The analysis of the dynamics of the system of bonding companies understood by the variables ***PREMIUM*** and ***PROFIT*** , departs from a system of differential equations that need to be solved simultaneously. For example, with ***x = PREMIUM*** and ***z = PROFIT*** being a system with two first-order ordinary differential equations:

$$\frac{dx}{dt} = \dot{x} = f(x, z, t) \quad (1)$$

$$\frac{dz}{dt} = \dot{z} = g(x, z, t) \quad (2)$$

The solution functions are time-differentiated functions ***(t)*** for the system of differential equations represented by (1) and (2). The initial conditions for the system are then proposed, which take the form of:

$$x_0 = x(t_0) \quad \text{and} \quad z_0 = z(t_0) \quad (3)$$

In addition, it is based on a condition of equilibrium: with (x^*, z^*) being a point where simultaneously $f(x^*, z^*) = 0$ and $g(x^*, z^*) = 0$, thus $\frac{dx}{dt} = 0$ and $\frac{dz}{dt} = 0$ are fulfilled.

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The aforementioned indicates that the considered variables do not change in time. In that sense, the system has a fixed point or point of equilibrium, also known as stationary or stable state.

Returning to the system set out in (1) and (2), the differential equations can be obtained for the variables, i.e., for both the ***PREMIUM (x)*** and for the ***PROFIT (z)*** In this sense, the dynamic system is expressed in the following manner:

$$\dot{x} = \theta x + \beta z + j \qquad (4)$$

$$\dot{z} = \gamma x + \delta z + k \qquad (5)$$

Where equations in (4) and (5) describe the motion of endogenous variables over time while **θ,β,γ** and **δ** are the coefficients associated to the endogenous variables ***x,z***. In both equations, ***j*** and ***k*** represent the constants of the system. In equation (4), the variations of ẋ or the premiums of the insurers depend positively on it and positively on the endogenous variable ***PROFIT***, analogous to **ż**, which also depends positively on the endogenous variables[4].

To comply with the equilibrium condition where $\frac{dx}{dt} = 0$ and $\frac{dz}{dt} = 0$, equations (4) and (5) are zeroed and the second endogenous variable is cleared in terms of ***x*** so that the equations can be solved simultaneously. Therefore, we have:

$$z = \frac{\theta}{\beta}x + \frac{j}{\delta}$$

$$z = \frac{\gamma}{\delta}x + \frac{k}{\delta}$$

All the information contained in the differential equations is united in a single graph, in this case, in a phase diagram. These diagrams have the particularity that it is possible to observe if a system moves towards a fixed point or equilibrium, or if it moves away from it. Specifically, a path approaches a fixed point if $x(t) \rightarrow x^*$ when $t \rightarrow \infty$. If so, then an "attractor" is present. When ***x(t)*** moves away from x^* when $t \rightarrow \infty$, then a "repellant" is present.

However, the situation where the trajectory first moves towards the fixed point and then moves away from the stable state may also arise, this is known as "saddle point" or "stability corridor". Figure 1 illustrates the three different cases that phase diagrams can present.

[4] The variations of **x** and **z** do not always depend positively on themselves, this situation is defined by the sign associated with the coefficients of the endogenous variables; this point is addressed later on in the section on system stability.

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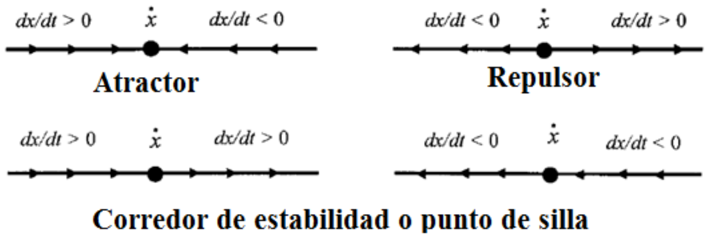

Figure 1.Fixed points, attractors, and repellants

Source: Adaptation from (Shone, 2002).

In this way, the arrows presented in Figure 1 in the phase diagrams show the regions of the trajectories that behave as attractors, repellants, or fixed points with respect to x^*,, i.e., the direction of change from $x(t)$ when t increases. This is a fundamental part of the study because it makes it possible to know if the dynamic system is globally stable (when any point $x(t)$ in the diagram behaves as an attractor), globally unstable (if any point $x(t)$ of the diagram behaves as a repellant), or if there is a stability corridor (when the directions are different with regard to x^*). Once the system of differential equations proposed for the study has been explained, the following sub-section analyses the stability of the dynamic system for each bonding company.

System Stability

In order to know if the dynamic systems proposed for the bonding companies present stable or unstable trajectories, it is considered convenient to specify the differential equations in terms of matrices and vectors. Therefore, we have:

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = A \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} j \\ k \end{bmatrix} \quad (8)$$

Where the matrix that contains the j and k coefficients refers to the constants of the system, while A is the matrix of coefficients associated to the endogenous variables x,z .
Said coefficients are the same ones expressed in equations (4) and (5).

$$A = \begin{bmatrix} \theta & \beta \\ \gamma & \delta \end{bmatrix} \quad (9)$$

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Taking the initial conditions into consideration, the equilibrium point is reached when $\frac{dx}{dt} = 0$ and $\frac{dz}{dt} = 0$, that is:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{10}$$

To examine the stability of the system we proceed to calculate the eigenvalues associated with the matrix system posed in equation (8). For homogeneous linear systems, if the determinant of $A \neq 0$, then the only point of equilibrium is obtained when $x^* = 0$ and $z^* = 0$, that is:

$$Det\,[A - \lambda I] = 0$$

$$Det = \left[\begin{bmatrix} \theta & \beta \\ \gamma & \delta \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right] = Det \begin{bmatrix} \theta - \lambda & \beta \\ \gamma & \delta - \lambda \end{bmatrix} \tag{11}$$

From the above we obtain an equation in the form of $\lambda^2 - \lambda(\theta + \delta) + (\theta\delta - \beta\gamma)$ where the solution is:

$$\lambda_1, \lambda_2 = \frac{-(\theta + \delta) \pm \sqrt{(\theta + \delta)^2 - 4(\theta\delta + \beta\gamma)}}{2} \tag{12}$$

The sign associated to the roots λ_1, λ_2 depends on two aspects: on the previous sign to the square root $-(\theta + \delta)$ and on the sign that is observed once the root has been resolved. It is emphasized that the key of the eigenvalues is the sign of the root, in a concrete way, the cases that can be obtained from the result of the signs associated to the eigenvalues are the following ones:

$\begin{bmatrix} - & \beta \\ \gamma & - \end{bmatrix}$ Globally stable system (13)

$\begin{bmatrix} + & \beta \\ \gamma & + \end{bmatrix}$ Globally unstable system (14)

$\begin{bmatrix} + & \beta \\ \gamma & - \end{bmatrix}$ o $\begin{bmatrix} - & \beta \\ \gamma & + \end{bmatrix}$ Fixed point or stability corridor (15)

A globally stable system is associated to negative signs in the main diagonal, in this case, any point of the phase diagram presents an attractor. Furthermore, when eigenvalues have associated positive values then any point of the phase diagram moves away from the stable state

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and repellants appear. Finally, case (15) alludes to the stability corridor, where the directions are different with respect to x^*.

It should be noted that the concept of stability does not insist that the trajectory followed by the variables has to approach the fixed point of equilibrium. As mentioned above, a fixed or equilibrium point (x^*, z^*) that satisfies condition $f(x, z) = 0$ and $g(x, z)= 0$ is stable or an attractor given a value (x_0, z_0) and close to the point (x^*, z^*). In this way, the signs associated with the eigenvalues determine the stability or instability of the system. The following section presents the results obtained for each bonding company.

Analysis Of Results: Phase Diagrams

After having developed the system of differential equations proposed for each of the bonding companies, the phase diagrams that correspond to each one and which comply with the equilibrium conditions presented are shown. It should be noted that the matrix of coefficients A present in equation (9) does not always who positive signs in all its components. This aspect is relevant as it defines both the inclination of the slopes and the intersection of the lines, in addition to the sign associated to the eigenvalues.

Since the proposed dynamic system is made up of linear equations, for the pairs of data associated with the variables (x, z) of each bonding company, the standard deviation of **x** and *z* is calculated and represented by the symbols σ_x and σ_y , respectively. Subsequently, the p linear correlation coefficient is calculated, which measures the strength and direction of a linear relation between ***PREMIUM*** and ***UTILITY***, so that each coefficient associated to the A matrix represented by equation (9) generally follows the formula of the slope:

$$\theta, \beta, \gamma, \delta = \rho \frac{\sigma_Z}{\sigma_x} \qquad (16)$$

Table (2) shows the coefficients obtained[5] in matrix A for each of the bonding companies, i.e., the coefficients associated with $\theta, \beta, \gamma, \delta$. When looking closely at the signs associated with the coefficients θ and δ, it is possible to know in advance whether the dynamic system of each entity is stable or unstable, given that θ and δ are the terms associated to obtain the eigenvalues posed in equation (12).

[5] All the calculations presented in the study are done with the R-Project software version 3.3.0. Available at https://www.r-project.org/.

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Table 5

Coefficients associated with matrix A

Bonding Companies	θ	β	γ	δ	j	k
Monterrey	0.6475	0.4851	0.2875	0.8890	-0.6792	-0.7286
Aserta	0.3177	0.6478	-0.1024	0.8050	-0.2745	-0.1624
Fiducia	0.2604	0.1203	-0.0790	0.9654	-0.0030	-0.0174
Insurgentes	1.1248	-0.2282	0.4342	0.2439	-0.2975	-0.1691
Sofimex	0.0011	1.2762	-0.3858	1.5047	-0.5190	-0.4658
Chubb	0.5692	0.0904	-0.0174	0.7136	-0.0721	-0.0516
Mexicana	0.2318	0.2951	-0.3217	1.1296	-0.0109	-0.0218
Asecam	-0.1002	0.7983	-0.6691	1.4433	-0.0170	-0.0177
Atlas	0.8772	0.2795	0.3686	0.7893	-0.1172	-0.1183
Dorama	0.8719	0.0708	0.4690	0.4784	-0.1193	-0.1218
Inbursa	0.8418	0.2484	0.0967	0.4235	-0.4085	-0.0834
Mapfre	0.4822	0.0882	0.0026	0.4686	-0.0136	-0.0047
Primero	0.9698	0.1589	0.4515	0.3864	-0.1714	-0.1139

Source: Own elaboration with R-Project.

Given the results of the coefficients presented in Table 5 and observing the signs of the eigenvalues of the differential equations that comprise the dynamic system of each bonding company, the result is that only the Asecam bonding company presents a stability corridor; the other bonding companies present an unstable behavior as to their premium-profit relationship.

The following are the phase diagrams that correspond to the behavior of the dynamic system of each bonding company. The arrows of the diagrams refer to the direction of change of and when . In Figure 2 the bonding company Monterrey is presented, the red line and the blue line satisfy the behavior of equations (8) and (9) where $\frac{dx}{dt} = 0$ and $\frac{dz}{dt} = 0$, i.e., they start from the condition of equilibrium; the diagram shows a globally unstable behavior, as all the arrows in the diagram are repellants.

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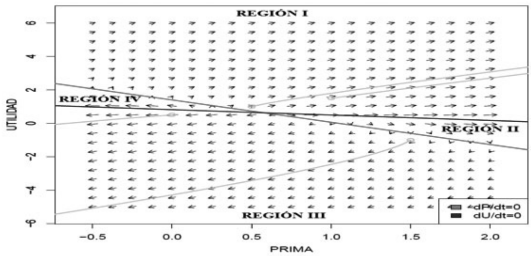

Figure 2.Phase diagram for Monterrey
Source: Own elaboration based on (Grayling, 2014) R-Project.

It should be noted that the points with the yellow curves have random coordinates shown by the attractors/repellants. As mentioned above, Monterrey has repellants in all its points, so it always moves away from the point of equilibrium. However, this does not necessarily mean something negative. For example, the corresponding premium values in region I would lead to increases in profit, but if premium amounts moved in region II, we would have higher premiums that would lead to a decrease in profit. In region III and IV it is observed that a strategy of reduction of premiums would significantly harm the profit of Monterrey. As for the trajectories of the curves, it is suggested that when the bonding company takes a more aggressive position or is more averse to risk (by increasing the amount of its premiums) it is not reflected in a greater profit, on the contrary, it causes it to decrease.

Additionally, the phase diagrams show the strategy used by the bonding companies from the trajectories that follow the differential equations, for example, in the case of Monterrey the slope of the profit curve is negative and takes both positive and negative values; the slope of the premium curve is also slightly negative and what is observed is that as the value of the premium so do the profits. Under the same logic, Table 3 presents four cases to analyze based on the slopes of the trajectories of the differential equations:

Table 6

Cases based on the slopes of the differential equations $\dot{x}_1$ and $\dot{x}_2$

Case 1	Slope	Case 3	Slope
Profit	Positive	Profit	Positive
Premium	Positive	Premium	Negative
Case 2	Slope	Case 4	Slope
Profit	Negative	Profit	Negative
Premium	Positive	Premium	Negative

Source: Own elaboration.

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In the first case we have the Asecam bonding company, which is the only one that has positive slopes in its trajectories; in this case, it is observed that the utility is elastic before the variation of the premiums, when Asecam is more averse to the risk, its utilities increase. This can be seen in Figure 3:

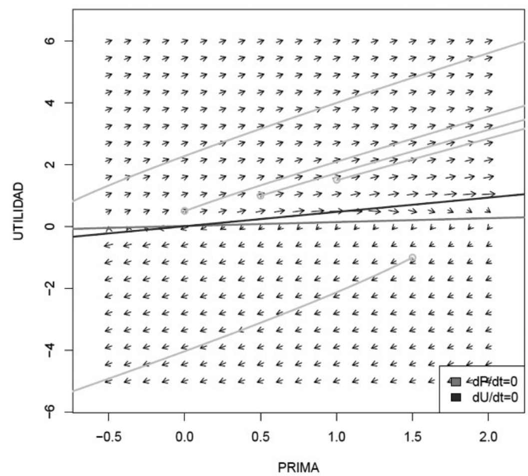

Figure 3.Phase diagram for Asecam

Source: Own elaboration based on (Grayling, 2014) R-Project.

For the following case, where the slope of the profit is negative and the slope of the premium is positive, only the Insurgentes bonding company presents such behavior. It can be observed that when Insurgentes assumes a more adverse position to risk (raising its premiums), its profits tend to fall sensibly, although, when the premiums reach the end of region I, these can take the system to a higher level of profits. Figure 4 shows this behavior.

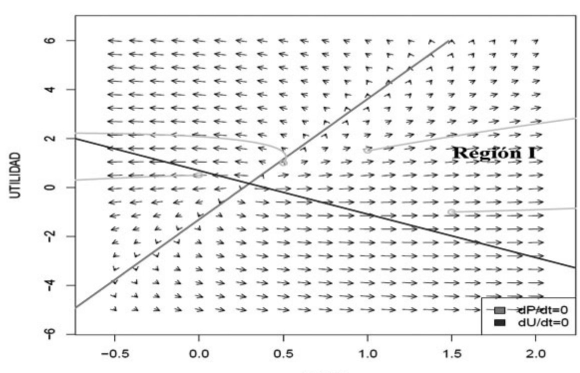

Figure 4.Phase Diagram for Insurgentes

Source: Own elaboration based on (Grayling, 2014) R-Project.

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Following the strategies that the bonding companies may present according to the slope comparison presented in Table 6, when the slope of the profit is positive and the slope of the premium is negative, the majority of the bonding companies present such behavior: Aserta, Fiducia, Sofimex, Chubb, Mexicana, and Mapfre. Specifically, when the slopes of the curves of the aforementioned bonding companies are more averse to risk, it can be seen translated into a higher level of profit. However, the companies Chubb, Fiducia, and Mapfre, although they have the same slopes as case three, differ significantly in their behavior since the trajectory that follows the profit is close to being perfectly elastic, therefore, these bonding companies are more sensitive to variations in their premiums. The corresponding phase diagrams are presented in Figure 5 below.

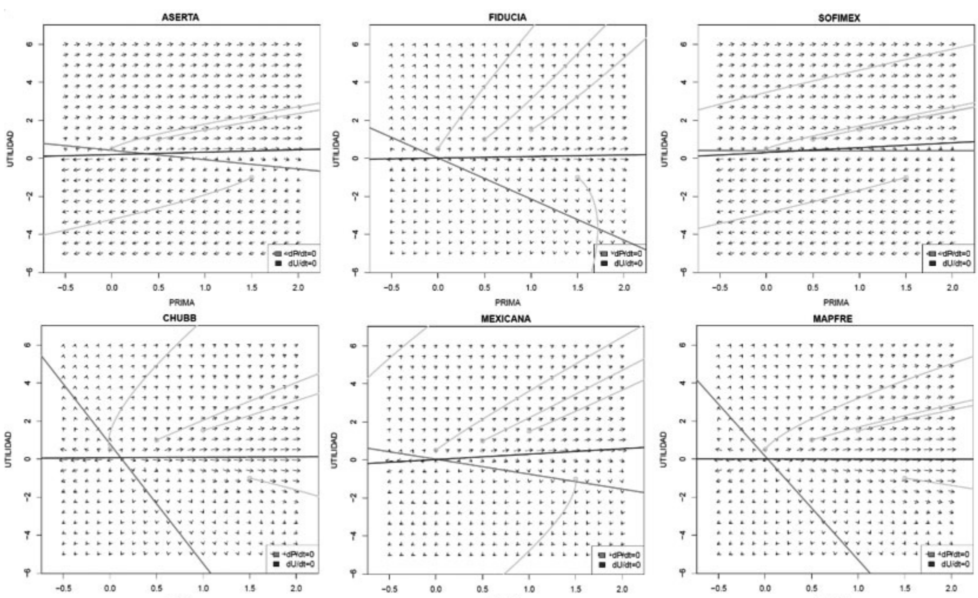

Figure 5.Phase diagram for case 3: Aserta, Fiducia, Sofimex, Chubb, Mexicana, and Mapfre

Source: Own elaboration based on (Grayling, 2014) R-Project.

Finally, the bonding companies: Monterrey, Atlas, Dorama, Inbursa, and Primero represent the fourth case in which both the slope of the profit trajectory and the premiums are negative, although Inbursa has a slope close to perfect elasticity in its profit trajectory. In case 4 it is observed that when there is an aggressive strategy of premium increase, the profits start to fall. Although all the bonding companies of this case have the particularity that, in region I, the increases of premiums located in that region move the system in such a way that any

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amount of premium or strategy that is followed would provoke a positive variation in its profits. Figure 6 shows case 4; it is worth noting that the Monterrey graph is not shown since it was initially presented to describe the interpretation of the phase diagrams.

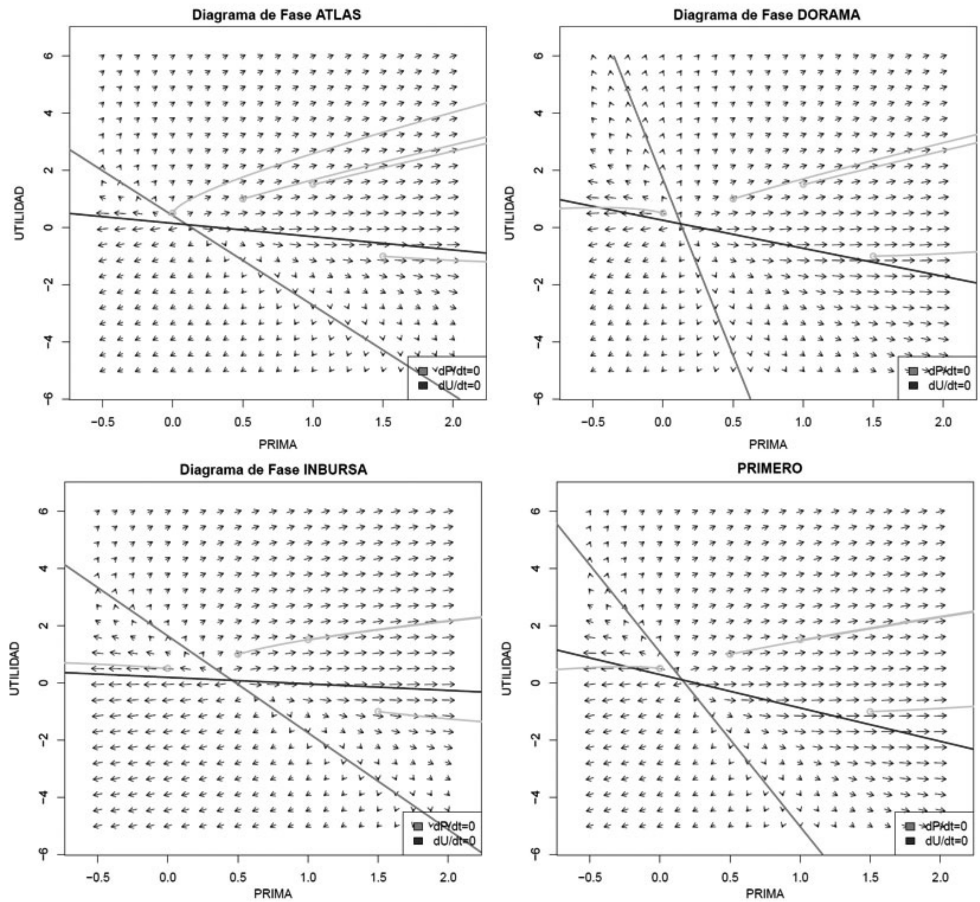

Figure 6.Phase diagram for case 3: Monterrey, Atlas, Dorama, Inbursa, and Primero

Source: Own elaboration based on (Grayling, 2014) R-Project.

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Conclusions

The objective of this work was to analyze the dynamics between the total premiums issued by the bonding sector and the gross profit reported by them. Specifically, the use of phase diagrams was proposed in order to understand the temporal dynamics between the variables and to know their trajectories in relation to the stable state. Based on the operating rules of Solvency II, it is assumed that small bonding companies have the greatest impact when operating in accordance with the new regulation, having a direct impact on the cost of premiums and the generation of profits.

Based on the proposed methodology, it is pointed out that Mexican bonding companies, despite their profits, present globally unstable behaviors (with the exception of Asecam, where there are regions that allow the bonding company to return to its stable state). Furthermore, there are cases where a greater aversion to risk of the bonding companies (represented by the premiums issued) does not always reflect higher profits unless they do so aggressively, and this is the case of Monterrey, Atlas, Dorama, Inbursa, and Primero. Likewise, inelastic behaviors were observed such as Inbursa, Chubb, Fiducia, and Mapfre, showing a trajectory in their profits with a near slope of being perfectly elastic, making them more sensitive to variations in their premiums.

It is worth mentioning that this work is limited to the operation of bonding companies in Mexico, which have substantial differences in the way insurance companies operate. The relevance of the work is to encourage the bonding companies to pursue a better business model that allows them to increase their profits by the good financial result they can obtain and is intended to evidence the viability of possible mergers or acquisitions between them, migrating to bonding insurers, as it is clear that current legislation does not allow them to work with premiums to have competitive operating margins. The main contribution of this work is the use of optimal control (phase diagrams) for the analysis of financial situations such as the case of bonding companies, which can naturally extend to the insurance market and the financial sector in general.

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