

## **On the multiple-cell thermohaline circulation**

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### RESUMEN

El modelo clásico de circulación termohalina de Stommel (1961) predice la existencia de dos modos de movimiento del océano para un sistema polo-ecuador: el modo térmico de una celda y el modo halino de una celda. En contraste, los experimentos de laboratorio realizados recientemente con el fin de reproducir este modelo conceptual dan tres estados estacionarios en el espacio de control de los parámetros del modelo: el modo térmico en dos capas, el modo halino en dos capas y un estado híbrido de tres capas. Se aplicó un modelo teórico bidimensional para investigar la formación de celdas múltiples. La solución del orden más bajo analizada usando el método de expansión sugiere la razón de aspecto como un parámetro pequeño. Se demuestra que una asimetría en la distribución vertical de la temperatura y la salinidad es responsable de la formación de una circulación termohalina con celdas múltiples.

### ABSTRACT

The classic Stommel (1961) model of thermohaline circulation has predicted the existence of two modes of ocean motion for the pole-equator system. These are, one-cell thermal and one-cell saline modes. In contrast, the laboratory experiments conducted recently to duplicate this conceptual model have found three steady states in the control space of the model parameters: a two-layer thermal mode, a two-layer saline mode, and a three-layer hybrid state. A two-dimensional theoretical model is applied to investigate the multiple-cell formation. The lowest order solution is analysed using an expansion method and suggesting the aspect ratio as a small parameter. It is shown that asymmetry of vertical distribution between temperature and salinity is responsible for forming the multiple-cell thermohaline circulation.

**Key words:** Thermohaline circulation, theoretical and laboratory modelling

## 1. Introduction

In the famous Stommel's model (Stommel, 1961) two steady states of thermohaline circulation (THC) were predicted for the pole-equator system. One state is characterised by dominating the temperature for the density difference and creating one cell of water movement with water rising at the equator and sinking at the pole (*T*-mode). The second one (*S*-mode), is dominated by the salinity force, providing one cell characterised by opposing vertical circulation.

Further theoretical studies of this problem, including the box models (Rooth, 1982), the 2-D theoretical (Marotzke *et al.*, 1988), global oceanic general circulation (Bryan, 1986) and coupled ocean-atmosphere (Manabe and Stouffer, 1988) models, have confirmed the existence of two principal modes of THC. The third steady state (as pole-to-pole asymmetric one-cell circulation) was found for the pole-equator-pole system (see reviews by Welander, 1986; Marotzke, 1993; Whitehead, 1995; Dijkstra, 2001). Various transient states of THC were noted in the numerical calculations. In particular, a multiple-cell circulation was achieved during the cycle of flow oscillations in the numerical calculations by Quon and Ghil (1995) with the imposed non-monotonic (asymmetric) boundary conditions.

Laboratory studies of the doubly driven flows of THC were recently carried out by Whitehead *et al.* (2003), Bulgakov and Skiba (2003). Various experimental apparatus were applied to duplicate the conceptual Stommel's box model. The primary finding was that these experiments have not only produced an anticipated two-layer circulation typical for the one-cell modes of THC. In addition, the laboratory runs have revealed an existence of a steady 3-layer flow system, so-called hybrid (H3) states. That is, two-cell circulation was achieved (Fig. 1) for the surface and bottom forcing experiments in the control space of the salt-to-heat buoyancy ratio  $0.9 < R = \beta\Delta S / \alpha\Delta T < 1.2$  and height-to-length aspect ratio  $0.2 < \varepsilon = \pi H / L < 0.9$ .

Presently, we have applied the 2-dimensional theoretical model proposed by Cessi and Young (1992) to give an interpretation of these phenomena as observed in the laboratory runs.

## 2. 2-D formulation

The two-dimensional formulation of the problem of the zonally averaged meridional circulation driven by thermohaline forcing was done by Cessi and Young (1992) in dimensionless form as:

$$\text{Pr}^{41} \Psi_{\varepsilon, \div \dots 2} J(\dots, \div \dots) \beta | \varepsilon_y T 4 \varepsilon_y S 2 \div^2 \dots \quad (1)$$

$$\varepsilon_y T 2 J(\dots, T) | \div T \quad (2)$$

$$Le^{41} (\varepsilon_y S 2 J / \dots, S(\div) | \div S \quad (3)$$

where  $\psi$  is stream function ( $\mathcal{Y} | 4 \varepsilon_z \dots, w | \varepsilon_y \dots$ ),  $T$  is temperature,  $S$  is salinity,  $\text{Pr} = \mu / k_T$  is the Prandtl number,  $Le = k_s / k_T$  is the Lewis number or diffusivity ratio,  $\varepsilon = \pi H / L$  is the height-

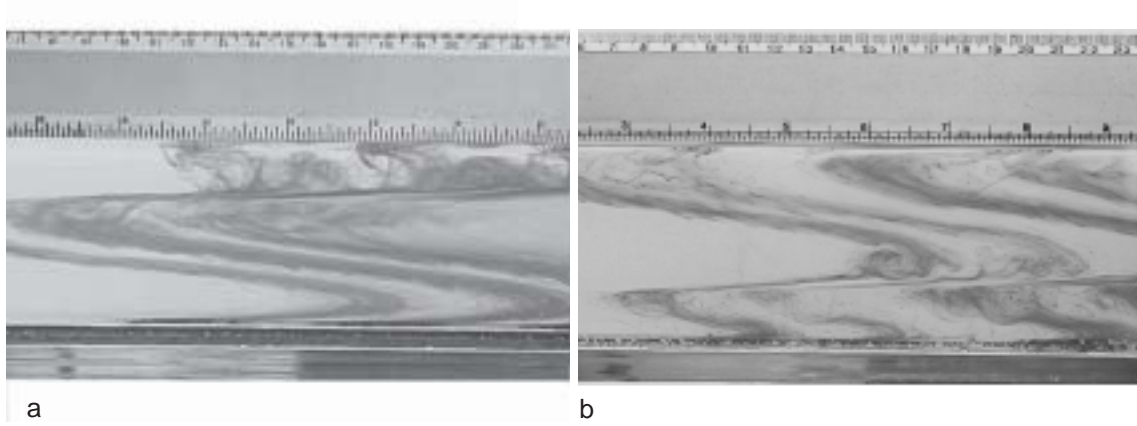


Fig. 1. Photographs illustrating two-cell THC between pole (left) and equator (right) under surface (a) and bottom (b) forcing experiments when the buoyancy ratio  $R = \beta \Delta S / \alpha \Delta T = 1$  and aspect ratio  $\varepsilon = \pi H / L = 0.4$  (from Bulgakov and Skiba, 2003).

to-length aspect ratio or wave number,  $\nabla^2 = \nabla_z^2 + 2\kappa^2 \nabla_y^2$  is the Laplacian operator,  $\mu$  is kinematic viscosity,  $k_T$  and  $k_S$  are the coefficients of heat and salt diffusivities respectively.

Forcing in a rectangular ocean basin ( $0 \leq z \leq 1$ ,  $-\pi \leq y \leq \pi$ ) is provided by the mixed boundary conditions for temperature and salinity, specifying the temperature and the salinity flux at the ocean surface. At the bottom and on the sides of the ocean both the temperature and salinity fluxes vanish, as follows:

$$z | 1: T | a \cos(y), \nabla_z S | b \cos(y), \dots | \nabla_z^2 \dots | 0$$

$$z | 0: \nabla_z T | \nabla_z S | \dots | \nabla_z^2 \dots | 0$$

$$y | \partial \phi : \nabla_y T | \nabla_y S | \dots | \nabla_y^2 \dots | 0$$

The non-homogeneous boundary conditions imposed at the surface for the temperature and salinity fields are supposed to be in equatorially symmetric form proposed by Thual and McWilliams (1992). Because mechanical forcing has been neglected, the boundary conditions for the stream function are homogeneous at all boundaries.

Thus, the problem contains five dimensionless parameters:  $Pr$ ,  $Le$ ,  $a$ ,  $b$  and  $\varepsilon$ . Here,  $a$  and  $b$  are parameters characterising the magnitudes of the temperature and salt flux forcing related as  $a \sim \varepsilon^2 b$ . In such a case the temperature and salinity contribution to the density field should be of the same order ( $R = \varepsilon^2 b / a \approx 1$ , see below the Results section) in contrast to Cessi and Young (1992) suggesting that it should be when  $b \sim \varepsilon^2 a$ .

The solvability of the problem (1)-(6) was investigated by Cessi and Young (1992). It was noted that this problem could be examined analytically using the expansion method for all the variables and suggesting  $\varepsilon$  as a small parameter.

Collecting terms of the same order, the following lowest order problem could be determined:

$$\epsilon_z^2 T_0 | 0, \quad (7)$$

$$\epsilon_z^2 S_0 | 0, \quad (8)$$

$$\epsilon_z^4 \dots_0 | \epsilon_y (S_0 \text{ and } T_0), \quad (9)$$

$$z=l: T_0 = a \cos(y), \epsilon_z S_0 | \dots_0 | \epsilon_z^2 \dots_0 | 0, \quad (10)$$

$$z=0: \epsilon_z T_0 | \epsilon_z S_0 | \dots_0 | \epsilon_z^2 \dots_0 | 0, \quad (11)$$

$$y = \pm \pi: \epsilon_y T_0 | \epsilon_y S_0 | \dots_0 | \epsilon_y^2 \dots_0 | 0. \quad (12)$$

The fixed-temperature boundary condition determines the zero-order temperature while the salinity  $S_0 = S_0(y)$  is undetermined at this order. The choice of  $S_0 = 0$  can be argued by saying that temperature diffusion is faster, and salinity evolution is started just in the third order problem. Then, the corresponding zero-order solution can be expressed analytically:

$$T_0(y) | a \cos(y), \dots_0(y, z) | \pi_0(z) \sin(y) \quad (13)$$

where

$$\pi_0(z) | \frac{a}{24} / z^4 \text{ and } 2z^3 \text{ and } 2z \quad (14)$$

The first order problem

$$\epsilon_z^2 T_1 | J(\dots_0, T_0) | 4 \epsilon_z \dots_0 \epsilon_y T_0 \quad (15)$$

$$\epsilon_z^2 S_1 | Le^4 J(\dots_0, S_0) | 0 \quad (16)$$

$$\epsilon_z^4 \dots_1 | / S_1 \text{ and } T_1 \text{ and } 0 \text{ and } Pr^4 J / \dots_0, \epsilon_z^2 \dots_0 \quad (17)$$

has the zero conditions at all the boundaries

$$z=l: T_1 | \epsilon_z S_1 | \dots_1 | \epsilon_z^2 \dots_1 | 0, \quad (18)$$

$$z=0: \epsilon_z T_1 | \epsilon_z S_1 | \dots_1 | \epsilon_z^2 \dots_1 | 0, \quad (19)$$

$$y = \pm \pi: \epsilon_y T_1 | \epsilon_y S_1 | \dots_1 | \epsilon_y^2 \dots_1 | 0, \quad (20)$$

and its solution is searched as

$$T_1(y, z) | t_1(z) \sin^2(y) \quad (21)$$

where

$$t_1(z) \mid \frac{a^2}{240} / 2z^5 \ 45z^4 \ 25z^2 \ 420 \quad (22)$$

The salinity field  $S_1 \mid S_1(y)$  is undetermined as well, and

$$\dots_1(y, z) \mid \pi_1(z) \sin(2y) \quad (23)$$

where

$$\pi_1(z) \mid \frac{a^2 z}{1451520 \text{Pr}} \left\{ \begin{array}{l} 42z^8 (522 \text{Pr}) 29z^7 (522 \text{Pr}) 472z^6 242z^5 (142 \text{Pr}) \\ 2504z^3 \text{Pr} 46z^2 (12118 \text{Pr}) 2(12274 \text{Pr}) \end{array} \right\} \quad (24)$$

The second order problem

$$\epsilon_z^2 T_2 \mid 4\epsilon_y^2 T_0 2J(\dots_0, T_1) 2J(\dots_1, T_0) - 4\epsilon_y^2 T_0 \quad (25)$$

$$\epsilon_z^2 S_2 \mid 4\epsilon_y^2 S_0 2Le^{41} \Psi_{\epsilon_z} S_0 2J(\dots_0, S_1) 2J(\dots_1, S_0) \mid F(y) \quad (26)$$

$$\begin{aligned} \epsilon_z^4 \dots_2 \mid \epsilon_y (S_2 4T_2) 42\epsilon_y^2 \epsilon_z^2 \dots_0 2\text{Pr}^{41} (J(\dots_0, \epsilon_z^2 \dots_1) 2J(\dots_1, \epsilon_z^2 \dots_0)) \mid - \\ \epsilon_y (S_2 4T_2) 42\epsilon_y^2 \epsilon_z^2 \dots_0 \end{aligned} \quad (27)$$

allows further analytical progress for the small amplitudes of external forcing parameters ( $a=O(10)$ ) by neglecting the non-linear advection terms in equations (25) and (27). This problem with the boundary conditions (where non-zero salt flux appears)

$$z=1: T_2 \mid \dots_2 \mid \epsilon_z^2 \dots_2 \mid 0 \quad \epsilon_z S_2 \mid \frac{b}{K^2} \cos(y) \quad (28)$$

$$z=0: \epsilon_z T_2 \mid \epsilon_z S_2 \mid \dots_2 \mid \epsilon_z^2 \dots_2 \mid 0, \quad (29)$$

$$y = \pm \pi: \epsilon_y T_2 \mid \epsilon_y S_2 \mid \dots_2 \mid \epsilon_y^2 \dots_2 \mid 0 \quad (30)$$

has the following diffusion dominated solution

$$T_2(y, z) \mid a \frac{z^2 41}{2} \cos(y) \quad (31)$$

$$S_2(y, z) \mid \frac{b}{\kappa^2} \frac{z^2}{2} \cos(y) \quad (32)$$

$$\dots_2(y, z) \mid \pi_2(z) \sin(y) \quad (33)$$

where

$$\pi_2(z) \mid \frac{z}{720} \left( \begin{array}{l} z^5 (3a 4 \frac{b}{\kappa^2}) 4 6z^4 a 4 15z^3 a \\ 25z^2 (7a 2 \frac{b}{\kappa^2}) 4 (17a 2 4 \frac{b}{\kappa^2}) \end{array} \right) \quad (34)$$

### 3. Results

Contours of the temperature  $T(y, z) = T_0 + \varepsilon T_1 + \varepsilon^2 T_2$  and salinity  $S(y, z) = \varepsilon^2 S_2$  are displayed in Figure 2 for the equal external forcing parameters  $a=b=10$ . Here we have chosen  $\varepsilon = 0.4$  to match the laboratory conditions. As the 1-order solution for temperature is negligible in comparison with the other two terms for the diffusion dominated regime, the shape functions for the temperature and salinity could be expressed as  $\phi_1 = 1 + \varepsilon^2 \cdot (z^2 - 1) / 2$  and  $\phi_2 = z^2 / 2$  respectively. Their vertical structures are visible in Figure 3 (curves 1 and 2). In this case, temperature effect evidently dominates over the salinity flux through all the depths.

When the solution for salt is changed by a factor of 6 (curve 3 in Fig. 3), i.e.  $R = \varepsilon^2 b / a = 0.96$ , this produces a two-cell flow in the vertical direction with the salinity driven motion near the surface, and with the thermal cell of circulation near the bottom. The stream function  $\psi(y, z)$  distribution as multiple-cell circulation is shown in Figure 4 when  $Pr = \infty$  and  $Pr = 10$ . The latter case ( $Pr = 10$ ) corresponds to real sea water condition and reflects well an inclination of the zero-circulation line observed in the laboratory experiments under the surface forcing (Fig. 1a).

Solution of the problem for the bottom forcing could be obtained in a similar manner. It is evident that in such a case two-cell circulation is opposite to the flow patterns under the surface forcing. That is, the salinity driven circulation is achieved near the bottom, whereas the thermally driven motion prevails in the upper semiplane (for comparison see Fig. 1b).

The interval of the multiple cell circulation  $0.9 < R < 1.1$ , for both solutions under the surface and bottom forcing, is close to that observed in the laboratory conditions (Bulgakov and Skiba, 2003). As is visible from Figure 5, the lower ( $b = 50$ ) and higher ( $b = 70$ ) values of the salt flux amplitude produce one-cell thermal and saline circulation correspondingly.

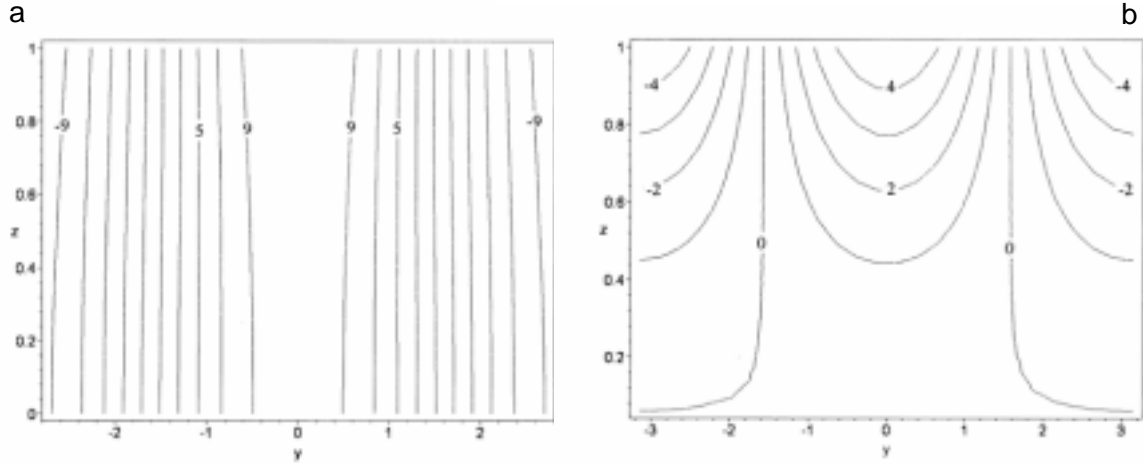


Fig. 2. Diffusion dominated regime for the pole-equator-pole system at small surface forcing ( $a=b=10$ ) and aspect ratio  $\varepsilon = 0.4$ : (a) temperature  $T(y,z)$  and (b) salinity  $S(y,z)$  fields.

It should be noted that the problem

$$\nabla \cdot T | 0, \nabla \cdot S | 0 \quad (35)$$

$$z | 1: T | a \cos(y), \nabla \cdot S | b \cos(y) \quad (36)$$

$$z | 0: \varepsilon_z T | \varepsilon_z S | 0, \quad (37)$$

$$y | \partial \phi : \varepsilon_y T | \varepsilon_y S | 0, \quad (38)$$

has an analytical solution

$$T | a \cos y \frac{ch\kappa z}{ch\kappa} \quad (39)$$

$$S | b \cos y \frac{1}{\kappa} \left\{ e^{4\kappa} \frac{ch\kappa z / z 41}{sh\kappa} 2 \kappa^{x/z 410} \right\}. \quad (40)$$

Equation (35) differ from (2)-(3) mainly by the absence of Jacobian operators describing differences between the temperature- and salinity-driven circulation. In such a way diffusion of the temperature and salinity is achieved in a similar manner. Thus, the shape functions have the same form. Here the three-layer circulation does not exist, and transitions between  $T$ - and  $S$ -modes occur directly. This means that asymmetry between the temperature and salinity shape functions (or different ways of temperature and salinity intrusion) is crucial to form the multiple-cell THC.

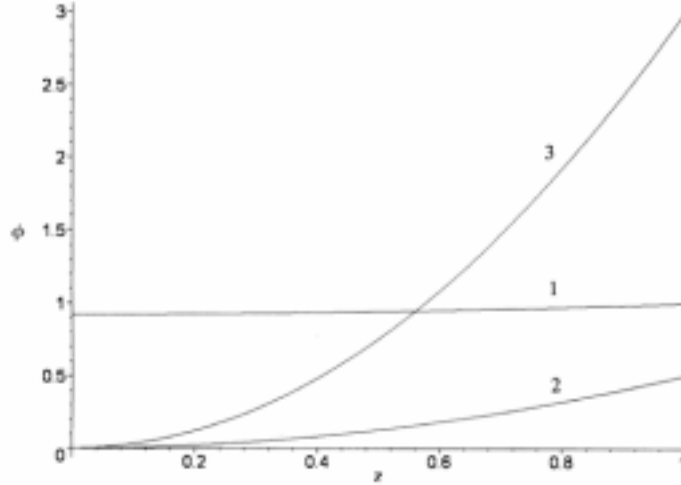


Fig. 3. The shape functions for temperature, curve 1:  $\phi_1 = 1 + \varepsilon^2 (z^2 - 1) / 2$ , and curve 2:  $\phi_2 = z^2 / 2$ , and for salinity, curve 3:  $\phi_3 = 3z^2$ .

#### 4. Discussion and conclusions

Two steady states of THC were predicted by Stommel (1961) for the pole-equator system near the state of equilibrium, which represent one-cell thermal driven and one-cell saline driven water motion.

Quon and Ghil (1995) have paid attention to the multiple-cell formation in their numerical experiments. When they applied non-monotonic (asymmetric about the centre of the pole-equator-pole system) boundary conditions for the temperature and salt flux, the four cells were prominent during the cycle of flow oscillations (see Fig. 8g from Quon and Ghil (1995)).

The laboratory experiments carried out recently (Whitehead *et al.*, 2003; Bulgakov and Skiba, 2003) to duplicate the Stommel's theory have shown that both one- and two-cell THC could be achieved as the steady states. That is, a three-layer current structure was observed in the laboratory runs when the buoyancy ratio was close to a unity ( $0.9 < R < 1.2$ ). To give an interpretation of the phenomena observed, we have applied 2-D theoretical model proposed by Cessi and Young (1992).

In this model, the external forcing for the pole-equator-pole system is provided by the mixed boundary conditions for the temperature and salinity in equatorially symmetric form. The solution of the problem was analysed with the expansion method by using the aspect ratio ( $\varepsilon$ ) as a small parameter. We assumed that for small amplitudes of the external forcing parameters, diffusion dominates at the lowest-order balance, and the temperature diffusion is faster than the salinity one ( $a \sim \varepsilon^2 b$ ). In this case, the problems arising at the three first lower orders could be examined analytically.



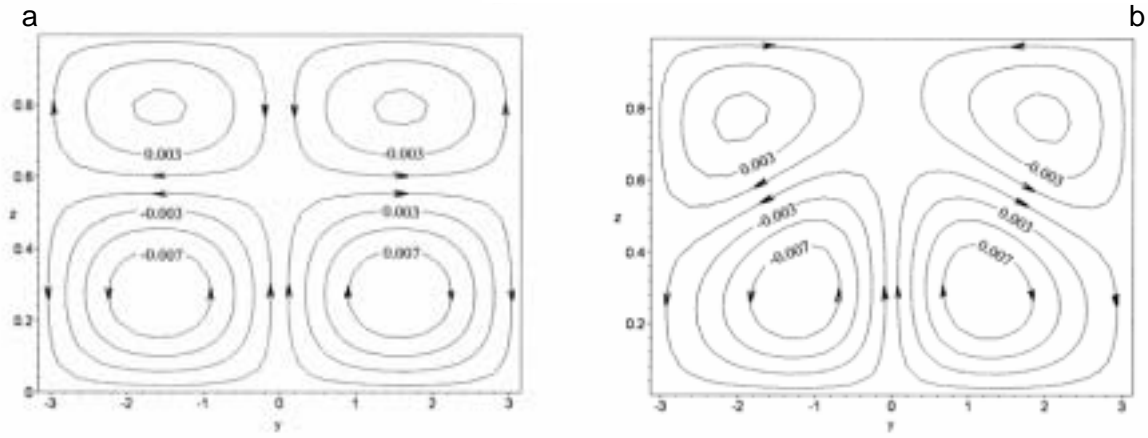


Fig. 4. Stream function  $\psi(y, z)$  distribution as multiple-cell circulation. The parameters are  $a = 10, b = 60, \varepsilon = 0.4$  when  $Pr = \infty$  (a) and  $Pr = 10$  (b)

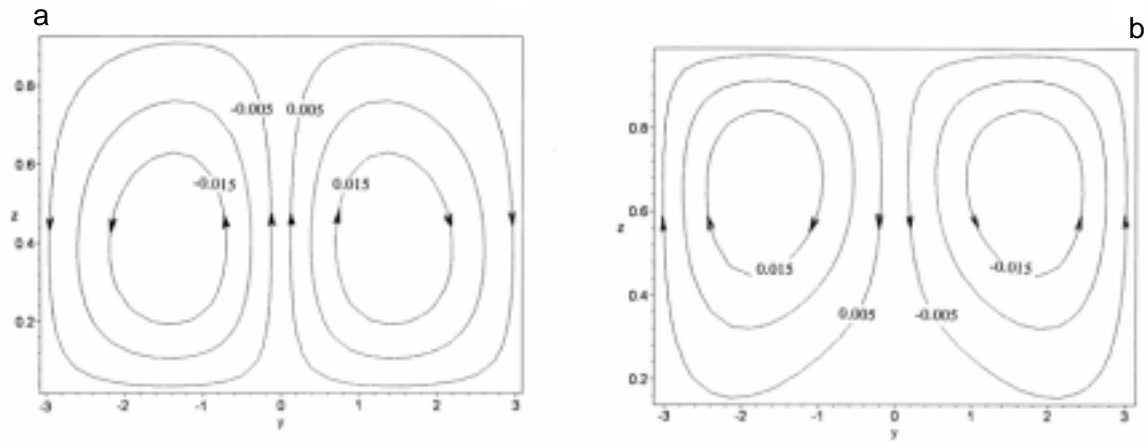


Fig. 5. Stream function  $\psi(y, z)$  distribution as one-cell circulation. The parameters are  $a = 10, \varepsilon = 0.4, Pr = 10$  when  $b = 50$  (a) and  $b = 70$  (b)

The results have shown that these assumptions eliminate the one-to-one correspondence between the temperature and salinity vertical distributions, destroying their symmetry and forming the multiple-cell THC. On the contrary, when temperature and salinity profiles are identical, it is evident that hybrid 3-layer circulation does not exist.

It is well known that the salinity and temperature fields in the ocean are not uniform in the vertical, even to a rough approximation. The different dynamics of the heat and salt transport was

documented in the laboratory experiments (Whitehead *et al.*, 2003; Bulgakov and Skiba, 2003) as well. The temperature-salinity symmetry was probably destroyed by the non-monotonic boundary conditions in the numerical experiments by Quon and Ghil (1995) where multiple-cell THC appeared. Therefore, we argue that asymmetry in the vertical distributions of the temperature and salinity is a necessary condition for the multiple-cell phenomena to occur.

### Acknowledgments

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